

## First-order Reactant in Homogeneous Dusty Fluid Turbulence Prior to the Ultimate Phase of Decay for Four-Point Correlation in a Rotating System

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**Abstract:** The aim of this study is to find the decay of energy in homogeneous dusty fluid turbulence at times preceding to the ultimate phase for the concentration fluctuation of a dilute contaminant due to a first order chemical reaction in a rotating system for the case of four-point correlation. Following Deissler's approach, two-, three- and four-point correlation equations have obtained and these equations are converted to spectral form by their Fourier-transform, the terms containing quintuple correlations are neglected in comparison to the third and fourth order correlation terms. Finally, integrating the energy spectrum over all wave numbers, the energy decay law of homogeneous dusty fluid turbulence for the concentration fluctuations ahead of the ultimate phase in a rotating system is obtained.

**Keywords:** Deissler's method, dusty fluid, first-order reactant, rotating system, turbulent flow

### INTRODUCTION

In recent year, the motion of dusty viscous fluids in a rotating system has developed rapidly. The motion of dusty fluid occurs in the movement of dust-laden air, in problems of fluidization, in the use of dust in a gas cooling system and in the sedimentation problem of tidal rivers. The behavior of dust particles in a turbulent flow depends on the concentrations of the particles and the size of the particles with respect to the scale of turbulent fluid. In geophysical flows, the system is usually rotation with a constant velocity, such large Scale flows are generally turbulent. Gustave Coriolis, French engineer and mathematician, who showed that a force could be used to allow the use of the ordinary laws of motion in a rotating reference frame. The Coriolis Effect is caused by the rotation of the Earth and the inertia of the mass experiencing the effect. The most commonly encountered rotating reference frame in the Earth. Because the Earth completes only one rotation per day, this force causes moving objects on the surface of the Earth to appear to change direction to the right in the northern hemisphere and to the left in the southern. This effect is responsible for the rotation of large cyclones. The acceleration entering the Coriolis force arises from two sources of change in velocity that result from rotation: the first is the change of the velocity of an object in time; the second is the change of velocity in space. Different positions in a rotating frame of reference have different velocities. The nature and size of the Coriolis effects depend upon where we are. The Coriolis Effect has great significance in

Astrophysics and Stellar dynamics. It is also significant in the Earth sciences, especially Meteorology, Physical geology and Oceanography.

Deissler (1958, 1960) developed a theory 'on the decay of homogeneous turbulence before the final period.' Using Deissler theory, Kumar and Patel (1974) studied the first-order reactant in homogeneous turbulence before the final period of decay for the case of multi-point and single-time correlation. Kumar and Patel (1975) extended their problem for the case of multi-point and multi-time concentration correlation. Loeffler and Deissler (1961) studied the decay of temperature fluctuations in homogeneous turbulence before the final period. Batchelor (1953) studied the theory of homogeneous turbulence. Chandrasekhar (1951) obtained the invariant theory of isotropic turbulence in magneto-hydrodynamics. Sarker and Kishore (1991) studied the decay of MHD turbulence before the final period. Sarker and Islam (2001) obtained the decay of dusty fluid MHD turbulence before the final period in a rotating system. Sultan (2008) studied the energy decay law of dusty fluid turbulent flow in a rotating system. Aziz *et al.* (2009) studied the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time in a rotating system. Aziz *et al.* (2010) also extended their previous problem in presence of dust particle.

It is noted that in all above cases, the researcher had considered three-point correlations and through their study they obtained the energy decay law ahead of the ultimate phase, by analyzing the above theories we have studied the first-order reactant in homogeneous dusty

fluid turbulence prior to the ultimate phase of decay for four-point correlation in a rotating system. Through this study we would like to find the decay law of first order reactant in homogeneous dusty fluid turbulence in a rotating system. Later we shall try to show in this study when the reaction rate  $R = 0$  causes the concentration fluctuation of decay more rapidly than for  $R \neq 0$ , non pure mixing dusty fluid turbulence.

### MATERIALS AND METHODS

**Basic equation:** The differential equation governing the concentration of a dilute contaminant undergoing a first-order chemical reaction in homogeneous dusty fluid turbulence in a rotating system could be written as:

$$\frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial x_k} = D \frac{\partial^2 u_i}{\partial x_k \partial x_k} - Ru_i - 2\epsilon_{mki} \Omega_m u_i + f(u_i - v_i) \quad (1)$$

where,  $u_i(\hat{x})$  is a random function of position and time at a point  $p$ ,  $u_k(\hat{x}, t)$  is the turbulent velocity,  $R$  is the constant reaction rate,  $D$  is the diffusivity,  $t$  is the time,  $\epsilon_{mki}$  is the alternating tensor,  $\Omega_m$  is the angular velocity components,  $f = \frac{kN}{\rho}$ , dimension of frequency;  $N$ , constant number density of dust particle,  $m_s = \frac{4}{3}\pi R_s^3 \rho_s$  is the mass of single spherical dust particle of radius  $R_s$ ,  $\rho_s$  is the constant density of the material in dust particle,  $u_i$  is the turbulent velocity component,  $v_i$  is the dust particle velocity component,  $x_k$  is the space coordinate and repeated subscript in a term indicates a summation of terms, with the subscripts successively taking on the values 1, 2 and 3.

**Two-point correlation and spectral equations:** Under the restrictions that the turbulence and the concentration fields are homogeneous, the local mass transfers have no effect on the velocity field and the reaction rate and the diffusivity are constant. The differential equation of a dilute contaminant undergoing a first-order chemical reaction in homogeneous system could be written as:

$$\frac{\partial \Gamma}{\partial t} + u_k \frac{\partial \Gamma}{\partial x_k} = D \frac{\partial^2 \Gamma}{\partial x_k \partial x_k} - R\Gamma \quad (2)$$

Subtracting the mean of (2) from Eq. (2), we obtain:

$$\frac{\partial X}{\partial t} + u_k \frac{\partial X}{\partial x_k} = D \frac{\partial^2 X}{\partial x_k \partial x_k} - RX \quad (3)$$

where,  $X(\hat{x}, t)$  is the fluctuation of concentration about the mean at a point  $p(\hat{x})$  and time  $t$ . The two-point correlation for the fluctuating concentration can be written, as:

$$\frac{\partial \langle XX' \rangle}{\partial t} + u_k \frac{\partial \langle XX' \rangle}{\partial x_k} = D \frac{\partial^2 \langle XX' \rangle}{\partial x_k \partial x_k} - R \langle XX' \rangle \quad (4)$$

where,  $X'_j$  is the fluctuating concentration at the point  $p'(\hat{x})$  and the point  $p'$  is at point a distance  $r$  from the point  $p$ . The symbol  $\langle \rangle$  is the ensemble average. Putting the Fourier transforms:

$$\langle XX'(r) \rangle = \int_{-\infty}^{\infty} \theta(\hat{K}, t) \exp[i(\hat{K} \cdot \hat{r})] d\hat{K} \quad (5)$$

$$\langle u_k XX'(r) \rangle = \int_{-\infty}^{\infty} \varphi_k(\hat{K}, t) \exp[i(\hat{K} \cdot \hat{r})] d\hat{K} \quad (6)$$

$$\langle u'_k XX'(r) \rangle = \int_{-\infty}^{\infty} \varphi'_k(\hat{K}, t) \exp[i(\hat{K} \cdot \hat{r})] d\hat{K} \quad (7)$$

into Eq. (4), one obtains:

$$\frac{\partial \theta}{\partial t} + (Dk^2 + 2R)\theta = ik_k [\varphi_k(\hat{K}, t) - \varphi_k(-\hat{K}, t)] \quad (8)$$

**Three-point correlation and spectral equations:** The same procedure can be used to find the three-point correlation, i.e., by taking the Navier-Stokes equations for a first-order chemical reaction in homogeneous dusty fluid turbulence in a rotating system at  $p$  is:

$$\frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial x_k} = D \frac{\partial^2 u_i}{\partial x_k \partial x_k} - Ru_i - 2\epsilon_{mki} \Omega_m u_i + f(u_i - v_i)$$

and the fluctuation equations at  $p'$  &  $p''$  one can find the three-point correlation equation as:

$$\begin{aligned} & \frac{\partial \langle u_i X X'' \rangle}{\partial t} + \frac{\partial \langle u_k u'_k X X'' \rangle}{\partial x_k} + \frac{\partial \langle u_k u'_k X X'' \rangle}{\partial x'_k} + \frac{\partial \langle u_k u'_k X X'' \rangle}{\partial x''_k} \\ & = - \frac{1}{\rho} \frac{\partial \langle p X X'' \rangle}{\partial x_i} + \nu \frac{\partial^2 \langle u_i X X'' \rangle}{\partial x_k \partial x_k} \\ & + D \left( \frac{\partial^2}{\partial x_k \partial x_k} + \frac{\partial^2}{\partial x'_k \partial x'_k} \right) \langle u_i X X'' \rangle - 2R \langle u_i X X'' \rangle - 2\epsilon_{mki} \Omega_m \langle u_i X X'' \rangle \\ & + f(\langle u_i X X'' \rangle - \langle v_i X X'' \rangle) \end{aligned} \quad (9)$$

Using the transformations:

$$\frac{\partial}{\partial x_k} = - \left( \frac{\partial}{\partial r_k} + \frac{\partial}{\partial r'_k} \right), \frac{\partial}{\partial x'_k} = \frac{\partial}{\partial x_k}, \frac{\partial}{\partial x''_k} = \frac{\partial}{\partial r'_k}$$

In order to convert Eq. (9) to spectral form, using six-dimensional Fourier transforms (Kumar and Patel, 1974) and:

$$\langle v_i X'X'' \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu_i(\hat{K}, \hat{K}', t) \exp[i(\hat{K}.r + \hat{K}'.r')] d\hat{K}dK'$$

with the fact that:

$$\langle u_i u_k' X'X'' \rangle = \langle u_i u_k' X''X' \rangle$$

one can write Eq. (9) in the form:

$$\begin{aligned} \frac{\partial \beta_j}{\partial t} + [v(\hat{K} + \hat{K}')^2 + D(\hat{K} + \hat{K}') + 2R + 2\varepsilon_{mki}\Omega_m - Qf] \beta_j \\ = -i(k_k + k_k') \psi_{jk} + i(k_k + k_k') \psi_{jk'} + \frac{1}{\rho} i(k_j + k_j') \gamma \end{aligned} \quad (10)$$

where,  $\mu_i(\hat{k}, \hat{k}', t) = L\beta(\hat{k}, \hat{k}', t)$  and  $1-L = Q$ , with  $L$  and  $Q$  arbitrary constants. If the momentum Eq. (3) at  $p$  is multiplied by  $X'X''$  and divergence of the time averages is taken, the resulting equation will be:

$$\frac{\partial^2 \langle u_i u_k X'X'' \rangle}{\partial x_i \partial x_k} = -\frac{1}{\rho} \frac{\partial^2 \langle p X'X'' \rangle}{\partial x_i \partial x_j} \quad (11)$$

In Fourier space it can be written as:

$$-(k_k + k_k') \psi_{ik} = \frac{1}{\rho} (k_i + k_i') \gamma \quad (12)$$

Substituting this into Eq. (10), we obtain

$$\begin{aligned} \frac{\partial \beta_j}{\partial t} + [v(\hat{K} + \hat{K}')^2 + D(\hat{K} + \hat{K}') + 2R + 2\varepsilon_{mki}\Omega_m - Qf] \beta_j \\ = -i(k_k + k_k') \psi_{jk} \end{aligned} \quad (13)$$

**Four-point correlation and spectral equations:** Again by taking the Navier-Stokes equations at  $p$  and  $p'$  and the concentration equations at  $p''$ ,  $p'''$  and following the same procedure as before, one can get the four-point correlations as:

$$\begin{aligned} \frac{\partial \langle u_i u_j X''X''' \rangle}{\partial t} + \frac{\partial \langle u_i u_k u_j X''X''' \rangle}{\partial x_k} + \frac{\partial \langle u_i' u_k' u_j X''X''' \rangle}{\partial x_k'} \\ + \frac{\partial \langle u_i u_j u_k' X''X''' \rangle}{\partial x_k''} + \frac{\partial \langle u_i u_j u_k'' X''X''' \rangle}{\partial x_k'''} \\ = -\frac{1}{\rho} \left[ \frac{\partial \langle p u_j X''X''' \rangle}{\partial x_i} + \frac{\partial \langle p' u_i X''X''' \rangle}{\partial x_j'} \right] \\ + v \left( \frac{\partial^2}{\partial x_i \partial x_k} + \frac{\partial^2}{\partial x_k' \partial x_i'} \right) \langle u_i u_j X''X''' \rangle \end{aligned}$$

$$-2R \langle u_i u_j X''X''' \rangle - 2\varepsilon_{mki}\Omega_m \langle u_i u_j X''X''' \rangle$$

$$+ D \left( \frac{\partial^2}{\partial x_k'' \partial x_k''} + \frac{\partial^2}{\partial x_k''' \partial x_k'''} \right) \langle u_i u_j X''X''' \rangle$$

$$+ f \left( \langle u_i u_j X''X''' \rangle - \langle v_i u_j X''X''' \rangle \right) \quad (14)$$

Using the transformations:

$$\begin{aligned} \frac{\partial}{\partial x_k} &= - \left( \frac{\partial}{\partial r_k} + \frac{\partial}{\partial r_k'} + \frac{\partial}{\partial r_k''} \right), \\ \frac{\partial}{\partial x_k'} &= \frac{\partial}{\partial x_k}, \quad \frac{\partial}{\partial x_k''} = \frac{\partial}{\partial r_k'}, \quad \frac{\partial}{\partial x_k'''} = \frac{\partial}{\partial r_k''} \end{aligned}$$

In order to convert Eq. (14) to spectral form, using nine-dimensional Fourier transforms (Kumar and Patel, 1974) and:

$$\begin{aligned} \langle v_i u_j X''X'''(\hat{r}, \hat{r}', \hat{r}'') \rangle &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu_{ij}(\hat{K}, \hat{K}', \hat{K}'', t) \\ &\exp[i(\hat{K}'.\hat{r} + \hat{K}''.\hat{r}'')] d\hat{K}d\hat{K}'d\hat{K}'' \end{aligned}$$

and with the fact that:

$$\langle u_k' u_i u_j X''X'''(\hat{r}, \hat{r}', \hat{r}'') \rangle = \langle u_k' u_i u_j X''X'''(\hat{r}, \hat{r}', \hat{r}'') \rangle$$

Equation (14) can be written as:

$$\begin{aligned} \frac{\partial g_{ij}}{\partial t} + [v(\hat{K} + \hat{K}' + \hat{K}'')^2 + v\hat{K}^2 \\ + D(\hat{K} + \hat{K}')^2 + 2R + 2\varepsilon_{mki}\Omega_m - Sf] g_{ij} \\ = -i(k_k + k_k' + k_k'') [h_{ijk}(\hat{K}, \hat{K}', \hat{K}'') - ik_k [h_{ijk}(-\hat{K} - \hat{K}' - \hat{K}'', \hat{K}', \hat{K}'')]] \\ - i(k_k' + k_k'') h_{ijk}(\hat{K}, \hat{K}', \hat{K}'') - \frac{1}{\rho} [(k_i + k_i' + k_i'') \Omega_j(\hat{K}, \hat{K}', \hat{K}'') \\ + k_j' \Omega_i(-\hat{K} - \hat{K}' - \hat{K}'', \hat{K}', \hat{K}'')] \end{aligned} \quad (15)$$

where,  $\mu_{ij}(\hat{K}, \hat{K}', \hat{K}'', t) = M g_{ij}(\hat{K}, \hat{K}', \hat{K}'', t)$  and  $1-M = S$ , with  $M$  and  $S$  are an arbitrary constants. Following the same procedure as was used in obtaining (12), we get:

$$\Omega_j(\hat{K}, \hat{K}', \hat{K}'') = -\rho \frac{(k_k + k_k' + k_k'')}{(k_i + k_i' + k_i'')} h_{ijk}(\hat{K}, \hat{K}', \hat{K}'') \quad (16)$$

Equation (15) and (16) are the spectral equations corresponding to the four-point correlations. To get a better picture of the of the first-order reactant of homogeneous dusty fluid turbulence decay in a rotating

system from its initial period to its final period, four-point correlations are to be considered. The same ideology could be applied to the concentration phenomenon. Here, we neglect the quintuple correlations since the decay faster than the lower-order correlations. As pointed out by Deissler (1958, 1960) when the quintuple correlations are neglected, the corresponding pressure-force terms which are related to them are also neglected. Under these assumptions, Eq. (15) and (16) give the solution as:

$$= g_{ij}(\hat{K}, \hat{K}', \hat{K}'', t) = \left[ g_{ij}(\hat{K}, \hat{K}', \hat{K}'', t_0) \exp\left\{-D\left[2N_s k^2 + (1+N_s)(k'^2 + k''^2) + 2N_s(k_k k'_k + k'_k k''_k + k_k k'_k)\right] - 2R + 2\varepsilon_{mki}\Omega_m - Sf\right\}(t-t_0) \right] \quad (17)$$

where,  $N_s = v/D$ . A relation between  $g_{ij}(\hat{K}, \hat{K}', \hat{K}'', t_0)$  and  $\varphi_{ij}(\hat{K}, \hat{K}')$  can be found by taking  $\hat{r}''=0$  in the expression for  $\langle u_i u'_j X'' X'''(\hat{r}, \hat{r}', \hat{r}'') \rangle$  and comparing it with the expression for  $\langle u_i u'_k X X''(\hat{r}, \hat{r}') \rangle$ :

$$\varphi_{ik}(\hat{K}, \hat{K}') = \int_{-\infty}^{\infty} g_{ij}(-\hat{K} - \hat{K}', \hat{K}', \hat{K}'', t) d\hat{K}'' \quad (18)$$

Substituting this in to Eq. (13), we obtains:

$$\frac{\partial \beta_j}{\partial t} + D\left[(1+N_s)(K^2 + K'^2) + 2N_s \hat{K} \cdot \hat{K}' + \frac{2R}{D} + \frac{2\varepsilon_{mki}\Omega_m}{D} - \frac{Sf}{D}\right] \beta_j = -i(k_k + k'_k) \int_{-\infty}^{\infty} g_{ij}(-\hat{K} - \hat{K}', \hat{K}', \hat{K}'', t) d\hat{K}''$$

which with the help of Eq. (17), gives:

$$\frac{\partial \beta_j}{\partial t} + D\left[(1+N_s)(K^2 + K'^2) + 2N_s \hat{K} \cdot \hat{K}' + \frac{2R}{D} + \frac{2\varepsilon_{mki}\Omega_m}{D} - \frac{Sf}{D}\right] \beta_j = -2i(k_k + k'_k) \left[ g_{ij}(-\hat{K} - \hat{K}', \hat{K}', \hat{K}'', t_0) \right]_0 \times \frac{a\pi^{3/2}}{D^{3/2}(1+N_s)^{3/2}(t-t_0)^{3/2}} \times \exp\left\{-D\left[\frac{N_s(2+N_s)k^2}{(1+N_s)} + 2N_s k k' + (1+N_s)k'^2 + \frac{2R}{D} + \frac{2\varepsilon_{mki}\Omega_m}{D} - \frac{Qf}{D}\right]\right\}(t-t_0) \quad (19)$$

Now, the solution of Eq. (19) is:

$$\beta_j(\hat{K}, \hat{K}', t_0) = \left[ \beta_j(\hat{K}, \hat{K}', t_0) \right]_0 \times \exp\left\{-D\left[(1+N_s)(K^2 + K'^2) + 2N_s \hat{K} \cdot \hat{K}' + \left(\frac{2R}{D} + \frac{2\varepsilon_{mki}\Omega_m}{D} - \frac{Qf}{D}\right)\right](t-t_0)\right\}$$

$$- \frac{2i(k_k + k'_k) b_i \pi^{3/2}}{D^{3/2}(1+N_s)^{3/2}} \times \left[ \frac{-2}{(t-t_0)^{1/2}} + \frac{4kD^{1/2}F(\omega)}{(1+N_s)^{1/2}} \right] \times \exp\left\{-D\left[\frac{N_s(2+N_s)k^2}{(1+N_s)} + N_s k k' + (1+N_s)k'^2 + \left(\frac{2\varepsilon_{mki}\Omega_m}{D} - \frac{Sf}{D}\right)\right]\right\}(t-t_0) \times \exp[-2R(t-t_0)] \quad (20)$$

where,

$$F(\omega) = \exp(-\omega^2) \int_0^{\infty} \exp(n^2) dn, \omega = k \left[ \frac{(t-t_0)D}{9(1+N_s)} \right]^{1/2}$$

and

$$\left[ g_{ij}(-\hat{K} - \hat{K}', \hat{K}', \hat{K}'', t_0) \right]_0 = b_i \quad (21)$$

Now, by taking  $\hat{r}''=0$  in the expression for  $\langle u_i X X''(\hat{r}, \hat{r}') \rangle$  is performed, we obtain and comparing the result with the expression  $\langle X u'_k X'(\hat{r}') \rangle$ :

$$\varphi_k(\hat{K}) = \int_{-\infty}^{\infty} \beta_k(\hat{K}, \hat{K}') dk' \quad (22)$$

Substituting Eq. (21) into (8), we obtain:

$$\frac{\partial G}{\partial t} + (2Dk^2 + 2R)G = W \quad (23)$$

where,

$$G = 2\pi k^2 \theta$$

and

$$W = 2\pi i k^2 \exp[-2R(t-t_0)] \times \int_{-\infty}^{\infty} k'_k \left[ \beta_k(\hat{K}, \hat{K}') - \beta_k(-\hat{K}, \hat{K}') \right]_0 \times \exp\left\{-D\left[(1+N_s)(k^2 + k'^2) + 2N_s \hat{K} \cdot \hat{K}' + \left(\frac{2\varepsilon_{mki}\Omega_m}{D} - \frac{Qf}{D}\right)\right](t-t_0)\right\} + \frac{2i\pi^{5/2}}{D^{3/2}(1+N_s)^{3/2}} \int_{-\infty}^{\infty} \left[ b_i(\hat{K}, \hat{K}') - b_i(-\hat{K}, -\hat{K}') \right] \times \left[ \frac{-2}{(t-t_0)^{1/2}} + \frac{4kF(\omega)D^{1/2}}{(1+N_s)^{1/2}} \right] \times \exp\left\{-D\left[\frac{N_s(2+N_s)k^2}{(1+N_s)} + 2N_s k k' + (1+N_s)k'^2 + \left(\frac{2\varepsilon_{mki}\Omega_m}{D} - \frac{Sf}{D}\right)\right]\right\}(t-t_0) dk' \quad (24)$$

In analogy with the turbulent energy spectrum function, the quantity G, in Eq. (23) can be called

concentration energy spectrum function and W, the energy transfer function, is responsible for the transfer of concentration from one wave number to another.

In order to find solution completely and following Deissler (1958, 1960) we assume that:

$$(2\pi)^2 ik_k \left[ \beta_0(\hat{K}, \hat{K}') - \beta_0(-\hat{K}, \hat{K}') \right]_0 \\ = -\xi_0 (k^2 k'^4 - k^4 k'^2)$$

and

$$\frac{8\pi^2}{D^{\frac{3}{2}}(1+N_s)^{\frac{3}{2}}} i \left[ b(\hat{K}, \hat{K}') - b(-\hat{K}, \hat{K}') \right]_0 \\ = -\xi_1 (k^4 k'^6 - k^6 k'^4)$$

where,  $\xi_0$  and  $\xi_1$  are arbitrary constants depending on the initial conditions. If these results are used in Eq. (24) and the integration is performed, we obtain:

$$W = - \frac{\xi_0 \pi^{\frac{1}{2}} N_s}{2(t-t_0)^{\frac{3}{2}} D^{\frac{3}{2}} (1+N_s)^{\frac{5}{2}}} \\ \times \exp \left\{ -D \left[ \left( \frac{1+2N_s}{1+N_s} \right) k^2 + \left( \frac{2\epsilon_{mki} \Omega_m - Qf}{D} \right) \right] (t-t_0) \right\} \\ \times \left\{ \frac{15k^4}{4D^2 N_s^2 (t-t_0)^2} \left( \frac{N_s}{1+N_s} \right) + \left[ 5 \left( \frac{N_s}{1+N_s} \right)^2 - \frac{3}{2} \right] \right. \\ \left. \frac{k^6}{N_s D (t-t_0)} + \left[ \left( \frac{N_s}{1+N_s} \right)^3 - \left( \frac{N_s}{1+N_s} \right) \right] k^8 \right\} \\ + \frac{\xi_1 \pi^{\frac{1}{2}} N_s}{(t-t_0)^{\frac{7}{2}} D^{\frac{3}{2}} (1+N_s)^{\frac{7}{2}}} \\ \times \exp \left\{ -2D \left[ \left( \frac{N_s}{1+N_s} \right) k^2 + \left( \frac{\epsilon_{mki} \Omega_m - Sf}{D} - \frac{Sf}{2D} \right) \right] (t-t_0) \right\} \\ \times \left\{ \frac{105k^6}{8D^3 N_s^2 (t-t_0)^3} \left( \frac{N_s}{1+N_s} \right) + \frac{15k^8}{4D^3 N_s (t-t_0)^2} \right. \\ \times \left[ 7 \left( \frac{N_s}{1+N_s} \right)^2 - 1 \right] + \left( \frac{N_s k^{10}}{D(1+N_s)(t-t_0)} \right) \\ \times \left[ \frac{21}{2} \left( \frac{N_s}{1+N_s} \right)^2 - 5 \right] + \frac{N_s^2 k^{12}}{(1+N_s)^2 (t-t_0)} \left[ \left( \frac{N_s}{1+N_s} \right)^2 - 1 \right] \right\} \\ - \frac{2\xi_1 \pi^{\frac{1}{2}} N_s}{(t-t_0)^{\frac{3}{2}} D(1+N_s)^4} \\ \times \exp \left\{ -D \left[ \left( \frac{1+2N_s}{1+N_s} \right) + \left( \frac{2\epsilon_{mki} \Omega_m - Sf}{D} \right) \right] (t-t_0) \right\} \\ \times \left\{ \frac{105k^7}{8D^3 N_s^2 (t-t_0)^3} \left( \frac{N_s}{1+N_s} \right) + \frac{15k^9}{4D^3 N_s (t-t_0)^2} \right.$$

$$\times \left[ 7 \left( \frac{N_s}{1+N_s} \right)^2 - 1 \right] + \left( \frac{N_s k^{11}}{D(1+N_s)(t-t_0)} \right) \\ \times \left[ \frac{21}{2} \left( \frac{N_s}{1+N_s} \right)^2 - 5 \right] + \frac{N_s^3 k^{13}}{(1+N_s)^2} \left[ \left( \frac{N_s}{1+N_s} \right)^2 - 1 \right] \\ \times \int_0^\infty \exp(n^2) dn \exp[-2R(t-t_0)] \quad (25)$$

It is very interesting that:

$$\int_{-\infty}^\infty W \, dk = 0 \quad (26)$$

This indicates that the expression for W satisfies the condition of continuity and homogeneity. Physically it was to be expected, since W is the measure of transfer of energy and the total energy transformed to all wave numbers is to be zero, which is what Eq. (26) gives. The linear Eq. (23) can be solved to give:

$$G = J(k) \exp[-(2Dk^2 + 2R)(t-t_0)] \\ + \exp[-(2Dk^2 + 2R)(t-t_0)] W \\ \times \exp[-(2Dk^2 + 2R)(t-t_0)] dt \quad (27)$$

where,  $J(k) = \frac{N_0 k^2}{\pi}$  is a constant of integration and can be obtained as by Corrsin (1951). Thus, by integrating the right-hand side of equation, we obtain:

$$G = \exp[-2R(t-t_0)] \left\{ \frac{N_0(1+N_s)\epsilon^2}{D\pi(t-t_0)} \right. \\ \times \exp \left[ -2(1+N_s)\eta^2 + \left( \frac{2\epsilon_{mki} \Omega_m - Qf}{D} - \frac{Qf}{D} \right) \right] \\ + \frac{\xi_0 \pi^{\frac{1}{2}} N_s}{2(t-t_0)^{\frac{9}{2}} D^{\frac{11}{2}} 2(1+N_s)^{\frac{3}{2}}} \\ \times \exp \left[ -(1+2N_s)\epsilon^2 + \left( \frac{2\epsilon_{mki} \Omega_m - Qf}{D} - \frac{Qf}{D} \right) \right] \\ \times \left[ \frac{3\epsilon^4}{2N_s} + \frac{(7N_s-6)\epsilon^6}{3} - f_1 \epsilon^8 + 2f_1 \epsilon^9 F(\epsilon) \right] \\ - \frac{\xi_1 \pi^{\frac{1}{2}}}{2(t-t_0)^7 D^{\frac{15}{2}} (1+N_s)^{\frac{3}{2}}} \\ \times \left[ \frac{105\epsilon^6}{32} + \frac{5}{16} (11-24N_s^2 + 8N_s)\epsilon^8 + f_2 \epsilon^{10} \right. \\ \left. + f_3 \epsilon^{12} - 2f_3 \epsilon^{14} \exp(-2\epsilon^2) \left[ E_i(2\epsilon^2) - 0.5772 \right] + \left( \frac{2\epsilon_{mki} \Omega_m - Sf}{D} - \frac{Sf}{D} \right) \right] \\ - \frac{2\xi_1 \pi^{\frac{1}{2}} N_s}{2(t-t_1)^7 D^{\frac{15}{2}} 2(1+N_s)^{\frac{3}{2}}} \\ \times \exp \left[ - \left[ (1+2N_s)\epsilon^2 + 2 + \left( \frac{2\epsilon_{mki} \Omega_m - Sf}{D} - \frac{Sf}{D} \right) \right] \right]$$

$$\begin{aligned}
 & \times \left\{ \frac{105}{8N_s} \left[ -\frac{\varepsilon^8}{3} - \frac{2\varepsilon^{10}}{3} - \frac{23\varepsilon^{12}}{30} + \frac{498\varepsilon^{14}}{630} \right] \right. \\
 & \times \exp \left[ (-2\varepsilon^2) + \left( \frac{2\varepsilon_{mki}\Omega_m}{D} - \frac{Sf}{D} \right) \right] \left[ E_i(2\varepsilon^2) - 0.5772 \right] \\
 & + 3.1808 \times 10^{-2} \varepsilon^{14} - 2.7178 \times 10^{-2} \varepsilon^{16} \\
 & + 1.3968 \times 10^{-2} \varepsilon^{18} - 4.6204 \times 10^{-3} \varepsilon^{20} \\
 & + 1.1584 \times 10^{-3} \varepsilon^{22} - 2.3139 \times 10^{-4} \varepsilon^{24} \\
 & + 3.8589 \times 10^{-5} \varepsilon^{26} - 5.5108 \times 10^{-4} \varepsilon^{28} \\
 & + 1.1584 \times 10^{-3} \varepsilon^{22} - 2.3139 \times 10^{-4} \varepsilon^{24} \\
 & + 6.8898 \times 10^{-7} \varepsilon^{30} \dots \left. \right\} \\
 & - \frac{15(N^2s - 2N_s - 1)}{4N_s} \left[ -\frac{\varepsilon^{10}}{2} - \frac{5\varepsilon^{12}}{6} + \frac{14\varepsilon^{14}}{15} \right] \\
 & \times \exp \left[ (-2\varepsilon^2) + \left( \frac{2\varepsilon_{mki}\Omega_m}{D} - \frac{Sf}{D} \right) \right] \left[ E_i(2\varepsilon^2) - 0.5772 \right] \\
 & + 2.0260 \times 10^{-2} \varepsilon^{14} + 3.5487 \times 10^{-3} \varepsilon^{16} \\
 & + 5.4047 \times 10^{-4} \varepsilon^{18} + 7.2369 \times 10^{-5} \varepsilon^{20} \\
 & + 8.6172 \times 10^{-6} \varepsilon^{22} + 9.22057 \times 10^{-7} \varepsilon^{24} \\
 & + 8.9477 \times 10^{-8} \varepsilon^{26} + 7.9374 \times 10^{-9} \varepsilon^{28} \\
 & + 6.4814 \times 10^{-10} \varepsilon^{30} + \dots \left. \right\} \\
 & + (11N^2s - 20N_s - 10)N_s \left[ -\varepsilon^{12} + \frac{4}{3}\varepsilon^{14} \right] \\
 & \times \exp \left[ (-2\varepsilon^2) + \left( \frac{2\varepsilon_{mki}\Omega_m}{D} - \frac{Sf}{D} \right) \right] \left[ E_i(2\varepsilon^2) - 0.5772 \right] \\
 & + 8.2505 \times 10^{-10} \varepsilon^{14} + 1.7495 \times 10^{-2} \varepsilon^{16} \\
 & + 3.1572 \times 10^{-3} \varepsilon^{18} + 4.90805 \times 10^{-4} \varepsilon^{20} \\
 & + 6.66926 \times 10^{-5} \varepsilon^{22} + 8.0290 \times 10^{-6} \varepsilon^{24} \\
 & + 8.6642 \times 10^{-7} \varepsilon^{26} + 8.4643 \times 10^{-8} \varepsilon^{28} \\
 & + 7.5495 \times 10^{-9} \varepsilon^{30} + \dots \left. \right\} \\
 & - (1 + 2N_s)N_s^3 \left[ \varepsilon^{14} \exp \left[ (-\varepsilon^2) + \left( \frac{2\varepsilon_{mki}\Omega_m}{D} - \frac{Sf}{D} \right) \right] \right] \\
 & \times \left[ E_i(2\varepsilon^2) - 0.5772 \right] \\
 & + 2.2654 \times 10^{-1} \varepsilon^{14} + 1.0679 \times 10^{-1} \varepsilon^{16} \\
 & - 3.3996 \times 10^{-3} \varepsilon^{18} + 9.0694 \times 10^{-3} \varepsilon^{20} \\
 & - 1.1099 \times 10^{-3} \varepsilon^{22} + 3.7350 \times 10^{-4} \varepsilon^{24} \\
 & - 4.444 \times 10^{-5} \varepsilon^{26} + 8.2388 \times 10^{-6} \varepsilon^{28} \\
 & - 8.47486 \times 10^{-7} \varepsilon^{30} + \dots \left. \right\} \} \} \} \quad (28)
 \end{aligned}$$

where,

$$\begin{aligned}
 \varepsilon^2 &= D \frac{(t-t_1)k^2}{(1+N_s)}, f_1 = \frac{4}{3}(3N^2s - 2N_s + 3) \\
 f_2 &= \left( \frac{15 - 40N^2s(2N_s + 1) + 4N^2s(11N^2s - 20N_s)}{16(1+N_s)^2D} \right) \\
 f_3 &= \left( \frac{15 + 20(4N^2s - 2N_s) + 4N^2s(9N^2s - 20N_s) - 16N^2s}{8(1+N_s)^3} \right)
 \end{aligned}$$

In a turbulent phenomenon, such as turbulent kinetic energy, we associate the so-called concentration energy with the fluctuating concentration, defined by the relation:

$$\frac{1}{2} \langle XX' \rangle = \int_{-\infty}^{\infty} G(k,t) dk \quad (29)$$

The substitution of Eq. (28) and subsequent integration with respect to k leads to the result:

$$\begin{aligned}
 \frac{1}{2} \langle X^2 \rangle &= \exp[-2R(t-t_0)] \\
 & \times \left[ \begin{aligned} & A(t-t_0)^{-\frac{3}{2}} + B(t-t_0)^{-5} \exp[-R_2(t-t_0)] \\ & C(t-t_0)^{-\frac{35}{2}} - D_1(t-t_0)^{-\frac{15}{2}}(t-t_1)^{\frac{1}{2}} - E(t-t_0)^{-15} \\ & - F(t-t_0)^{-\frac{29}{2}} - G(t-t_0)^{-15}(t-t_1)^{\frac{1}{2}} \end{aligned} \right] \exp[-R_3(t-t_0)] \quad (30)
 \end{aligned}$$

This is the decay law of first order reactant in homogeneous dusty fluid turbulence in a rotating system, where,

$$\begin{aligned}
 A &= \frac{N_0}{D^2 2^2 \pi^2}, B = \frac{\xi_0 R_1}{D^6}, C = cd, D_1 = ce, \\
 E &= cf'(gh + gj_1 - k_1 l_1 - k_1 n + pq + ps - t'vw), \\
 F &= cf'(gi_1 - k_1 m - ht'), G = cf'pr_1, \\
 c &= \frac{15\pi\xi_1}{D^8 2^2 N_s^2 (1+N_s)} \\
 d &= \frac{105}{32} + \frac{35(11 - 24N^2s + 8N_s)}{64N_s} + \frac{63f_2}{64N_s^2} + \frac{693f_3}{64N_s^3} - \frac{1808f_3}{256} + \frac{415910f_3 N^2 s}{(1+N)^{\frac{7}{2}}} \\
 e &= 2f_3 \frac{D^2 I(-2N_s \varepsilon^2, \varepsilon)^2}{(1+N_s)^{\frac{1}{2}} \pi^2}, f' = \frac{37.12311 \xi_1 N_s \pi^2}{D^8 \varepsilon^2 (1+N_s)}, g = \frac{105}{2N_s} \\
 h &= -\frac{1}{3} - \frac{6}{\varepsilon_1} - \frac{75.90}{\varepsilon_1^2} - \frac{2.52052 \times 10^{-3} \varepsilon_1^{\frac{9}{2}}}{(1+N_s)^{\frac{15}{2}}} \\
 i_1 &= 1.118 \varepsilon_1^{\frac{9}{2}} \frac{(1+N_s)^{\frac{1}{2}}}{D} \times I \left[ (1+2N_s) \varepsilon, \varepsilon^2 \right] \\
 j_1 &= \frac{163}{\varepsilon_1^3} \left( 3.1808 \times 10^{-2} - 4.0767 \times 10^{-1} \varepsilon_1^{-1} + 3.561804 \varepsilon_1^{-2} \right. \\
 & \left. - 2.2386 \times 10 \varepsilon_1^{-3} + 1.1786 \times 10^2 \varepsilon_1^{-4} - 5.4148 \times 10^2 \varepsilon_1^{-5} \right. \\
 & \left. + 2.02576 \times 10^3 \varepsilon_1^{-6} - 8.7048 \times \varepsilon_1^{-7} + 3.1561 \times 10^4 \varepsilon_1^{-8} \dots \right)
 \end{aligned}$$

$$k_1 = \frac{135(6N^2s - 2N_s - 1)}{N_s \varepsilon_1}, l_1 = -\frac{1}{2} - \frac{55}{6\varepsilon_1} - \frac{2.3510 \times 10^{-3} + 163\varepsilon_1^{\frac{11}{2}}}{2^{15}(1+N_s)^{\frac{15}{2}}}$$

$$m = 7.4469 \varepsilon_1^{\frac{11}{2}} \times \frac{(1+N_s)^{-\frac{1}{2}}}{D} I \left[ (1+2N_s) \varepsilon^2, \varepsilon^2 \right]$$

$$n = \frac{163}{\varepsilon_1^2} (2.0261 + 5.3230 \times 10^{-2} \varepsilon_1)$$

**RESULTS AND DISCUSSION**

In Eq. (30) we obtained the decay law of first order reactant in homogeneous dusty fluid turbulence for four-point correlation in a rotating system after neglecting quintuple correlation terms. The equation contains the terms  $(t-t_0)^{-3/2}$ ,  $(t-t_0)^{-5}$ ,  $(t-t_0)^{-15/2}$ . Thus, the terms associated with the higher-order correlation die out faster than those associated with the lower-order ones; therefore, the assumption that the higher-order correlations can be neglected in comparison with lower-order correlations seems to be valid in our case. If the system is non-rotating and the fluid is clean (i.e.,  $\Omega_m = 0$ ,  $f = 0$ ) the Eq. (30) becomes:

$$+ 1.3782 \times 10^{-1} \varepsilon_1^2 + 3.5063 \times 10^{-1} \varepsilon_1^{-3} \\ + 8.7676 \times 10^{-1} \varepsilon_1^{-4} + 2.57710^{-1} \varepsilon_1^{-5} \\ + 5.2347 \varepsilon_1^{-6} + 1.2538 \varepsilon_1^{-7} + 2.9690 \times 10 \varepsilon_1^{-8} + \dots)$$

$$p = 396(11N_s^2 - 20N_s - 10)N_s \varepsilon_1^{-2} \\ q = -1 - 3.0532 \times 10^{-4} \varepsilon_1^{13/2} (1 + N_s)^{-15/2} \\ r_1 = 1.0638 \varepsilon_1^{13/2} (t-t_1)^{1/2} \frac{(1+N_s)^{1/2}}{D} \times I[(1+2N_s)\varepsilon^2, \varepsilon^2] \\ s = 13 \varepsilon_1^{-1} (8.2504 + 2.6243 \times 10^{-1} \varepsilon^{-1} + 8.0509 \times 10^{-1} \varepsilon^{-2}) \\ + 2.3780 \times \varepsilon_1^{-3} + 6.7856 \varepsilon_1^{-4} + 1.8789 \times 10 \varepsilon_1^{-5} \\ + 5.0688 \times 10^1 \varepsilon_1^{-6} + 1.3370 \times 10^2 \varepsilon_1^{-7} \\ + 3.4583 \times 10^2 \varepsilon_1^{-8} + \dots)$$

$$t' = -1.4219 N_s^2 \varepsilon_1^{11/2} (1 + N_s)^{1/2} \\ u = \left\{ \left( -3.1886 \times 10^{-3} + 1.4443 \times 10^2 (t-t_0)^{1/2} \right. \right. \\ \left. \left. \times D^{1/2} (1 + N_s)^{-1/2} I[(1+2N_s)\varepsilon^2, \varepsilon^2] \right) \right\} \\ v = 1.8102 \times 10^2 (1 + 2N_s)^{1/2} \varepsilon_1^{-15/2} \\ w = (2.2653 \times 10^{-1} + 1.60195 \varepsilon_1^{-1} \\ - 8.6664 \times 10^{-1} \varepsilon^{-2} + 4.3941 \times 10 \varepsilon_1^{-3} \\ - 1.1293 \times 10^2 \varepsilon_1^{-4} + 8.7405 \times 10^2 \varepsilon_1^{-5} \\ - 2.6001 \times 10^3 \varepsilon_1^{-6} + 1.3013 \times 10^4 \varepsilon_1^{-7} \\ - 3.8822 \times 10^4 \varepsilon_1^{-8} + \dots)$$

$$R_1 = \frac{\pi}{2(1+N_s)(1+2N_s)^{5/2}} \left[ \frac{9}{16} + \frac{5N_s(7N_s-6)}{16(1+2N_s)} \right. \\ \left. - \frac{105f_1N_s}{32(1+2N_s)} + \frac{f_1N_s(1+2N_s)^{5/2}}{2^2(1+N_s)^{11/2}} \right] \\ \times \sum_{n=0}^{\infty} \frac{1.35 \dots (2n+9)}{n!(2N_s+1)2^{2n}(1+N_s)^n} \\ \varepsilon_1 = 2(1+2N_s), R_2 = \left( \frac{2\varepsilon_{mki}\Omega_m}{D} - \frac{Qf}{D} \right), R_3 = \left( \frac{2\varepsilon_{mki}\Omega_m}{D} - \frac{Sf}{D} \right)$$

and

$$I[-\alpha\varepsilon^2, \beta\varepsilon^2] = \int_0^{\infty} \exp(-\alpha\varepsilon^2) \left[ \varepsilon^{14} \exp(\beta\varepsilon^2) E_i(\beta\varepsilon^2) \right] dk$$

The first term of Eq. (30) corresponds to the concentration energy for two-point concentration; the second term represents first order reactant in homogeneous dusty fluid turbulence in a rotating system for three-point correlation.

The expression  $\exp[-R_2(t-t_0)]$  represents the dusty fluid turbulence in a rotating system for three-point correlation; the expression  $\exp[-R_3(t-t_0)]$  and the remainder are due to dusty fluid turbulence for four-point correlation in a rotating system.

$$\frac{1}{2} \langle X^2 \rangle = \exp[-2R(t-t_0)] \times \left[ A(t-t_0)^{-3/2} + B(t-t_0)^{-5} - \right. \\ \left. \left\{ C(t-t_0)^{-15/2} - D_1(t-t_0)^{-15/2} (t-t_1)^{1/2} - E(t-t_0)^{-15} \right. \right. \\ \left. \left. F(t-t_0)^{-29/2} - G(t-t_0)^{-15} (t-t_1)^{1/2} \right\} \right] \quad (31)$$

which is obtained by Kumar and Patel (1974). Here  $A, B, C, D_1, E, F, G$  are an arbitrary constants. With  $R = 0$  and the contaminant replaced by the temperature, the results agrees with the result obtained by Loeffler and Deissler (1960) for the decay of temperature fluctuation in homogeneous turbulence before the final period for three-point correlations. For large times, the last terms become negligible and give the  $-3/2$  power decay law for the final period. In figures, Eq. (30) represented by the curves  $y_1, y_2, y_3$  and (31) by  $y_4, y_5, y_6$  at  $t_0 = .3, t_1 = .5; t_0 = .8, t_1 = 1$  and  $t_0 = 1.3, t_1 = 1.5$ , respectively the

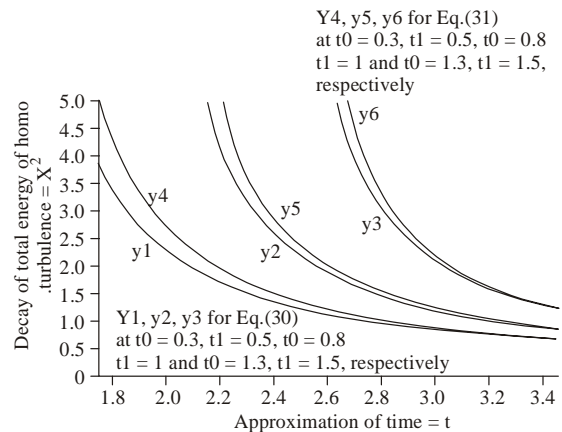


Fig. 1: Comparison between Eq. (30) and (31) if  $R = 0.025$

CONCLUSION

From the above figures and discussions, this study shows that the chemical reaction rate increases than the concentration fluctuation to decay more decreases and vice versa. At the chemical reaction rate  $R = 0$  causes the concentration fluctuation of decay more rapidly than they would for ( $R \neq 0$ ) non pure mixing homogeneous dusty fluid turbulence. Also we conclude that due to the effect of rotation of homogeneous dusty fluid turbulence in the flow field of the first order chemical reaction for four-point correlation the turbulent energy decays more slowly than the energy decay for the first order reactant in homogeneous turbulence.

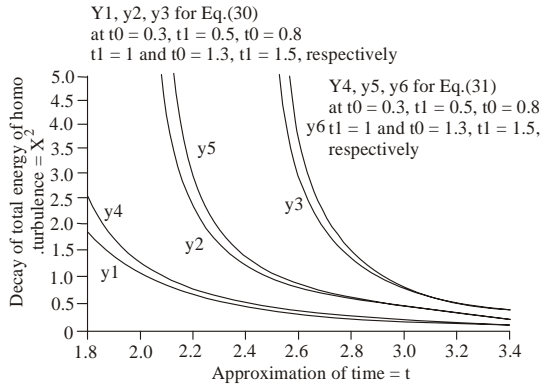


Fig. 2: omparison between Eq. (30) and (31) if  $R = 0.5$

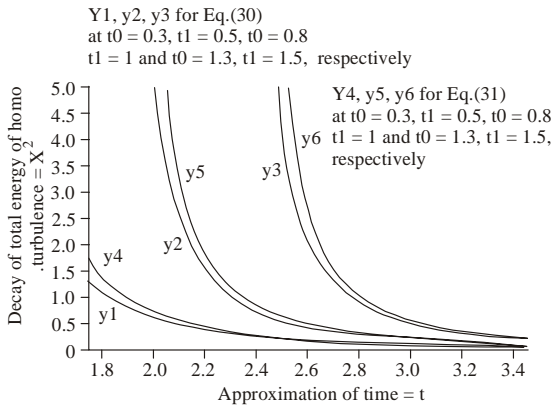


Fig. 3: Comparison between Eq. (30) and (31) if  $R = 0.8$

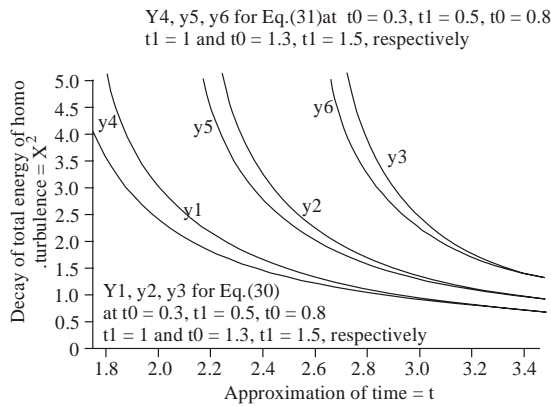


Fig. 4: Comparison between Eq. (30) and (31) if  $R = 0$

energy decays more slowly. For  $R_2 = 0.25$  and  $R_3 = 0.5$ , the comparison between the Eq. (30) and (31) are shown in Fig. 1, 2, 3 and 4 corresponding to the values  $R = 0.025, 0.5, 0.8$  and  $0$ , respectively, the energy decays slowly than non rotating clean system which indicates in the figures clearly.

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