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Global Equilibrium Stability of Hepatitis B Model and Vaccination Impact

[1]A.A. Momoh, [1]M.O. Ibrahim, [2]B.A. Madu and [1]K.K. Asogwa
[1]Department of Mathematics, Usmanu Danfodiyo University, Sokoto State of Nigeria
[2]Department of Mathematics and Statistics, University of Maiduguri, Borno State of Nigeria

Abstract: In this study, we propose an MSEIR model with passive immunisation, exposed individual therapy and infectious hepatitis B therapy to describe the dynamics of HBV transmission. We established the disease free equilibrium and epidemic equilibrium states of the model and analyse for stability of the epidemic equilibrium state using Bellman and Cooke's theorem. We found out the epidemic equilibrium state was stable, which implies that the MSEIR model can be used for predicting the long-term effectiveness of the immunisation programme combined with Exposed individuals therapy and infectious hepatitis therapy in sustaining a population.

Keywords: Epidemic, equilibrium state, hepatitis, passive immunisation, stability, vaccines

INTRODUCTION

Hepatitis B virus (HBV) is a serious public health problem worldwide and major cause of chronic hepatitis, Cirrhosis, and Hepatocellular Carcinoma (HCC). It was estimated that approximately 2 billion people have serological evidence of past and present HBV infection. More than 350 million are chronic carriers of HBV (WHO, 2000). Approximately 75% of chronic carriers live in Asia and the western pacific (Gust, 1996). It was reported that 15-40% of HBV infected patients would develop cirrhosis, liver failure, or HCC (Lok, 2002), and 500,000 to 1.2 million people die of HBV infection annually Mahoney (1990) and Lee (1997). Because of the high HBV-related morbidity and mortality, the global disease burden of hepatitis B is substantial.

The prevalence of HBV infection varies remarkably throughout regions of the world Margolis *et al*. (1991). Hepatitis B is highly endemic in developing regions with large population such as South East Asia, China, Sub-Saharan Africa and the Amazon Basin, where at least 8% of the population are HBV chronic carrier. Most infections occur during infancy or childhood. Since most infections in children are asymptomatic, there is little evidence of acute disease related to HBV, but the rates of chronic liver disease and liver cancer in adult are high Alter (2003).

Passive immunoprophylaxis: Hepatitis B Immune Globulin (HBIG) is a sterile solution of ready-made antibodies against hepatitis B. HBIG is prepared from human blood from selected donors who already have a high level of anti-bodies to hepatitis B and used in passive immuneoprophylaxis. Passive immunoprophylaxis is used in four situations

- Newborns of mothers infected with hepatitis
- after needle stick exposure
- after sexual exposure
- after liver transplantation

Immunoprophylaxis is recommended for all infants born to HBsAg positive mothers. Current dosing recommendations are 0.13mL/kg HBIG immediately after delivery or within 12 h after birth in combination with recombinant vaccines. The combination results in a higher-than 90% level of protection against perinatal acquisition of HBV.

In this study, we propose an MSEIR model with passive immunisation, exposed individual therapy and infectious hepatitis B therapy to describe the dynamics of HBV transmission. We found out the epidemic equilibrium state was stable, which implies that the MSEIR model can be used for predicting the long-term effectiveness of the immunisation programme combined with Exposed individuals therapy and infectious hepatitis therapy in sustaining a population.

MATERIALS AND METHODS

Model description: The M-S-E-I-R model is partitioned into compartments of passively immune infants (M), susceptible individuals (S), exposed individuals in the latent period (E), infectious individuals (I) and removed individuals (R), respectively. The immunised compartment changes due to the coming in of the

Corresponding Author: A.A. Momoh, Department of Mathematics, Usmanu Danfodiyo University, PMB 2346 Sokoto State of Nigeria

immunised children into the population where we assumed that a proportion of B of the incoming individuals are immunised against hepatitis B infection. This compartment reduces due to expiration of duration of vaccine efficacy at the rate δ and also by natural death at the rate μ.

The susceptible population increases due to the coming of individuals from the immunised compartment as a result of the expiration of the duration of vaccines efficacy at the rate δ. The susceptible population also reduces due to natural death rate μ and infection with contact rate of infection β.

The population dynamics of the exposed class at the latent period grows with the incidence rate of β SI. This class reduces by natural death rate μ and occasional break down of the exposed individuals at latent period into infectious hepatitis B at the rate ε.

Similarly, the population dynamics of the infectious grows with the occasional breakdown of exposed individuals at the latent period into infectious hepatitis B at the rate ϵ. This class also reduces by natural death rate μ and successful cure of infectious hepatitis B patient at the rate γ and natural death at the rate μ.

Lastly the dynamics of the removed class increases with successful cure of hepatitis B patients at the rate γ and decreases by natural death rate μ. This research work was carried out in 2011 with Sokoto State of Nigeria selected has area of study.

Basic assumptions: The epidemiological features of the hepatitis B virus (HBV) lead to the following assumptions about the transmission of the disease.

- The population has a constant size, N and divided into four and five compartments: MSEIR.
- The birth and death occur at a constant and equal rate.
- The population is mixing in a homogeneous manner i.e. every person has the same chance to becoming in contact with an infected person.
- The transmission of the HBV infection occurs at a rate β.
- Recovery occurs at a rate constant rate γ.

Model equations: Keeping in view of these assumptions, our population dynamic, i.e., "passively immune infant-susceptible- infectious-removed" is governed by the following set of differential equations.

$$\frac{dM}{dT} = B - \delta MS - \mu M \tag{1}$$

$$\frac{dS}{dT} = \delta MS - \beta SI - \mu S \tag{2}$$

$$\frac{dE}{dT} = \beta SI - \varepsilon E - \mu E \tag{3}$$

$$\frac{dI}{dT} = \varepsilon E - \gamma I - \mu I \tag{4}$$

$$\frac{dI}{dT} = \gamma I - \mu R \tag{5}$$

where total population at any instant t is

$$N(t) = M(t) + S(t) + E(t) + I(t) + R(t).$$

Now, in the above system (1)-(5), use the following:

$$\frac{dM}{dt} = \frac{dS}{dt} = \frac{dE}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0 \text{ and } M(t) = V,\ S(t) = w,$$
$$E(t) = x,\ I(t) = y,\ R(t) = z$$

to get the following re-scaled system:

$$B - \delta vw - \mu v = 0 \tag{6}$$

$$\delta vw - \beta wy - \mu w = 0 \tag{7}$$

$$\beta wy - \varepsilon x - \mu x = 0 \tag{8}$$

$$\varepsilon x - \gamma y - \mu y = 0 \tag{9}$$

$$\gamma y - \mu z = 0 \tag{10}$$

In the next section, we will study the existence of disease free equilibrium state and epidemic equilibrium state.

Equilibrium states of the model:
The disease free equilibrium state when solved is:

$$E_1 = \left(\frac{B}{\mu}, 0, 0, 0, 0\right) \tag{11}$$

The epidemic equilibrium state when solved is:

$$w^* = \frac{(\varepsilon + \mu)(\gamma + \mu)}{\beta\varepsilon} \tag{12}$$

$$v^* = \frac{B\beta\varepsilon}{\delta[(\varepsilon + \mu)(\gamma + \mu)] + \mu\beta\varepsilon} \tag{13}$$

$$y^* = \frac{\delta B\beta\varepsilon - [\delta[(\varepsilon + \mu)(\gamma + \mu)] + \mu\beta\varepsilon]\mu}{[\delta[(\varepsilon + \mu)(\gamma + \mu)] + \mu\beta\varepsilon]\beta} \tag{14}$$

$$x^* = \frac{(\gamma + \mu)[\delta B\beta\varepsilon - [\delta(\varepsilon + \mu)(\gamma + \mu)] + \mu\beta\varepsilon]\mu]}{\beta\varepsilon[\delta[(\varepsilon + \mu)(\gamma + \mu) + \mu\beta\varepsilon]} \tag{15}$$

$$z^* = \frac{[\delta B\beta\varepsilon - [\delta[(\varepsilon + \mu)(\gamma + \mu)] + \mu\beta\varepsilon]\mu}{[\delta[(\varepsilon + \mu)(\gamma + \mu)] + \mu\beta\varepsilon]\beta\mu} \tag{16}$$

Hence the epidemic equilibrium state is given by:

$$E^* = (v^*, w^*, x^*, y^*, z^*) \tag{17}$$

Having established the equilibrium states, we now analyse for the stability of the epidemic equilibrium state

RESULTS AND DISCUSSION

The characteristics equation: The Jacobian matrix of this system of equations is

$$J_2 = \begin{pmatrix} -(\delta w + \mu) - \lambda & -\delta v & 0 & 0 & 0 \\ \delta w & \delta v - \beta y - \mu - \lambda & 0 & -\beta w & 0 \\ 0 & \beta y & -(\varepsilon + \mu) - \lambda & 0 & 0 \\ 0 & 0 & \varepsilon & -(\gamma + \mu) - \lambda & 0 \\ 0 & 0 & 0 & \gamma & -\mu - \lambda \end{pmatrix} \tag{18}$$

The characteristics equation is obtained from the Jacobian determinant with the eigen values λ

Where,

$$\det(J1 - \lambda I) \tag{19}$$

⇒

$$\det = \begin{pmatrix} -(\delta w + \mu) - \lambda & -\delta v & 0 & 0 & 0 \\ \delta w & \delta v - \beta y - \mu - \lambda & 0 & -\beta w & 0 \\ 0 & \beta y & -(\varepsilon + \mu) - \lambda & 0 & 0 \\ 0 & 0 & \varepsilon & -(\gamma + \mu) - \lambda & 0 \\ 0 & 0 & 0 & \gamma & -\mu - \lambda \end{pmatrix}$$

$$-(\delta v + \mu) - \lambda[(\delta v - \beta y - \mu - \lambda)(-(\varepsilon + \mu) - \lambda)(-(\gamma + \mu) - \gamma)(-\mu - \lambda)] + \delta^2 vw[(-(\varepsilon + \mu) - \lambda)(-(\gamma + \mu) - \lambda)(-\mu - \lambda)] = 0 \tag{20}$$

Eq. (20) takes the form

$$H(\lambda) = [-(\delta V + \mu) - \lambda][(\delta v - \beta y - \mu - \lambda)(-(\varepsilon + \mu) - \lambda)(-(y + \mu) - \lambda)(-\mu - \lambda)] + \delta^2 vw[(\varepsilon + \mu) - \lambda)(-(y + \mu) - \lambda)(-\mu - \lambda)] = 0 \tag{21}$$

Expanding and rearranging in ascending powers of λ we obtained

$$H(\lambda) = -\lambda^5 - [(\delta w+\mu)+\mu+(\gamma+\mu)+(\delta v-\beta y-\mu)-(\varepsilon+\mu)]\lambda^4 + [(\delta v-\beta y-\mu)(\delta w+\mu)-(\varepsilon+\mu)(\delta w+\mu)-(\gamma+\mu)(\delta w+\mu) - \mu(\delta w+\mu)-\mu(\gamma+\mu)+\mu(\delta v-\beta y-\mu)-\mu(\varepsilon+\mu)+(\delta v-\beta y-\mu)(\gamma+\mu)-(\gamma+\mu)(\varepsilon+\mu)+(\delta v-\beta y-\mu)(\varepsilon+\mu)-\lambda^2 vw]\lambda^3 + [\mu(\delta v-\beta y-\mu)(\delta w+\mu)-\mu(\gamma+\mu)(\delta w+\mu)+(\delta v-\beta y-\mu)(\gamma+\mu)(\delta w+\mu)-(\gamma+\mu)(\varepsilon+\mu)(\delta w+\mu)(\delta v-\beta y-\mu)(\varepsilon+\mu) (\delta w+\mu)+\mu(\delta v-\beta y-\mu)(\gamma+\mu)-\mu(\varepsilon+\mu)(\gamma+\mu)+\mu(\delta v-\beta y-\mu)(\varepsilon+\mu)+(\delta v-\beta y-\mu)(\varepsilon+\mu)(\gamma+\mu)-\mu\delta^2 vw-(\varepsilon+\mu) \delta^2 vw-(\gamma+\mu)\delta^2 vw]\lambda^2 + [\mu(\delta v-\beta y-\mu)(\gamma+\mu)(\delta w+\mu)-\mu(\varepsilon+\mu)(\gamma+\mu)(\delta w+\mu)+\mu(\delta v-\beta y-\mu)(\varepsilon+\mu)(\delta w+\mu) + (\delta v-\beta y-\mu)(\varepsilon+\mu)(\gamma+\mu)(\delta w+\mu)+\mu(\delta v-\beta y-\mu)(\varepsilon+\mu)(\gamma+\mu)-\mu(\gamma+\mu)\delta^2 vw-\mu(\gamma+\mu)\delta^2 vw-(\varepsilon+\mu)(\gamma+\mu)\delta^2 vw] \lambda + [\mu(\delta v-\beta y-\mu)(\varepsilon+\mu)(\gamma+\mu)(\delta w+\mu)-\mu(\varepsilon+\mu)(\gamma+\mu)\delta^2 vw] \quad (22)$$

Global asymptotic stability of epidemic equilibrium: In order to study the global stability of the MSEIR model, we apply Bellman and Cooke's theorem.

Let λ= iq (23)

Substituting Eq. (23) for λ in Eq. (22) we obtained

$$H(\lambda) = -(iq)^5 - [(\delta w+\mu)+\mu+(\gamma+\mu)+(\delta v-\beta y-\mu)-(\varepsilon+\mu)](iq)^4 + [(\delta v-\beta y-\mu)(\delta w+\mu)-(\varepsilon+\mu)(\delta w+\mu)-(\gamma+\mu)(\delta w+\mu)-\mu(\delta w+\mu) - \mu(\gamma+\mu)+\mu(\delta v-\beta y-\mu)-\mu(\varepsilon+\mu)+(\delta v-\beta y-\mu)(\gamma+\mu) - (\gamma+\mu)(\varepsilon+\mu)+(\delta v-\beta y-\mu)(\varepsilon+\mu)-\delta^2 vw](iq)^3 + [\mu(\delta v-\beta y-\mu)(\delta w+\mu) - \mu(\gamma+\mu)(\delta w+\mu)+(\delta v-\beta y-\mu)(\gamma+\mu)(\delta w+\mu)-(\gamma+\mu)(\varepsilon+\mu)(\delta w+\mu) (\delta v-\beta y-\mu)(\varepsilon+\mu)(\delta w+\mu)+\mu(\delta v-\beta y-\mu)(\gamma+\mu)-\mu(\varepsilon+\mu)(\gamma+\mu) + \mu(\delta v-\beta y-\mu)(\varepsilon+\mu)+(\delta v-\beta y-\mu)(\varepsilon+\mu)(\gamma+\mu)-\mu\delta^2 vw-(\varepsilon+\mu)\delta^2 vw - (\gamma+\mu)\delta^2 vw](iq)^2 + [\mu(\delta v-\beta y-\mu)(\gamma+\mu)(\delta w+\mu)-\mu(\varepsilon+\mu)(\gamma+\mu)(\delta w+\mu) + \mu(\delta v-\beta y-\mu)(\varepsilon+\mu)(\delta w+\mu)+(\delta v-\beta y-\mu)(\varepsilon+\mu)(\gamma+\mu)(\delta w+\mu) + \mu(\delta v-\beta y-\mu)(\varepsilon+\mu)(\gamma+\mu)-\mu(\gamma+\mu)\delta^2 vw-\mu(\gamma+\mu)\delta^2 vw - (\varepsilon+\mu)(\gamma+\mu)\delta^2 vw](iq)+[\mu(\delta v-\beta y-\mu)(\varepsilon+\mu)(\gamma+\mu)(\delta w+\mu) - \mu(\varepsilon+\mu)(\gamma+\mu)\delta^2 vw]$$

$$H(\lambda) = -iq^5-[(\delta w+\mu)+\mu+(\gamma+\mu)+(\delta v-\beta y-\mu)-(\epsilon+\mu)]q^4-[(\delta v-\beta y-\mu)(\delta w+\mu)-(\epsilon+\mu)(\delta w+\mu)-(\gamma+\mu)(\delta w+\mu) -\mu(\delta w+\mu)-\mu(\gamma+\mu)+\mu(\delta v-\beta y-\mu)-\mu(\epsilon+\mu)+(\delta v-\beta y-\mu)(\gamma+\mu)-(\gamma+\mu)(\epsilon+\mu)+(\delta v-\beta y-\mu)(\epsilon+\mu)-\delta^2 vw]iq^3- [\mu(\delta v-\beta y-\mu)(\delta w+\mu)-\mu(\gamma+\mu)(\delta w+\mu)+(\delta v-\beta y-\mu)(\gamma+\mu)(\delta w+\mu)-(\gamma+\mu)(\epsilon+\mu)(\delta w+\mu)(\delta v-\beta y-\mu)(\epsilon+\mu)(\delta w+\mu)+ \mu(\delta v-\beta y-\mu)(\gamma+\mu)-\mu(\epsilon+\mu)(\gamma+\mu)+\mu(\delta v-\beta y-\mu)(\epsilon+\mu)+(\delta v-\beta y-\mu)(\epsilon+\mu)(\gamma+\mu)-\mu\delta^2 vw-(\epsilon+\mu)\delta^2 vw(\gamma+\mu)\delta^2 vw]q^2+ [\mu(\delta v-\beta y-\mu)(\gamma+\mu)(\delta w+\mu)-\mu(\epsilon+\mu)(\gamma+\mu)(\delta w+\mu)+\mu(\delta v-\beta y-\mu)(\epsilon+\mu)(\delta w+\mu)+(\delta v-\beta y-\mu)(\epsilon+\mu)(\gamma+\mu) (\delta w+\mu)-(\epsilon+\mu)(\gamma+\mu)\delta^2 vw](iq)+[\mu(\delta v-\beta y-\mu)(\epsilon+\mu)(\gamma+\mu)(\delta w+\mu)-\mu(\epsilon+\mu)(\gamma+\mu)\delta^2 vw] \quad (24)$$

Resolving Eq. (24) into real and imaginary parts we obtained

H (iq) = F(q) + iG (q) (25)

$$Let\ F(0)G'(0) - F'(0)G(0) > 0 \quad (26)$$

$$F(q) = -[(\delta w+\mu)+\mu+(\gamma+\mu)+(\delta v-\beta y-\mu)-(\epsilon+\mu)]q^4-[\mu(\delta v-\beta y-\mu)(\delta w+\mu)-\mu(\gamma+\mu)(\delta w+\mu)+(\delta v-\beta y-\mu)\ (\gamma+\mu)(\delta w+\mu) -(\epsilon+\mu)(\gamma+\mu)(\delta w+\mu)(\delta v-\beta y-\mu)(\epsilon+\mu)(\delta w+\mu)+\mu(\delta v-\beta y-\mu)(\gamma+\mu)-\mu(\epsilon+\mu)(\gamma+\mu)+\mu(\delta v-\beta y-\mu)(\epsilon+\mu)+(\delta v-\beta y-\mu)(\epsilon+\mu)(\gamma+\mu)-\mu\delta^2 vw-(\epsilon+\mu)\delta^2 vw-(\gamma+\mu)\delta^2 vw]q^2+[\mu(\delta v-\beta y-\mu)(\epsilon+\mu)(\gamma+\mu)(\delta w+\mu)-\mu(\epsilon+\mu)(\gamma+\mu)\delta^2 vw\ \delta^2 vw]$$

$G'(q) = -q^5-[(\delta v-\beta y-\mu)(\delta w+\mu)-(\epsilon+\mu)(\delta w+\mu)-\mu(\delta w+\mu)\mu(\gamma+\mu)+\mu(\delta v-\beta y-\mu)-\mu(\epsilon+\mu)+(\delta v-\beta y-\mu)(\gamma+\mu)-(\epsilon+\mu)(\gamma+\mu)(\delta v-\beta y-\mu)(\epsilon+\mu)-\delta^2 vw]q^3+[\mu(\delta v-\beta y-\mu)(\gamma+\mu)(\delta w+\mu)-\mu(\epsilon+\mu)(\gamma+\mu)(\delta w+\mu)+\mu(\delta v-\beta y-\mu)(\epsilon+\mu)(\delta w+\mu)+(\delta v-\beta y-\mu)(\epsilon+\mu)(\gamma+\mu)(\delta w+\mu)+\mu(\delta v-\beta y-\mu)(\epsilon+\mu)(\gamma+\mu)-\mu(\epsilon+\mu)\delta^2 vw-\mu(\gamma+\mu)\delta^2 vw-(\epsilon+\mu)(\gamma+\mu)\delta^2 vw]q$

Therefore,

$F'(q) = -4[(\delta w+\mu)+\mu+(\gamma+\mu)+(\delta v-\beta y-\mu)-(\epsilon+\mu)]q^3-2[\mu(\delta v-\beta y-\mu)(\delta w+\mu)-\mu(\gamma+\mu)(\delta w+\mu)+(\delta v-\beta y-\mu)(\gamma+\mu)(\delta w+\mu)-(\epsilon+\mu)(\gamma+\mu)(\delta w+\mu)(\delta v-\beta y-\mu)(\epsilon+\mu)(\delta w+\mu)+\mu(\delta v-\beta y-\mu)(\gamma+\mu)-\mu(\epsilon+\mu)(\gamma+\mu)+\mu(\delta v-\beta y-\mu)(\epsilon+\mu)+(\delta v-\beta y-\mu)(\epsilon+\mu)(\gamma+\mu)-\mu\delta^2 vw-(\epsilon+\mu)\delta^2 vw-(\gamma+\mu)\delta^2 vw]q$ (27)

$G'(q) = -5q^4-3[(\delta v-\beta y-\mu)(\delta w+\mu)-(\epsilon+\mu)(\delta w+\mu)-(\gamma+\mu)(\delta w+\mu)-\mu(\delta w+\mu)\mu(\gamma+\mu)+\mu(\delta v-\beta y-\mu)-\mu(\epsilon+\mu)+(\delta v-\beta y-\mu)(\gamma+\mu)-(\epsilon+\mu)(\gamma+\mu)(\delta v-\beta y-\mu)(\epsilon+\mu)-\delta^2 vw]q^2+[\mu(\delta v-\beta y-\mu)(\gamma+\mu)(\delta w+\mu)-\mu(\epsilon+\mu)(\gamma+\mu)(\delta w+\mu)+\mu(\delta v-\beta y-\mu)(\epsilon+\mu)(\delta w+\mu)+(\delta v-\beta y-\mu)(\epsilon+\mu)(\gamma+\mu)(\delta w+\mu)+\mu(\delta v-\beta y-\mu)(\epsilon+\mu)(\gamma+\mu)-\mu(\epsilon+\mu)\delta^2 vw-\mu(\gamma+\mu)\delta^2 vw-(\epsilon+\mu)(\gamma+\mu)\delta^2 vw]$ (28)

Now letting q = 0 we obtained

$$F(0) = \mu(\varepsilon+\mu)(\gamma+\mu)[(\delta v-\beta y-\mu)(\delta w+\mu)-\delta^2 vw] \quad (29)$$

$$G(0) = 0 \quad (30)$$

$$F'(0) = 0 \quad (31)$$

$G'(0) = [\mu(\delta v-\beta y-\mu)(\gamma+\mu)(\delta w+\mu)-\mu(\epsilon+\mu)(\gamma+\mu)(\delta w+\mu)+\mu(\delta v-\beta y-\mu)(\epsilon+\mu)(\delta w+\mu)+(\delta v-\beta y-\mu)(\epsilon+\mu)(\gamma+\mu)(\delta w+\mu)+\mu(\delta v-\beta y-\mu)(\epsilon+\mu)(\gamma+\mu)-\mu(\epsilon+\mu)\delta^2 vw-\mu(\gamma+\mu)\delta^2 vw-(\epsilon+\mu)(\gamma+\mu)\delta^2 vw]$ (32)

Substituting Eq. (30) for G(0) and Eq. (31) for F′(0) in Eq. (26) we obtained

Table 1: Stability analysis of the epidemic equilibrium state in MSEIR model using Bellman and Cooke's theorem

B	δ	β	γ	ϵ	μ	J	Remarks
0.2	0.1	0.1	0.5	0.6	0.2	0.000260280	Stable
0.2	0.2	0.2	0.5	0.6	0.2	0.000572826	Stable
0.2	0.3	0.3	0.5	0.6	0.2	0.00093762	Stable
0.2	0.4	0.4	0.5	0.6	0.2	0.001354693	Stable
0.2	0.5	0.5	0.5	0.6	0.2	0.001824010	Stable
0.2	0.6	0.6	0.5	0.6	0.2	0.002345590	Stable
0.2	0.7	0.7	0.5	0.6	0.2	0.002919429	Stable
0.2	0.8	0.8	0.5	0.6	0.2	0.003545527	Stable
0.2	0.9	0.9	0.5	0.6	0.2	0.004223898	Stable
0.2	1.0	1.0	0.5	0.6	0.2	0.004954502	Stable

$$F(0)G'(0) > 0 \quad (33)$$

Let $J = F(0)G'(0) > 0$ (34)

Hence the condition for Eq. (34) to hold is

$$J > 0 \quad (35)$$

Substituting the epidemic state equations in MSEIR model into Eq. (29) we obtained

$$F(0) = \mu(\varepsilon+\mu)(\gamma+\mu)\begin{bmatrix}\delta^2 B(\varepsilon+\mu)(\gamma+\mu)\\ \delta[(\varepsilon+\mu)(\gamma+\mu)]+\beta\mu\varepsilon\end{bmatrix} \quad (36)$$

Substituting the epidemic state equations in MSEIR model into Eq. (32) we obtained

$$G'(0) = -\begin{bmatrix}\dfrac{\mu(\varepsilon+\mu)(\gamma+\mu)[\delta(\varepsilon+\mu)+\beta\mu\varepsilon]}{\beta\varepsilon}\\ +\dfrac{B\mu\delta^2(\varepsilon+\mu)^2(\gamma+\mu)}{[\delta(\varepsilon+\mu)(\gamma+\mu)+\beta\mu\varepsilon]}\\ \dfrac{B\mu\delta^2(\gamma+\mu)^2(\varepsilon+\mu)}{[\delta(\varepsilon+\mu)(\gamma+\mu)+\beta\mu\varepsilon]}\\ \dfrac{B\delta^2(\varepsilon+\mu)^2(\gamma+\mu)^2}{[\delta(\varepsilon+\mu)(\gamma+\mu)+\beta\mu\varepsilon]}\end{bmatrix} \quad (37)$$

Using hypothetical values for the parameters of J we use mathematical software (Maple) to evaluate J and the result is presented in Table (1)

CONCLUSION

We propose an MSEIR model with passive immunisation, exposed individual therapy and infectious hepatitis B therapy to describe the dynamics of HBV transmission. We found out the epidemic equilibrium state was stable, which implies that the MSEIR model can be used for predicting the long-term effectiveness of

the immunisation programme combined with Exposed individuals therapy and infectious hepatitis therapy in sustaining a population. However, despite the much progress in understanding the transmission dynamics of HBV infection, we still have a long way to go before we can conquer hepatitis B infection. Hence, further research and understanding in this sector may bring exciting new information and better understanding of the transmission dynamics of HBV.

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