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Heat and Mass Transfer of an MHD Free Convection Flow along a Stretching Sheet with Chemical Reaction, Radiation and Heat Generation in Presence of Magnetic Field

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Abstract: The present study comprises of steady two dimensional magnetohydrodynamic heats and mass transfer free convection flow along a vertical stretching sheet in the presence of magnetic field. The problem has been analyzed by applying Nachtsheim-Swigert shooting iteration technique with sixth order Runge-Kutta integration scheme. The nonlinear partial differential equations governing the flow fields occurring in the problems have been transformed to dimensionless nonlinear ordinary differential equations by introducing suitably selected similarity variables. The ensuing equations are simultaneously solved by applying numerical iteration scheme for velocity, temperature and concentration. The results are displayed graphically in the form of velocity, temperature and concentration profiles. The corresponding skin-friction coefficient, Nusselt number and Sherwood number are displayed in tabular form as well. The effects of several important parameters on the velocity, temperature and concentration profiles are investigated.

Keywords: Electric conductivity, molecular diffusivity and schmidt number, radiation parameter

INTRODUCTION

Transport processes occur either naturally or artificially and in both cases the differences of density made by temperature, chemical composition differences and gradients and material or phase constitution define or drive the flow. In recent years the boundary layer flow and heat transfer over a continuously stretched surface have drawn a considerable attention. For example, materials manufactured by extrusion processes and heat-treated materials traveling between a feed roll and a wind-up roll or on conveyor belt possess the characteristics of a continuous moving surface. The hydro-magnetic flow and heat transfer problems have become important industrially. To be more specific, it may be pointed out that many metallurgical processes involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid and that in the process of drawing, these strips are sometimes stretched. In all the cases the properties of the final product depend on the rate of cooling to a great extent. By drawing such strips in an electrically conducting fluid subjected to a magnetic field, the rate of cooling can be controlled and a final product of desired characteristics can be achieved. Finally it can be said that the study of boundary layer flow and heat transfer over a continuously stretched surface is very important because of its various possible applications in countless places such as hot rolling, wire drawing and plastic extrusion, continuous casting, glass fiber production, crystal growing, paper production and many other places that are important for our industrial developments and financial developments as well. Boundary layer flow over a continuously stretched surface was first studied by Sakiadis (1961a, b). He found a numerical solution by employing similarity transformation to the problem. Then the problem of mass transfer at the stretched surface came into light and it was solved by Erickson *et al*. (1966) who solved it extending the study of Sakiadis (1961a) combined analytical and experimental study of the flow and temperature fields in the boundary layer on a continuous stretching sheet was presented by Tsou *et al*. (1967). Chen and Char (1980) investigated the effects of variable surface temperature and variable surface heat flux on the heat transfer characteristics of a linearly stretching sheet subject to blowing or suction. Elbashbeshy (1997) studied heat transfer with variable and uniform surface heat flux subject to injection and suction. Crane (1970) investigated the boundary layer flow defined by a stretching sheet whose velocity varies linearly with a distance from a fixed point of surface. Gupta and Gupta (1977) studied the momentum, heat and mass transfer in the boundary layer with suction and blowing. The constant surface case with a power law temperature variation was investigated by Soundalgekar and Ramana (1980). In geophysics, astrophysics, engineering applications and other industrial areas the Magneto-Hydrodynamics (MHD) is encountered. There exists a significant importance of

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MHD in stellar and planetary magnetospheres, aeronautics. Chakrabarti and Gupta (1979) and recently Ishak *et al*. (2008) investigated MHD flow heat and mass transfer over a stretching sheet and Kumar *et al*. (2002) studied MHD flow and heat transfer on a continuously moving vertical plate. Heat generation or absorption problems are other important questions that have attracted the scientists. Vajravelu and Hadjinicalau (1997) have investigated the heat transfer characteristics in the laminar boundary layer of a viscous fluid over a stretching sheet. Chamka (1999) have studied hydro-magnetic three dimensional convective heat transfers from a stretching surface with heat generation or absorption. Recently Alam *et al*. (2009) have investigated steady MHD free and forced convection with large suction and constant heat and mass transfer and Rahman *et al*. (2009) investigated numerically MHD forced convection flow with variable viscosity. By conduction, convection and radiation heat can be transferred. The effect of radiation on MHD flow and heat transfer has now become an important fact industrially. At high operating temperature, the effect of radiation could be quite significant. Many tasks in engineering level are operated through high temperature and hence the effect of radiation is very significant and cannot be neglected. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. Takhar *et al*. (1996) studied the radiation effects on MHD free convection floe of a gas past a semi-infinite vertical plate. The study of radiation effect on forced convection flow over a stretching surface was accomplished by Elbashbeshy (2000). The radiation effect on hydro-magnetic convective heat transfer over a stretching sheet was considered and investigated by Ghaly (2002). Raptis and Massalas (1998) investigated the effect of radiation on the unsteady MHD flow of an electrically conducting viscous fluid past a plate. Using Rosseland diffusion approximation on mixed convection along a vertical plate with uniform free stream velocity and surface temperature (Hossain and Takhar, 1996) determined the radiation effect. Samad and Karim (2009) studied thermal radiation interaction with unsteady MHD free convection flow through a vertical flat plate with time dependent suction in the presence of magnetic field. El-Aziz (2009) analyzed the effect of radiation on the heat and fluid flow over an unsteady stretching surface. The investigation of magneto-hydrodynamic heat transfer over non-isothermal stretching sheet was accomplished by Chiam (1997) and the study of heat transfer in a fluid with variable thermal conductivity over stretching sheet was also done by Chiam (1998). Mahapatra and Gupta (2002) experimented the heat transfer in stagnation point towards a stretching sheet. Khan *et al*. (2003) investigated and studied viscoelastic MHD flow, heat and mass transfer over a porous stretching sheet with dissipation energy and stress work. Seddeek and Salem (2005) investigated and computed laminar mixed convection adjacent to a vertical continuously stretching sheet with variable viscosity and variable thermal diffusivity. The study of steady MHD through horizontal channel whose lower plate is a stretching sheet and the upper plate is a permeable plate bounded by porous medium was investigated by Sharma and Mishra (2001). Sharma and Singh (2009) studied the steady flow and heat transfer of a viscous incompressible fluid flow through porous medium over a stretching surface. With similarity method (Ostrach, 1953; Sparrow and Gregg, 1956) the problem of natural convection along a vertical isothermal or uniform flux plate has been solved. The viscous dissipation term was first taken into account by Gebhart (1962). Recently Pantokratoras (2005) studied the effect of viscous dissipation in natural convection in a new and modern way. Abo-EIdahab (2005) investigated flow and heat transfer in a micro polar fluid past a stretching surface embedded in a non-Darcian porous medium with uniform free stream.

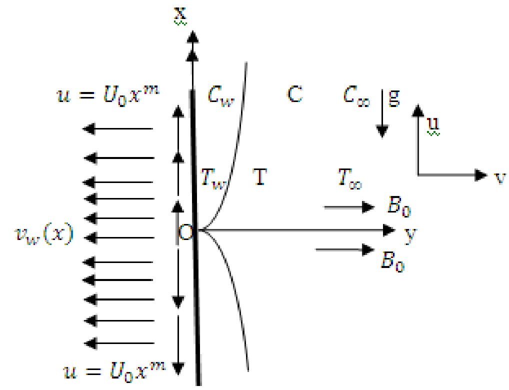

Fig. 1: Flow model with coordinate system

Mathematical formulation: We consider a steady two-dimensional magneto hydrodynamic heat and mass transfer flow of a various incompressible fluid along a vertical stretching sheet with constant heat generation absorption with radiation. We take the X-axis along the sheet and Y-axis is normal to it. Two equal and opposite forces are introduced along the X-axis so that the sheet is stretched keeping the origin fixed. A uniformed magnetic field of strength B_0 is imposed along the Y-axis. A radiation depending on the temperature is applied on the stretching sheet. The physical configuration considered here is shown in the following Fig. 1.

To formulated the problem, at first the two dimensional continuity equation will be introduced. On the momentum boundary layer equation the left hand side consists of the acceleration terms. The right hand side consists of several terms. The first term indicates the viscous term, the second term indicates the buoyancy term and the third term indicates the presence of magnetic field. On the energy equation, in addition with the usual terms a new term corresponding to radiation will have to be introduced in the right hand side referring to the radiative heat flux gradient. The corresponding equation consists of the usual terms.

Governing equations:

The governing equations representing the proposed flow field are:

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equations:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}+sg\beta[T-T_\infty]-\frac{\sigma {B_0}^2}{\rho}u \tag{2}$$

Energy equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho c_p}(T-T_\infty)-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{3}$$

Concentration equation:

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2} \tag{4}$$

where, (u, v) are the velocity components along x and y directions respectively, v is the kinematic viscosity of the fluid, g is the acceleration due to gravity, β is the volumetric co efficient of thermal expansion, B_0 is the uniform magnetic field strength(magnetic induction), T and $T_\infty$are the fluid temperature within the boundary layer and in the free stream respectively, C is the concentration of the fluid within the boundary layer, ρ is the density of the fluid, C_p is the specific heat at constant pressure, α is the electrical conductivity, $Q_0$is the volumetric rate of heat generation D_m is the Coefficient of mass diffusivity, $q_r$is the rate heat transfer and s is a dummy parameter stands for 0 for forced convection, +1 for heating problem and -1 for cooling problem.

Boundary conditions:

$$u=U_0x^m, v=v_w, T=T_w, C=C_w \ at \ y=0$$

$$u=0, T-T_\infty=T_w-T_\infty=ax^n, C-C_\infty=C_w-C_\infty=bx^l \ \ at \ y\to\infty$$

Here $v_w$is a velocity component at the wall having positive value to indicate suction. T_w is the uniform wall temperature and C_w, C_∞ are the concentration of the fluid at the sheet and far from the sheet respectively. The effect from second term on the right hand side of the Eq. (2) is due to buoyancy force and the governing Eq. (1-4) represent free convection flow when we take $s=1$. The free convection flow where the buoyancy effect is dominant.

Similarity analysis: To transform Eq. (2) to (4) into a set of ordinary differential equations the following dimensionless variables are introduced:

$$u=U_0x^m f'(\eta) \tag{5}$$

$$\eta(x,y)=y\sqrt{\frac{m+1}{2}}\sqrt{\frac{U_0x^m}{\nu x}}=y\sqrt{\frac{m+1}{2}}\sqrt{\frac{U_0x^{m-1}}{\nu}} \tag{6}$$

$$T-T_\infty=ax^n\theta(\eta) \tag{7}$$

$$C-C_\infty=bx^l\varphi(\eta) \tag{8}$$

where,

η = The dimensionless distance normal to the sheet
f' = The dimensionless primary velocity
θ = The dimensionless fluid temperature
φ = The dimensionless concentration

Finally we get the following local similarity equations:

$$f'''+ff''-\frac{2m}{m+1}f'^2+\frac{2\lambda}{m+1}\theta-\frac{2M}{m+1}f'=0 \tag{9}$$

$$\theta''+\frac{3NPr}{3N+4}f\theta'+\frac{6NPr}{(m+1)(3N+4)}(Q-nf')\theta=0 \tag{10}$$

$$\varphi''+Scf\varphi'-\frac{2l}{m+1}Scf'\varphi=0 \tag{11}$$

The transformed boundary conditions are:

$$\left.\begin{array}{c} f=f_w, f'=1, \theta'=1, \varphi'=1 \text{ at } \eta=0 \\ f'=0, \theta=0, \varphi=0 \text{ at } \eta\to\infty \end{array}\right\} \tag{12}$$

where, a prime denotes differentiation with respect to η and $fw=-v_w\sqrt{\frac{2}{m+1}}\sqrt{\frac{x^{1-m}}{\nu U_0}}$ is the suction parameter.

The dimensionless parameters appeared into the above equations are defined as follows:

$Q=\frac{Q_0}{\rho c_p U_0}\frac{1}{x^{m-1}}$ = The Local rotation parameter
$M=\frac{\sigma {\beta_0}^2}{\rho}\frac{x^{1-m}}{U_0}$ = The local magnetic parameter
$N=\frac{\kappa\kappa_1}{4\sigma_1 T_\infty^3}$ = The radiation parameter and
$P_r=\frac{\mu c_p}{\kappa}$ = The Prandtl number
$Sc=\frac{\nu}{D_m}$ = The Schmidt number

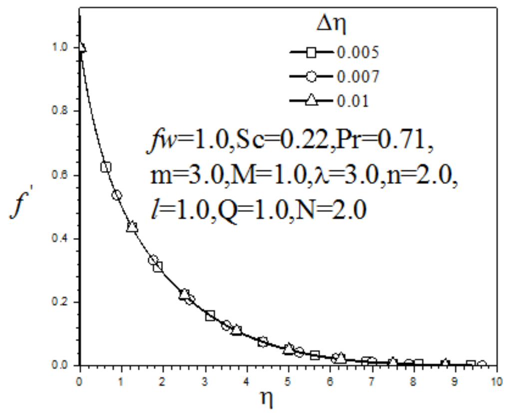

Fig. 2a: velocity profiles for various values of η

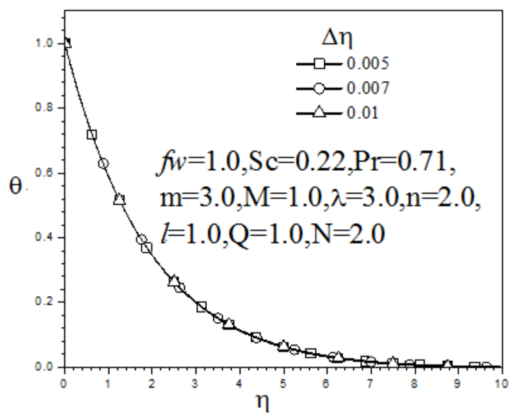

Fig. 2b: Temperature profiles for various values of η

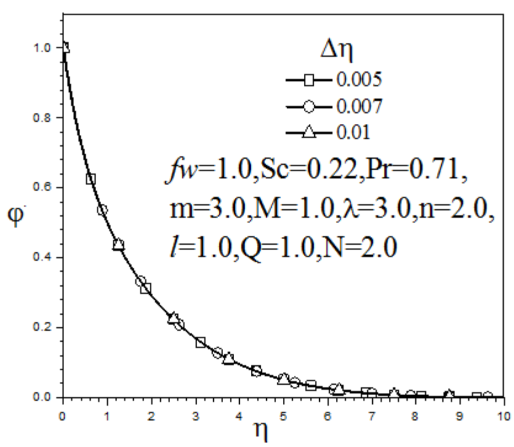

Fig. 2c: Concentration profiles for various values of η

The nonlinear differential Eq. (9-11) under the boundary condition (12) is solved numerically for various values of the parameters occurring in the problem.

Numerical computations: By applying a shooting method called Nachtsheim and Swigert (1965) iteration technique along with sixth order Runge-Kutta-Butcher iteration scheme the numerical computation of the nonlinear differential Eq. (9)-(11) under the boundary conditions (12) have been performed. Here the step size is taken as $\Delta\eta = 0.01$ to satisfy the convergence criterion of 10^{-6} in all cases. The value of η_∞ was found to each iteration loop by $\eta_\infty = \eta_\infty + \Delta\eta$. The maximum value of η_∞ to each group of parameters Pr, fw, M, m, n, l, Sc, Q, λ and N is determined when the value of unknown boundary conditions at $\eta = 0$ change to successful loop with error less than 10^{-6}.

In order to verify the effects of the step size $\Delta\eta$, the programming code was run for three different step sizes as $\Delta\eta = 0.005$, $\Delta\eta = 0.007$, $\Delta\eta = 0.01$ and an excellent agreement was found, which is shown in the Fig. 2. Figure 2 shows the velocity, temperature and concentration profiles for different step sizes.

Here we see that for step sizes, $\Delta\eta = 0.01, 0.007, 0.005$ the velocity profiles, temperature profiles and the concentration profiles are good agreement among them.

RESULTS AND DISCUSSION

We have shown the dimensionless velocity, temperature and concentration profiles to present the result s obtained in the numerical computations have been carried out for various values of parameters entering into the problem in compliance with the different physical conditions. These parameters are, the Prandtl number (Pr), suction parameter (f_w), magnetic field parameter (M), radiation parameter (N), Schmidt number (Sc), heat source parameter (Q), velocity index (m), temperature index (n) and concentration index (l).

The effect of Prandtl number (Pr) on velocity, temperature and concentration profiles is shown in the above illustrated Fig. 3. In the Fig. 3a and b we observe that the velocity and temperature profiles are decreasing with the increase of Prandtl number (Pr) and from the Fig. 3c it is clear that the concentration profile is increasing with the increase of Prandtl number (Pr). The velocity gradient is decreasing from positive to negative as we increase Prandtl number (Pr). At the value Pr = 0.9 (around) the velocity gradient is zero. For the value Pr = 3.0,7.0, there is a rise on the velocity boundary layer near the stretching sheet or in other words we can say that there exists a rise on the velocity boundary layer near the stretching sheet for Pr$\geq$3.0. On the other hand there exists no rise for any values of Pr in the temperature and concentration boundary layers. Physically Pr = 0.71, 1.0, 7.0 correspond to air at 20°C, to electrolyte solution such as salt water and to water respectively.

We can observe the effect of magnetic parameter (M) on the dimensionless velocity, temperature and concentration distribution from Fig. 4. The velocity profiles decrease whereas temperature and concentration profiles increase as magnetic parameter

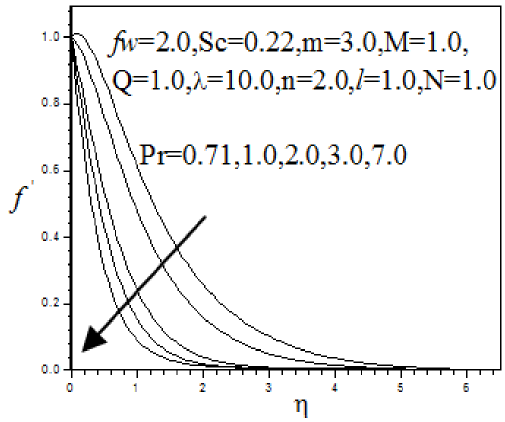

Fig. 3a: Velocity profiles for various value of prandtl number, Pr

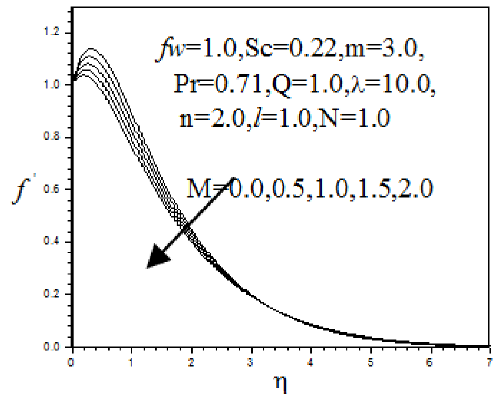

Fig. 4a: Velocity profiles for various values of magnetic number, M

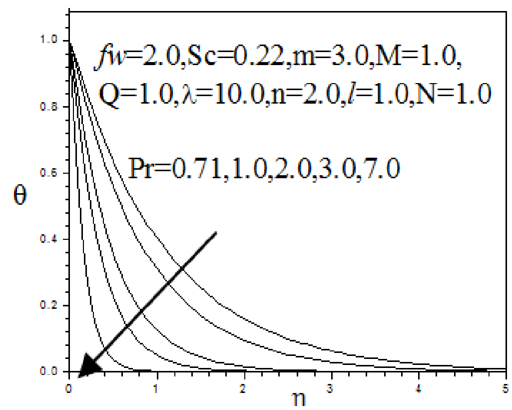

Fig. 3b: Temperature profiles for various value of prandtl number, Pr

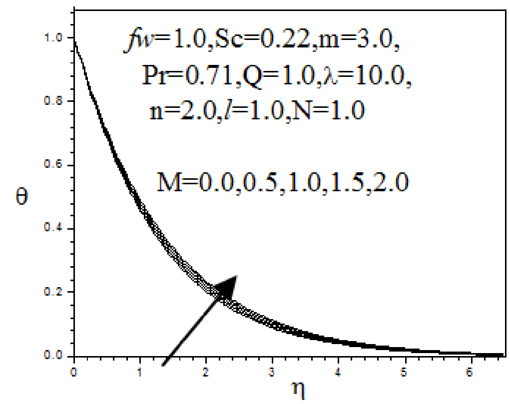

Fig. 4b: Temperature profiles for various values of magnetic number, M

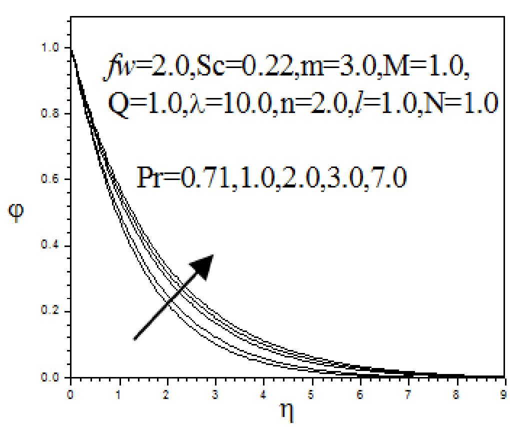

Fig. 3c: Concentration profiles for various value of prandtl number, Pr

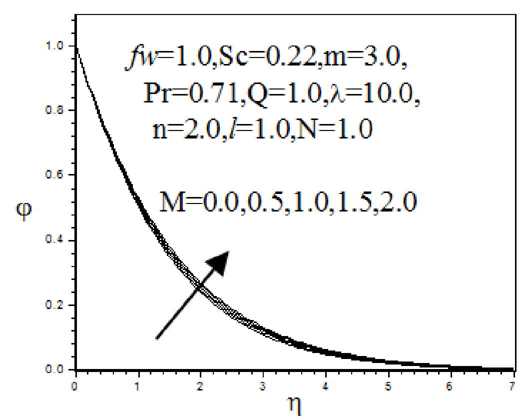

Fig. 4c: Concentration profiles for various values of magnetic number, M

M increases. The velocity gradient is positive near the surface, which indicates that the velocity of stretching sheet is smaller than that of the adjacent fluid. However from Fig. 4 it can be seen that there is a sharp rise in velocity profiles near the surface. Since magnetic field lines act as a string to retard the motion of fluid, the rate of heat and mass transfer increase.

The effect of radiation parameter (N) on the dimensionless velocity, temperature and concentration profiles is shown in Fig. 5. The velocity and temperature decrease with the increment of radiation

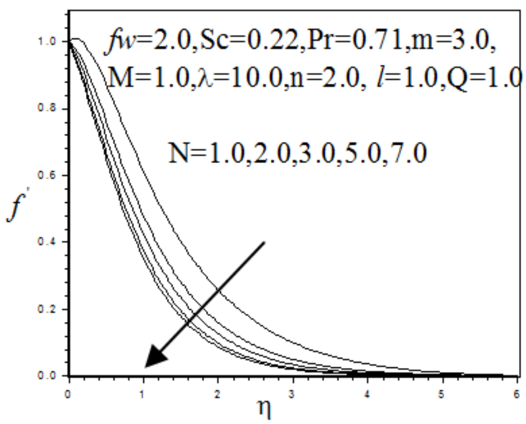

Fig. 5a: Velocity profiles for various values of radiation parameter, N

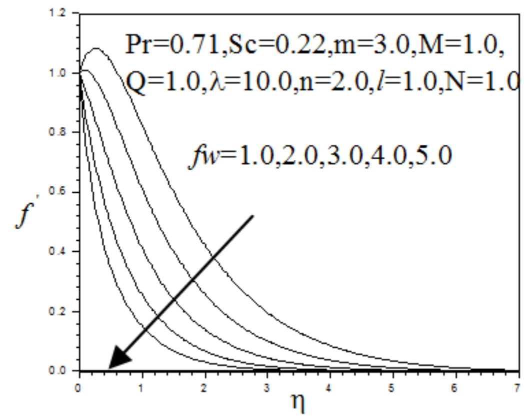

Fig. 6a: Velocity profiles for various values of suction parameter, *fw*

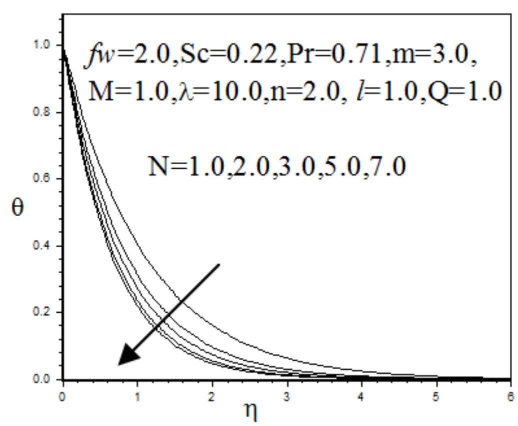

Fig. 5b: Temperature profiles for various values of radiation parameter, N

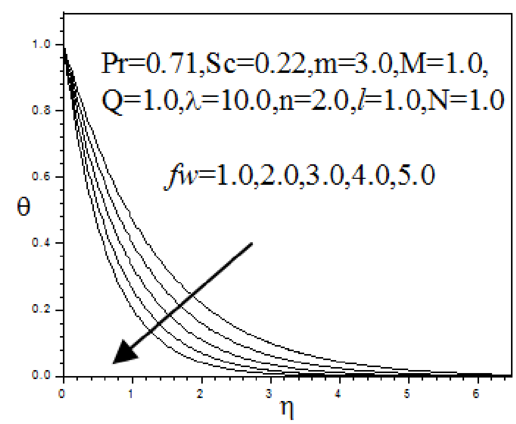

Fig. 6b: Temperature profiles for various values of suction parameter, *fw*

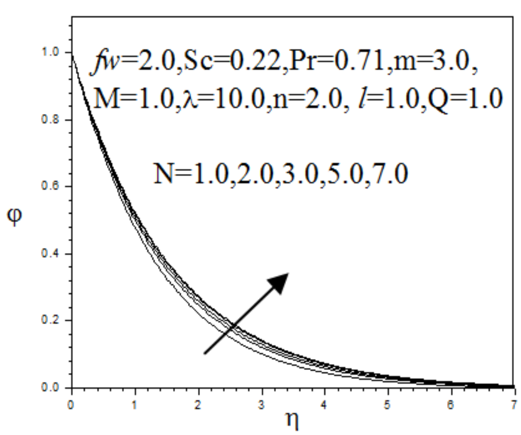

Fig. 5c: Concentration profiles for various values of radiation parameter, N

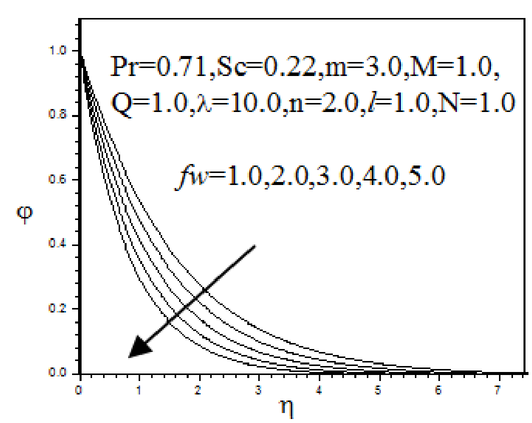

Fig. 6c: Concentration profiles for various values of suction parameter, *fw*

parameter (N). The velocity profiles rise near the surface for $N \leq 5.0$. Since there exist a sharp decrease in velocity and temperature profiles, radiation can be used to control velocity and temperature boundary layers. At N = 1.75 (around) the velocity gradient is zero. Figure 6 shows that the concentration profiles increase as the values of radiation number increase

The graphical representations of the effect of suction parameter (*fw)* on velocity, temperature and

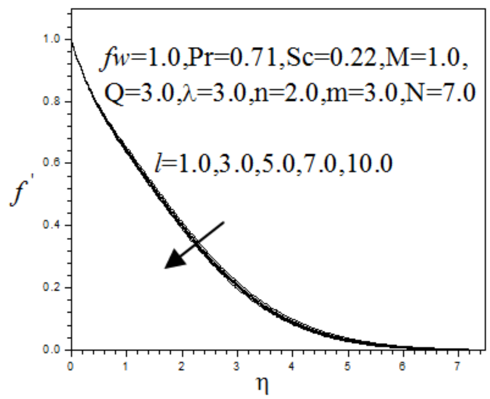

Fig. 7a: Velocity profiles for various values of concentration index, *l*

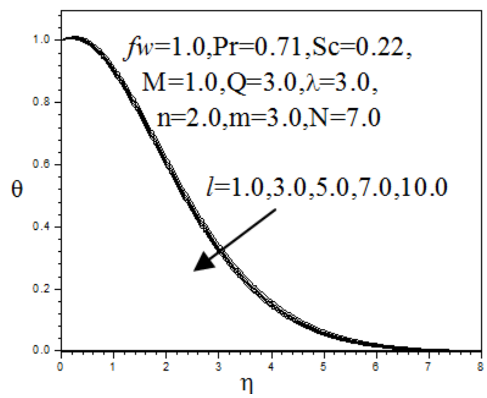

Fig. 7b: Temperature profiles for various values of concentration index, *l*

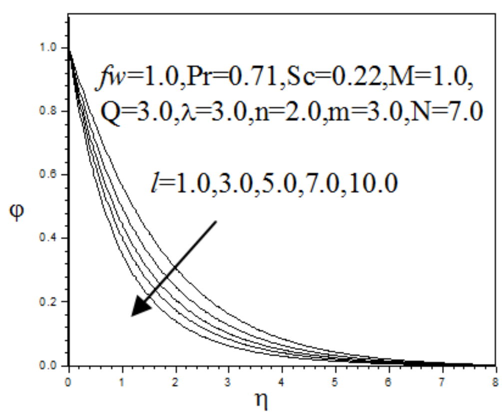

Fig. 7c: Concentration profiles for various values of concentration index, *l*

concentration profiles are illustrated in the Fig. 6. It can be seen that the velocity, temperature and concentration distributions decrease with the increment of the value of suction parameter (*fw*). Due to high buoyancy there is a sharp rise for the value $fw \leq 3.0$. The velocity profiles decrease with the increment of suction parameter (*fw*)

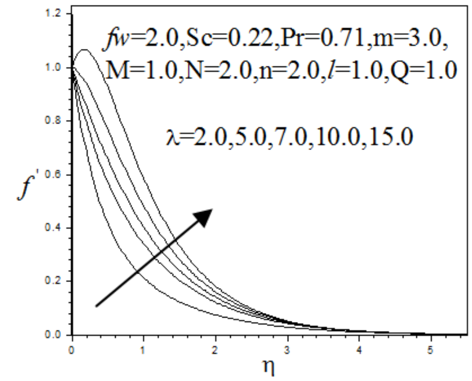

Fig. 8a: Velocity profiles for various values of buoyancy parameter, λ

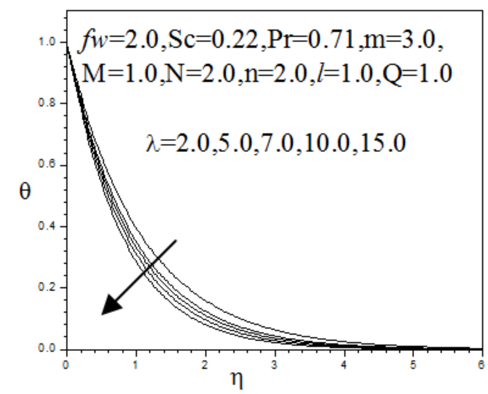

Fig. 8b: Temperature profiles for various values of buoyancy parameter, λ

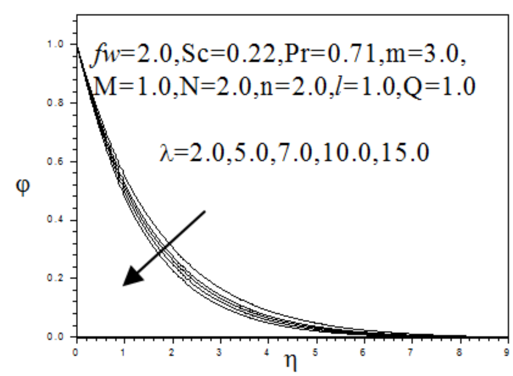

Fig. 8c: Concentration profiles for various values of buoyancy parameter, λ

expressing the fact that suction stabilizes the boundary layer. We find the value of velocity gradient zero at the point near to $fw = 2.36$. The reduction in the temperature and concentration profiles also indicates that suction stabilizes the temperature and concentration boundary layers.

Figure 7a describes the outcome of concentration index (*l*) on the dimensionless velocity, temperature and

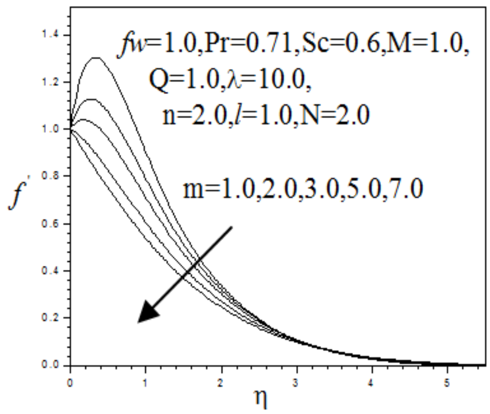

Fig. 9a: Velocity profiles for various values of velocity index, m

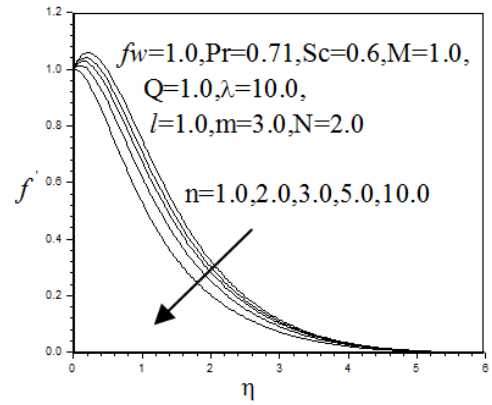

Fig. 10a: Velocity profiles for various values of temperature index, n

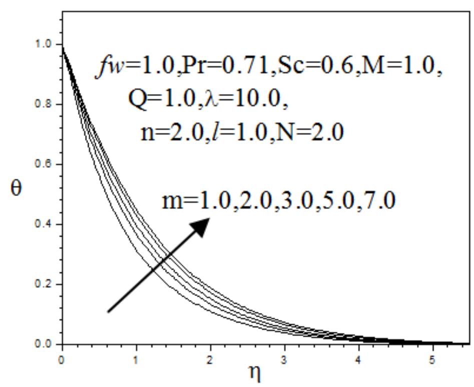

Fig. 9b: Temperature profiles for various values of velocity index, m

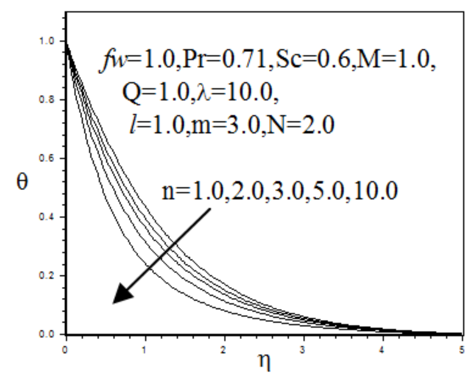

Fig. 10b: Temperature profiles for various values of temperature index, n

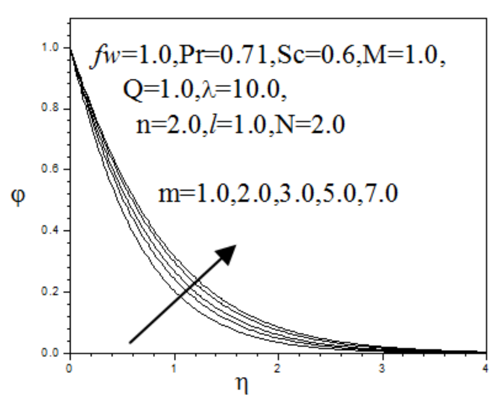

Fig. 9c: Concentration profiles for various values of velocity index, m

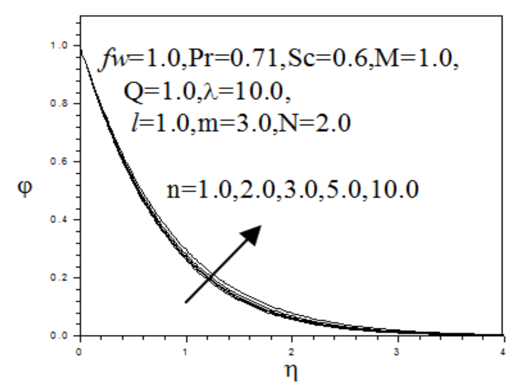

Fig. 10c: Concentration profiles for various values of temperature index, n

concentration profiles. The values of velocity, temperature and concentration decrease as we increase the values of concentration index (l). There is a rise in temperature profiles near the surface. From Fig. 7b and c it can be seen that there exists a little effect of concentration index on temperature and concentration profiles.

Figure 8 portrays the result of buoyancy parameter (λ) on velocity, temperature and concentration distributions. With the growth of buoyancy parameter (λ) the velocity profiles increase and the temperature and concentration profiles monotonically decrease. There exists sharp rises in the velocity profiles near the surface. The negativity of velocity gradient at the surface indicates that the stretching sheet velocity is greater than adjacent fluid velocity and while the velocity gradient increases from negative to positive, the velocity gradient diminishes near at λ = 10.43. The temperature gradient at the surface positive means that heat is transferred from the ambient medium to the sheet.

Figure 9 depicts the velocity, temperature and concentration profiles for different value of velocity index (m). With the rise of velocity index (m) the profiles of velocity decrease and the profiles of temperature and concentration increase. From Fig. 9a it is very apparent that there are some overshooting near the surface for m≤7.0. The velocity gradient changes from positive to negative near the surface. The positivity of velocity gradient near the surface means that the velocity of adjacent fluid is smaller than the velocity of stretching sheet. At m = 4.5 velocity gradient is zero. So velocity index (m) is an important parameter to control the velocity boundary layer

Figure 10 portrays the result of temperature index (n) on velocity, temperature and concentration distributions. With the growth of temperature index (n) the concentration profiles increase and the velocity and temperature profiles decrease. In Fig. 10a we can see there exists rises in the velocity profiles near the surface. The positivity of velocity and temperature gradient mean the stretching sheet velocity is lower than the fluid velocity and the stretching sheet is gaining temperatures respectively.

The profiles of velocity, temperature and concentration for different values of heat source parameter (Q) are shown in Fig. 11. It is observed that the dimensionless velocity increases but the concentration decreases uniformly as the heat source parameter (Q) increases. It is also observed that as Q increases the velocity and temperature increases rapidly and overshoot. The temperature gradient at the surface is decreasing from positive to negative, which means that the temperature is transferred from the fluid to the sheet. But as Q increases the transfer rate decreases and at Q = 4.5 (around) the temperature gradient is zero and the sheet starts to transfer heat to the environment.

Figure 12 shows the influence of Schmidt number (Sc) on dimensionless velocity, temperature and concentration distributions. We see that the velocity, temperature decrease with the rise of Schmidt number (Sc) till Sc = 0.60, which is for water vapor. When Sc gets large value exceeding 0.60 the velocity and temperature profiles remain same. However, there is a sharp rise in the velocity boundary layers near the stretching sheet. From Fig. 12c it can be seen that Schmidt number has a good effect on concentration profiles.

Skin-friction coefficients, the Nusselt number and the Sherwood number: The skin friction coefficients, (C_f) the Nusselt number (Nu_x) and the

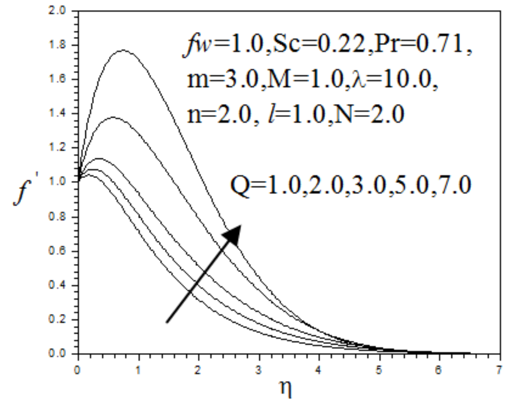

Fig. 11a: Velocity profiles for various values of heat source parameter, Q

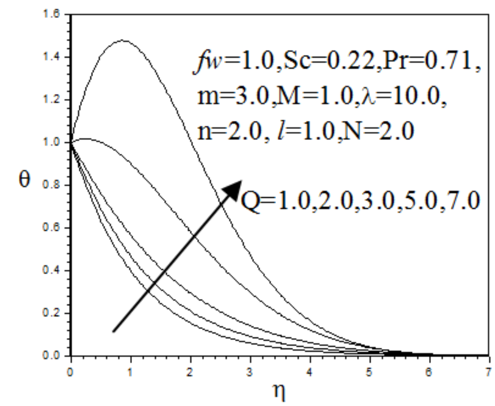

Fig. 11b: Temperature profiles for various values of heat source parameter, Q

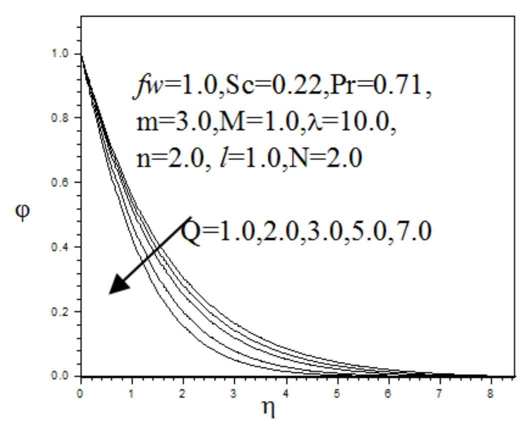

Fig. 11c: Concentration profiles for various values of heat source parameter, Q

Sherwood number (Sh) are significant in the engineering field .These parameters refer to the wall shear stress, local wall heat transfer rate and wall mass transfer rate respectively.

The equation defining skin frictions is:

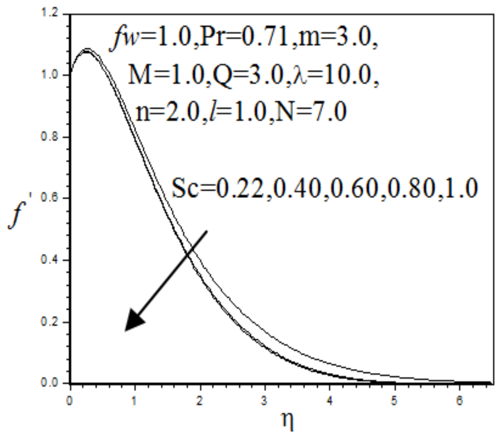

Fig. 12a: Velocity profiles for various values of Schmidt number, Sc

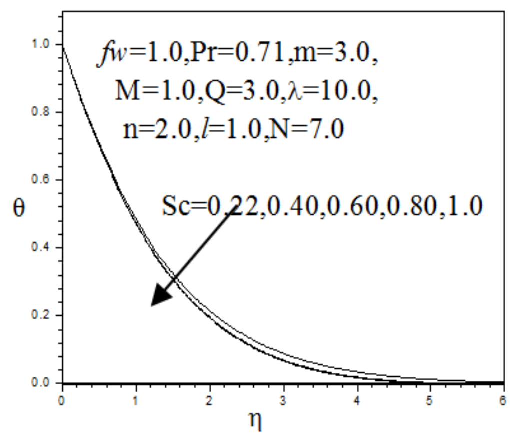

Fig. 12b: Temperature profiles for various values of Schmidt number, Sc

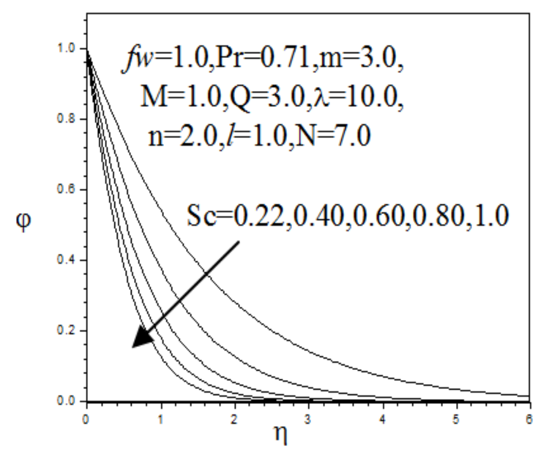

Fig. 12c: Concentration profiles for various values of Schmidt number, Sc

Table 1: C_f , Nu_x and Sh for different values of Pr, M and N

Pr	C_f	Nu_x	Sh	M	C_f	Nu_x	Sh	N	C_f	Nu_x	Sh
0.71	0.2626	0.8843	0.7086	0.0	0.9985	0.7346	0.5964	1.0	0.2626	0.8843	0.7086
1.0	-0.099	1.1422	0.6761	0.5	0.8481	0.721	0.5889	2.0	-0.0923	1.1369	0.6765
2.0	-0.8945	2.025	0.6115	1.0	0.7043	0.7078	0.5817	3.0	-0.2534	1.2718	0.6623
3.0	-1.3244	2.9132	0.585	1.5	0.5668	0.695	0.5747	5.0	-0.4045	1.4136	0.6494
7.0	-1.9778	6.446	0.5591	2.0	0.4352	0.6825	0.5681	7.0	-0.4765	1.4873	0.6435

Table 2: C_f, Nu_x and Sh for different values of fW, Q and Sc

f_W	C_f	Nu_x	Sh	Q	C_f	Nu_x	Sh	Sc	C_f	Nu_x	Sh
1.0	0.7043	0.7078	0.5817	2.0	-1.9979	0.9534	0.5944	0.22	0.7272	0.6262	0.5796
2.0	0.2626	0.8843	0.7086	5.0	-1.2085	1.0453	0.6338	0.40	0.6863	0.64	0.8868
3.0	-0.4925	1.0908	0.8542	7.0	-0.7412	1.0873	0.6529	0.60	0.6806	0.642	1.1889
4.0	-1.5116	1.3276	1.0196	10.0	-0.0923	1.1369	0.6765	0.80	0.6806	0.642	1.4665
5.0	-2.6799	1.5894	1.2025	15.0	0.8984	1.2001	0.7077	1.0	0.6806	0.642	1.7282

Table 3: C_f, Nu_x and Sh for different values of m, n and l

m	C_f	Nu_x	Sh	n	C_f	Nu_x	Sh	l	C_f	Nu_x	Sh
1.0	2.1923	1.1275	1.4134	1.0	0.5955	0.7384	1.1786	1.0	-0.6134	-0.0975	0.5377
2.0	1.0949	0.9574	1.2528	2.0	0.4936	0.8718	1.1667	3.0	-0.6227	-0.0851	0.6935
3.0	0.4936	0.8718	1.1667	3.0	0.4084	0.9877	1.1566	5.0	-0.6281	-0.078	0.8262
5.0	-0.1613	0.7871	1.0748	5.0	0.271	1.1849	1.1398	7.0	-0.6336	-0.0706	0.9425
7.0	-0.5172	0.7454	1.0254	10.0	0.0358	1.566	1.1102	10.0	-0.6396	-0.0627	1.0962

$$C_f = \sqrt{2(m+1)}Re_x^{-\frac{1}{2}}f''(0) \quad (13)$$

The local Nusselt number Nu_x is given by:

$$Nu_x = -\sqrt{\frac{m+1}{2}}\sqrt{Re_x}\,\theta'(0) \quad (14)$$

The local Sherwood number (Sh) is given by:

$$Sh = -\sqrt{\frac{m+1}{2}}\sqrt{Re_x}\,\varphi'(0) \quad (15)$$

Where, $Re_x = \frac{U_0 x^{m+1}}{\nu}$ is the local Reynolds number and $\frac{l}{\sqrt{2(m+l)}}$, $\sqrt{\frac{2}{m+1}}Re_x^{-\frac{1}{2}}$, $\sqrt{\frac{2}{m+1}}Re_x^{-\frac{1}{2}}$ are constant.

It is observed from Eq. (13 to 15) that the Skin-friction coefficient, Nusselt number and the Sherwood number are proportional to $f''(0), -\theta'(0)$ and $-\varphi'(0)$ respectively.

Table 1 to 3 comprising of the proportional valuesof C_f, Nu_x and h.

CONCLUSION

Free convection flow past an infinite or semi-infinite vertical plate is very important to numerous practical applications. The thermal diffusion is neglected in such kind of works and hence there exists not enough information regarding this fact. In the present thesis work an attempt is taken to focus on the above mentioned fact and to accomplish the task the effect of chemical reaction of an electrically conducting viscous incompressible fluid on the flow over a continuously stretching sheet in the presence of heat and mass transfer as well as uniform magnetic field normal to the sheet with heat generation or absorption and the effect of the pertinent parameters on the velocity, temperature and concentration distribution is then thoroughly studied. To be more specific here we have tried to investigate and study the MHD free convection heat and mass transfer along a continuously stretching vertical sheet with heat generation in the presence of radiation effect which has become an important phenomenon nowadays.

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NOMENCLATURE

B_0 : Uniform magnetic field strength
C : Species concentration in the flow field
C_w : Species concentration at the wall
C_∞ : Species concentration in the free stream
C_f : Skin friction coefficient
c_p : Specific heat at constant pressure
D_m : Coefficient of mass diffusivity
f : Dimensionless stream function
fw : Suction parameter
Gr : Grash of number
g : Acceleration due to gravity
κ : Thermal conductivity
κ_1 : Mean absorption coefficient
l : Concentration index
M : Magnetic field parameter
m : Velocity index
N : Radiation parameter
n : Temperature index
Nu_x : Local Nusselt number
Pr : Prandtl number
Q_0 : Heat source parameter
Q : Heat source parameter
q_r : Radiative heat flux (Rosseland approximation)
Re : Reynolds number
Re_x : Local Reynolds number
Sc : Schmidt number
Sh : Sherwood number
T : Temperature within the boundary layer
T_w : Temperature at the wall
T_∞ : Free stream temperature
u : Velocity along x-axis
v : Velocity along y-axis
v_w : Suction velocity
x : Coordinate along the plate
y : Coordinate normal the plate

Greek symbols:

α : Thermal diffusivity
β : Volumetric coefficient of thermal expansion
η : Similarity variable
$\Delta\eta$: Step size
θ : Dimensionless fluid temperature
μ : Coefficient of viscosity
ν : Coefficient of kinematics viscosity
ρ : Fluid density
σ : Electrical conductivity
φ : Dimensionless fluid concentration Subscript
ψ : Stream function
∞ : Outside the boundary layer condition superscript
s : Differentiation with respect to η

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