

Effect of Convolution between Two Complex Waves having the Same Angular Frequencies with Amplitude Spectrum Analysis in Frequency Domain

A.N.M. Rezaul Karim

Department of Computer Science and Engineering, International Islamic University
Chittagong, Bangladesh

Abstract: As digital signal processing continues to emerge as a major discipline in the field of electrical engineering, an even greater demand has evolved to understand the basic theoretical concepts involved in the development of varied and diverse signal processing systems. The most fundamental concepts employed are the sampling theorem, Fourier transforms, convolution, covariance, etc. The intent of this paper will be to address the concept of convolution and to present Convolved signal between Two Complex Waves having the same angular frequencies with Amplitude Spectrum Analysis in Frequency Domain The Fourier series gives us a very important and useful way of representing an image while it is a very powerful representation for many reasons, we will mainly introduce it as a way of understanding the effect of Convolution between two complex waves, how the losing local information of the spectrum.

Key words: Attenuation, convolution, fourier series, frequency spectrum

INTRODUCTION

Perhaps the easiest way to understand the concept of convolution would be an approach that initially clarifies a subject relating to the frequency spectrum of linear networks. Determining the frequency spectrum or frequency transfer function of a linear network provides one with the knowledge of how a network will respond to or alter an input signal. Conventional methods used to determine this entail the use of spectrum analyzers which use either sweep generators or variable-frequency oscillators to impress upon a network all possible frequencies of equal amplitude and equal phase. The response of a network to all frequencies can thus be determined. The characteristics of electrical (and other) signals can be explored in two ways. The first is to examine their waveforms in the time domain, which can be done using an oscilloscope. The second is to examine their spectra in the frequency domain, which can be done using a spectrum analyzer. Both descriptions are useful, although frequency domain analysis is the most natural tool for many tasks in electrical circuit design, speech and music analysis, and for video signals. In fact, frequency domain analysis is of fundamental importance in all branches of engineering, from the design of bridges and roads to wireless Communications. Many Signals are more convenient to process, analyze, Synthesize and/or compress in the Frequency Domain

- A signal can be described in two spaces: time domain and frequency domain. The techniques of signals analysis both in time domain and frequency domain

are called methods of time-frequency analysis (Leon, 1989; Leon and Patrick, 1998).

- Fourier Series (FS) (Okan, 1994) is an ideal tool of spectrum analysis for stationary signals for which the spectrum is not a function of the time. The power of Fourier series is that it allows the decomposition of a signal into individual frequency components.
- Frequency domain allows efficient representation of a good approximation to the original function
- **Frequency spectra:** The frequency spectrum is often displayed by plotting graphs of both the amplitudes and phases of the various harmonic components against angular frequency ω_n . A plot of amplitude against angular frequency is called the amplitude spectrum. While that of phase against angular frequency is called the phase spectrum. (GLYN, 1999)
- As the Fourier series expansion will consist of sinusoids having frequencies $n\omega$ that are multiples of the input signal frequency
- ^o(GLYN, 1999), the steady-state response will also have components having such frequencies. The concept of spectrum is similar to the one used in optics for light, or in material science for metals, each indicating the distribution of power or energy over frequency.
- A complex waveform may be treated as being composed of a number of sinusoid waveforms. These sinusoids are of various phases, frequencies and amplitudes. The description of the magnitude, phase and frequency of these various waves is known as the

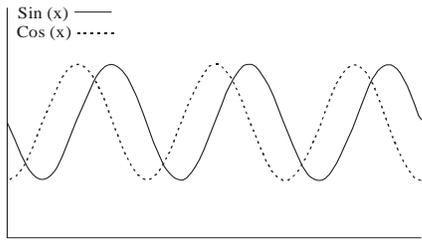


Fig. 1: Sine wave shifted in phase by 90°

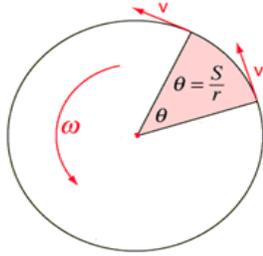


Fig. 2: Angular frequency

spectrum of the signal, by analogy with the spectrum of light

- The frequency spectrum is the plot showing each of the harmonic amplitudes against Frequency. These spectra are mirror images of each other; one of them (negative spectrum) can be ignored if the amplitude of the other one is doubled (Peter, 1980).
- **Amplitude spectrum:** The plot of amplitude C_n versus the (radian) frequency ω (use the Function

$$C_n = \sqrt{a_n^2 + b_n^2} \quad (n = 0, 1, 2, 3, \dots)$$

- Attenuation is the gradual loss in intensity of any kind of flux through a medium. For instance, sunlight is attenuated by dark glasses, X-rays are attenuated by lead, and light and sound are attenuated by water.

Definition and relation:

- A cosine wave is just a sine wave shifted in phase by 90° ($\theta = 90^\circ$) (Fig. 1)
- Angular Speed or Angular Frequency = $\omega; \omega = \frac{2\pi}{T}$ (Fig. 2) $\theta = 2\pi, \therefore \omega = \theta/t \Rightarrow \theta = \omega t$
- We have, $a \cos \theta \pm b \sin \theta \approx R \sin(\theta \pm \alpha)$

The above relationships are obtained from the trigonometric identity. Where, Phase $\alpha = \tan^{-1}(a/b)$ and $R = \sqrt{a^2 + b^2}$

Mathematical notation: Mathematically, a periodic signal may be represented by a Fourier series as follows:

$$f(t) = A_0 + A_1 \sin(\omega t + \phi_1) + A_2 \sin(2\omega t + \phi_2) + \dots + A_n \sin(n\omega t + \phi_n) + \dots \quad (1)$$

The term $A_1 \sin(\omega t + \phi_1)$ is called the first harmonic or the fundamental mode, and it has the same frequency ω as the parent function $f(t)$. The term $A_n \sin(n\omega t + \phi_n)$ is called the nth harmonic and it has frequency $n\omega$, which is n times that of the fundamental. A_n Denotes the amplitude of the nth harmonic and ϕ_n is its phase angle, measuring the lag or lead of the nth harmonic with reference to a pure sine wave of the same frequency (GLYN, 1999):

$$\begin{aligned} A_n \sin(n\omega t + \phi_n) &\equiv A_n [\sin n\omega t \cos \phi_n + \cos n\omega t \sin \phi_n] \\ A_n \sin(n\omega t + \phi_n) &\equiv (A_n \cos \phi_n) \sin n\omega t + (A_n \sin \phi_n) \cos n\omega t \\ A_n \sin(n\omega t + \phi_n) &\equiv (A_n \sin \phi_n) \cos n\omega t + (A_n \cos \phi_n) \sin n\omega t \\ A_n \sin(n\omega t + \phi_n) &\equiv a_n \cos n\omega t + b_n \sin n\omega t \end{aligned}$$

where, $a_n = A_n \sin \phi_n$ and $b_n = A_n \cos \phi_n$

The expansion (1) may be written as:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t) + \dots \quad (2)$$

$a_0/2$ The average (DC) value of the signal:

$$a_0 = \frac{1}{\pi} \int_{-x}^x f(t) dt$$

n The harmonic number: 1 = fundamental, 2 = 2nd harmonic, etc a_n Peak value of the magnitude of the the nth cosine harmonic, b_n Peak value of the magnitude of the nth sine harmonic, ω_0 Fundamental frequency, $\omega_0 = 2\pi/T$, T Period of, $f(t)$ The cosine and sine waves occur at multiples of the fundamental angular frequency $\omega_0 = 2\pi/T = 2\pi f_0$ [$f_0 = 1/T$]. The terms a_n and b_n determine the magnitude of each harmonic. These are the quantities that show up as vertical lines on a spectrum plot

where, amplitude and phase $C_n = \sqrt{a_n^2 + b_n^2}$

$$\phi = \tan^{-1}\left(\frac{a_n}{b_n}\right)$$

MATERIALS AND METHODS

Convolution is a mathematical operator which computes the point wise overlap between two functions Note that convolution in the time domain is equivalent to multiplication in the frequency domain (and vice versa) Convolution is basic operations that we will perform to extract information from images.

Convolving two waveforms in the time domain means that we are multiplying their spectra (i.e. frequency content) in the frequency domain. By "multiplying" the

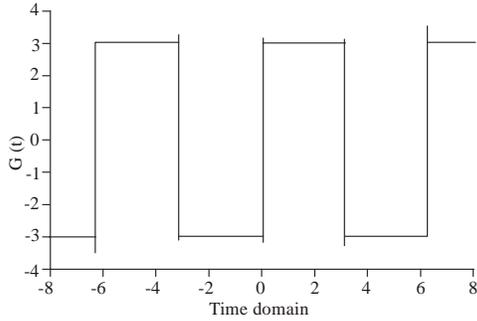


Fig. 3: A complex wave of g(t)

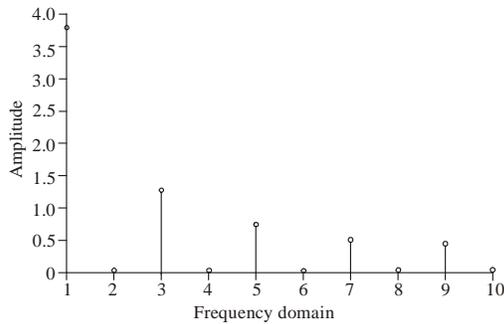


Fig. 4: Amplitude spectrum of g(t)

spectra we mean that any frequency that is strong in both signals will be very strong in the convolved signal, and conversely any frequency that is weak in either input signal will be weak in the output signal (Curtis Roads, 1996). In practice, a relatively simple application of convolution is where we have the "impulse response" of a space. However; the output duration of a convolved signal is the sum of the durations of the two inputs. Some basic principles and the means for performing the analysis with MATLAB have been considered in this study.

Experiment: • Let a periodic function g(t) of period 2π is defined by (Fig. 3, 4):

$$g(t) = \begin{cases} -3; & -\pi \leq t < 0 \\ 3; & 0 \leq t < \pi \end{cases} \quad g(t+2\pi) = g(t)$$

The function $g(t) = \frac{6}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} (1 - \cos n\pi) \sin nt$ represent a Fourier series for the above function, $a_0/2 = 0$, $a_n = 0$, $b_n = 6/n\pi (1 - \cos n\pi)$

$$\therefore c_n = \sqrt{a_n^2 + b_n^2} = \sqrt{0 + b_n^2} = b_n = 6/n\pi (1 - \cos n\pi)$$

$\theta = \omega t = nt$, $\therefore \omega = n$ that angular frequency are same of cosine and sine signal.

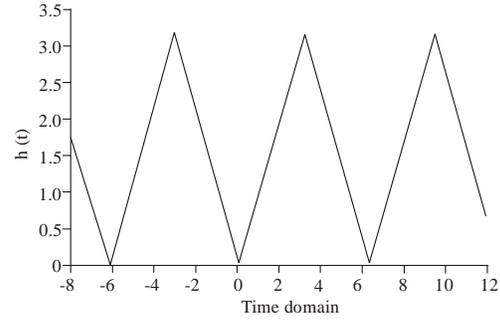


Fig. 5: A complex wave of h(t)

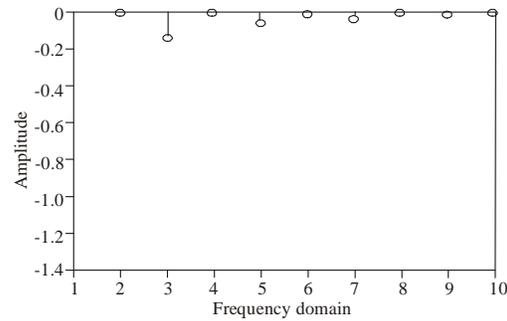


Fig. 6: Amplitude spectrum of h(t)

It has frequency components at frequencies

$$\omega_n = n = n.1 = n\omega_0 \quad (n = 1, 2, 3, \dots)$$

where, ω_0 is the frequency of the parent function g(t).

The amplitude C_n is drawn with respect to angular frequency $\omega = n$.

- Again, Let a periodic function h(t) of period 2π is defined by (Fig. 5, 6):

$$h(t) = \begin{cases} -t; & -\pi \leq t < 0 \\ t; & 0 \leq t < \pi \end{cases} \quad h(t+2\pi) = h(t)$$

The function represent a $h(t) = \frac{\pi}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n^2} (\cos n\pi - 1) \cos nt$

Fourier series for the above function, $a_0/2 = \pi/2$, $a_n = 2/\pi n^2 (\cos n\pi - 1)$, $b_n = 0$

$$\therefore c_n = \sqrt{a_n^2 + b_n^2} = \sqrt{a_n^2 + 0} = a_n = \frac{2}{\pi n^2} (\cos n\pi - 1)$$

$\theta = \omega t = nt$, $\therefore \omega = n$ that angular frequency are same of cosine and sine signal.

It has frequency components at frequencies

$$\omega_n = n = n.1 = n\omega_0 \quad (n = 1, 2, 3, \dots)$$

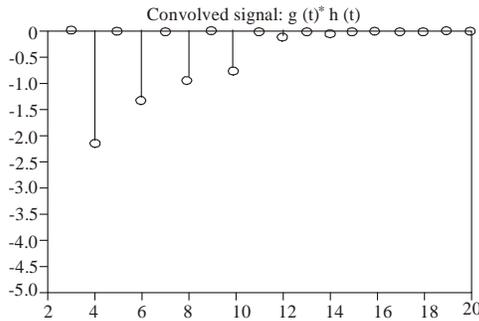


Fig. 7: Convolved signal

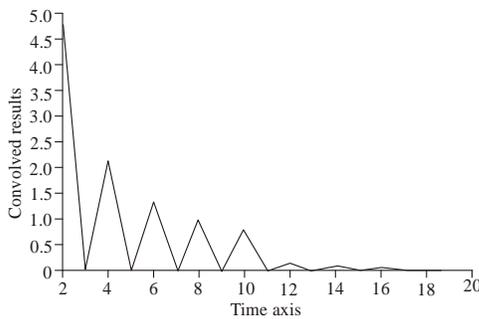


Fig. 8: Result of convolved signal shows losing local information of the spectrum

were ω_0 is the frequency of the parent function $h(t)$, The amplitude C_n is drawn with respect to angular frequency $\omega = n$.

RESULTS AND DISCUSSION

Conventional methods used to determine this entail the use of spectrum analyzers which use either sweep generators or variable-frequency oscillators to impress upon a network all possible frequencies of equal amplitude and equal phase (Fig. 7, 8). In fact, one can convolve any sound with another, not just an impulse response. In that case, we are "filtering" the first sound through the spectrum of the second, such that any

frequencies the two sounds have in common will be emphasized. A particular case is where we convolve the sound with itself, thereby guaranteeing maximum correlation between the two sources. In this case, prominent frequencies will be exaggerated and frequencies with little energy will be attenuated. This example showed how the response could be determined using the convolution. The spectrum after convolution of two complex waves shows the losing local information of the spectrum

CONCLUSION

This study attempted to implement the convolution between two complex waves in frequency domain. These examples showed how the response occurred of amplitude spectrum of complex waves by convolution. The spectrum after convolution of Fourier series (complex waves) shows the losing local information of the spectrum. To do research convolution between complex waves having different angular frequency is our future work.

REFERENCES

- Glyn, J., 1999. Advanced Modern Engineering Mathematics. 2nd Edn., pp: 281, 283, 338.
- Curtis Roads, 1996. The Computer Music Tutorial. MIT Press.
- Leon, C., 1989. Time-frequency distributions: A review. Proc. IEEE, 77(7): 941-981.
- Leon, C. and L Patrick, 1998. Recent Development in Time-Frequency Analysis. Kluwer Academic Publishers. Old Good Books, Waterloo, ON, Canada Hingham, MA, USA.
- Okan, K.E., 1994. A comparative review of real and complex Fourier-related transforms. Proc. IEEE, 82(3): 429-447.
- Peter, H., 1980. Audio Spectrum Analyse. Wireless World Magazine: An Audio-Spectrum Analyser that is Suitable for Construction by Hobbyists, pp: 55-60.