

## Research Article

### Classification of Gas-dynamic Discontinuities and their Interference Problems

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**Abstract:** The aim of the study is to give a common classification of gasdynamic discontinuities, shock-wave structures and shock-wave processes. We have considered the classification of gas-dynamic discontinuities, shock-wave processes, shock-wave structures, discontinuity interaction problems. We have considered different classification criteria: thermodynamic, cinematic, transiency, discontinuity direction, arriving and outgoing discontinuities. A comprehensive list of discontinuity interference problems is given, as well as classification of possible transformations and wave front reorganization up to the case of the problem two-dimensional transient definition.

**Keywords:** Gas-dynamic discontinuity interference, riemann waves, shock, shockwave, simple waves

#### INTRODUCTION

Objects of study are gas-dynamic discontinuities, shock-wave structure and shock-wave processes, as well as their classification.

The Shock-Wave Process (SWP) should be understood as the process of transformation of gas-dynamic variables in waves and discontinuities:

$$f \rightarrow f_0 \quad (1)$$

Variables  $f$  are a great number of cinematic ( $u$ -speed,  $w$ -acceleration), thermodynamic ( $p$ -pressure,  $\rho$ -density,  $t$ -temperature),  $f_0$ -parameters of deceleration, entropy change  $\Delta S = C_v \ln \theta / \theta_0$ , where  $\theta = p/p_0$ -Laplace-Poisson invariant and  $h$  and  $h_0$ -enthalpy, as well as thermal and physical parameters (thermal capacity  $c_p$  and  $c_v$ ,  $\gamma = c_p/c_v$ -adiabatic index, viscosity index etc.), which can change during the SWP.

We have set the problem to classify all possible types of interaction of all types of waves and gas-dynamic discontinuities. Let us remind briefing information about the Gas-Dynamic Discontinuities (GDD). As we know, supersonic streamline can include areas where parameters change leap, abruptly. In such case, within the perfect gas model they say about existence of gas-dynamic discontinuities.

Gas-dynamic discontinuities in supersonic streamline can be zero-order  $\Phi_0$ : the depression/compression wave center, shock wave and sliding surface where the streamline gas-dynamic parameters endure discontinuity (pressure  $P$ , total pressure  $P_0$ , speed  $u$ , velocity vector angle  $\theta$ ) and of the first order called as weak discontinuities (discontinuity

characteristics, weak tangential discontinuities)  $\Phi_1$ , where first-order derivatives of gas-dynamic variables endure discontinuity. It is possible to specify features (discontinuities)  $\Phi_i$  of the space of any order gas-dynamic variables.

Dynamic compatibility conditions (DCC) at GDD  $\Phi_0$  (Uskov and Mostovkykh, 2010a), connecting the streamline parameters before discontinuity and after it, are deduced from the conservation laws for streamline of matter, power streamline, momentum flux component written before discontinuity and after it. This ratio's parameter is discontinuity intensity  $J$  (more often it is specified as the ratio of pressure after discontinuity and pressure before it).

Differential conditions of dynamic compatibility (DCDC)  $\Phi_0$  connect the streamline irregularity before discontinuity and after it (Uskov and Mostovkykh, 2012):

$$N_i = c_i \sum_{j=1}^5 A_{ij} N_j \quad (2)$$

Coefficients  $A_{ij}$ ,  $c_i$  are published in the works by Uskov and Mostovkykh (2012) and Uskov *et al.* (1995). For generality, equation included  $N_4 = \delta/y$  ( $\delta = 0$  in two-dimensional streamline) and  $N_5 = K_\sigma$  (compression shock curvature). DCDC for known streamline field before discontinuity, discontinuity intensity and curvature allow to calculate derivatives from gas-dynamic variables after the discontinuity (Uskov and Mostovkykh, 2010b). If one of irregularities is known, you can find the discontinuity curvature in the given point. It allows, in some cases, to calculate the

streamline field with clear selection of gas-dynamic discontinuities and to calculate their geometry by means of DCDC. For example, at the supersonic jet (arriving into the atmosphere from the Laval nozzle) boundary,  $N_1 = 0$ , this allows to calculate the curvature of the streamline boundary at the nozzle edge (Bulat *et al.*, 1993; Uskov and Chernyshev, 2006).

DCC and DCDC allow to make the total list of possible configurations of interacting GDD and to investigate their existence domain. The basic monograph (Uskov *et al.*, 1995) is devoted to the problem and, for the first time, it defines the modern full-blown theory of the stationary GDD interference.

**MATERIALS AND METHODS**

**Discontinuity classification:**

**Classification according to the thermodynamic principle:** The important thermodynamic difference of simple waves and discontinuities is the entropy behavior in the streamlines going through them. The basic parameter of such waves is static pressure ratio (discontinuity intensity)  $J = \hat{p}/p$  after and before the wave.

Density ratio  $J = \hat{p}/p$  is connected with the wave intensity by Laplace-Poisson is entropy (invariant) ( $\vartheta = \text{const}$ ), that is:

$$g/\vartheta = JE^\gamma = 1 \tag{3}$$

Or by means of Rankine-Hugoniot shock adiabat:

$$E = \frac{1 + \varepsilon J}{J + \varepsilon} \tag{4}$$

The first case defines is entropic acoustic waves like Riemann waves ( $\bar{R}$ ) or Prandtl-Meyer ( $\bar{\omega}$ ). The second case defines shock non is entropic waves ( $\bar{D}$ ).

Depression and compression waves. Waves are classed into depression waves ( $J \leq 1$ ) and compression waves ( $J > 1$ ). The last ones cover isentropic compression waves ( $\bar{\omega}_c$  and  $\bar{R}_c$ ) and shock waves ( $\bar{D}$ ).

**Intense and weak waves:** If the intensity values  $J \neq 1$ , the wave is intense. In degenerate wave ( $J = 1$ ), the gas-dynamic variables do not change, but at such a

Table 1: Possible waves and discontinuities

	$\Delta S = 0$			$\Delta S \neq 0$		
	$J < 1$	$J = 1$	$J > 1$	$J < 1$	$J = 1$	$J > 1$
$\Delta h_0 = 0$	$\bar{\omega}_r$	$v$	$\bar{\omega}_c$	-	-	$\bar{\sigma}$
$\Delta h_0 \neq 0$	$\bar{R}_r$	-	$\bar{R}_c$	-	$\bar{\tau}, \bar{K}$	$\bar{D}$

weak discontinuity these derivatives can change. Weak discontinuities (discontinuity characteristics) are the front and trailing edges of isentropic waves and degenerating into then intense discontinuities ( $J \rightarrow 1$ ). At weak discontinuities, not only first-order derivatives ( $f'$ ) can change, but higher-order derivatives. All possible waves and discontinuities are tabulated (Table 1).

**Discontinuity classification:**

**Classification according to the cinematic principle:** Under the cinematic principle, waves and GDD are divided into stationary and transient ones. The first cover waves generated in supersonic streamlines (Fig. 1a), like Prandtl-Meyer ( $\bar{\omega}$ ) waves and compression shocks ( $\bar{\sigma}$ ) (standing shock waves, Fig. 1b). Wave front and tailing edges ( $\bar{\omega}$ ) and surface of discontinuity in the gas supersonic streamlines going through them.

The Riemann progressive wave edges and shock wave edges move in space and time. For stationary and transient discontinuities ratio of total pressure  $J_0 = p_{02}/p_{01}$  and total enthalpy  $H_0 = h_{02}/h_{01}$  looks differently (Kochin *et al.*, 1963). In steady streamlines  $H_0 = 1$  the ratio goes over:

$$J_0 = (JE^\gamma)^{\frac{1}{\gamma-1}} \tag{5}$$

Formula (5) describes the total pressure loss factor in supersonic steady waves (compression shocks ( $\bar{\sigma}$ ) and in isentropic Prandtl-Meyer waves where  $JE^\gamma = 1$ ).

In transient streamlines, the total enthalpy changes ( $H_0 \neq 1$ ), so, in Riemann waves and progressive shock waves changes  $J_0$  and  $H_0$  are related by formula:

$$J_0 = \left( \frac{H_0^\gamma}{JE^\gamma} \right)^{\frac{1}{\gamma-1}} \tag{6}$$

And the most easy in simple  $\bar{R}$  waves:

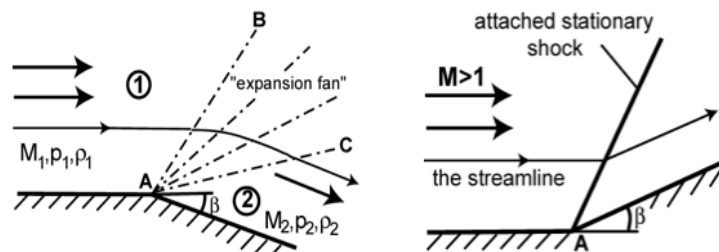


Fig. 1: Prandtl-Meyer wave (a) and stationary shock

$$J_0 = H_0^{\frac{\gamma}{\gamma-1}} \quad (7)$$

**Discontinuity classification:**

**Classification by the wave edge travel direction:** One-dimensional wave travel directions for the original gas streamline is possible to characterize by factor ( $\chi = \pm 1$ ) of the edge travel direction.

For  $\chi = +1$  the edges travel with the original streamline travel and the waves are called as wake waves. The edge speeds of isentropic waves relative to the streamline particles are sonic ones and have the propagation speed  $u+a$  (a-sound speed). The normal shock wave edge has speed  $D>a$  and overruns the original streamline particles.

Values  $\chi = -1$  meet the oncoming waves which fronts travel towards the original streamline. As the front edges of simple waves relative to gas particles travel at the sound speed, in the supersonic original streamline ( $u>a$ ) they meet the streamline particles, but drift downstream. We call such waves as drift waves.

Thereby, in the direction of travel, the waves relative to the streamline are possible to class into: wake waves, streamline co-directional waves and oncoming waves. Oncoming simple waves spreading to the supersonic streamline are drift waves.

If the shock wave speed propagation is less than the speed of incoming supersonic streamline, such a wave will be also a drift one.

Particularly, a sample of drift wave is normal shock wave which is called as standing shock wave.

**Discontinuity classification:**

**Shock-Wave structure Classification (SWC):** In the shock-wave process not only single waves and discontinuities can take part, but also their systems and Shock-Wave Structures (SWC). Shock-wave systems (structures) are considered a set of some waves and discontinuities though which current lines or particle trajectories sequentially go in stable or transient streams. SWC is often used for control of gas streamline gas-dynamic parameters and with their help you can solve different aero gas dynamics applied problems. For this, in original streamlines wave or compression shock systems with optimal properties are specially created. Atypical sample is compression shock systems (Fig. 2) in plane's supersonic air inlets (Ovsyannikov, 2003).

Shock-wave structures occur as a result of interaction (crossing, interference) of waves or discontinuities among themselves, with tangential, contact, free or solid surfaces.

We can distinguish follow, oncoming discontinuities, discontinuity decay or branching (Uskov and Mostovyykh, 2010a).

Relative to the point where SWC forms, discontinuities is classed onto incoming and outgoing

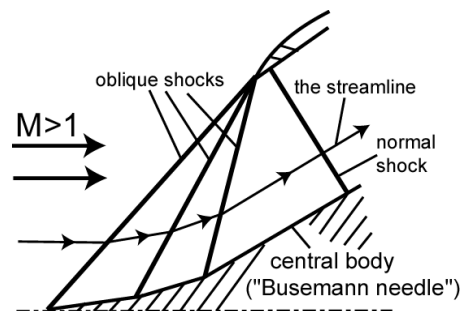


Fig. 2: Optimal SWC in the air inlet

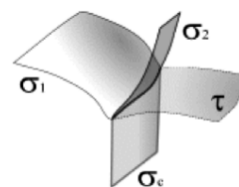


Fig. 3: Shock-wave triple configuration

ones, which is especially important for investigation of causes of random discontinuity decay. The reason of the structure formation is waves and discontinuities arriving in the same point. Outgoing waves (discontinuities) are the result of interaction of incoming waves (discontinuities). Because of interaction of isentropic waves, the outgoing waves can be only centered depression waves ( $\bar{\omega}_r$  or  $\bar{R}_w$ ), which centers coincide with the cross point of incoming waves. The compression wave centers are formed due to crossing characteristics of one family with formation of a shock-wave structure in this point.

In these structures, the current lines go through different systems of incoming and outgoing waves. A typical sample is Triple Configurations (TC) composed of one incoming  $\sigma_1$  and two outgoing  $\sigma_2, \sigma_c$  normal discontinuities (Fig. 3) divided with tangential discontinuity ( $\tau$ ).

Figure 3  $\sigma_1$  shows compression shocks, from which  $\sigma_1$  is incoming shock branched into shocks (2) and (c), of the following streamlines are supersonic. In shock-wave structures some current lines serially go through the wave system (1) and (2) and another part-only through wave (c). The structures are usually calculated based on the dynamic compatibility conditions at tangential (or contact) discontinuity.

**RESULTS AND DISCUSSION**

**Classification of wave and discontinuity interaction problems:**

**Classification by the gas-dynamic indication:** According to the gas-dynamic indication, waves can be divided into two types: isentropic waves and discontinuities. Then you can separate three types of wave interaction (Uskov, 2000):

- Crossing gas-dynamic discontinuities (including shock waves and contact discontinuities) among themselves
- Interaction of Riemann isentropic waves
- Interaction of isentropic waves with gas-dynamic discontinuities

Besides, waves and discontinuities can interact with solid surfaces.

Entropy waves (contact discontinuities) cannot cross, therefore, the first type covers shock-waves interaction or shock-wave-contact discontinuities interaction. Interaction process of  $\bar{D}$  or  $\bar{R}$  wave with contact discontinuity is called refraction and its reason is possible to indicate as a sum of cymmy incoming waves:  $\bar{D} + \bar{K}$  or  $\bar{R} + \bar{K}$ . This process is accompanied with wave refraction and wave reflection at contact discontinuity. Refraction  $\bar{D} + \bar{K}$  covers gas-dynamic discontinuities interaction and refraction  $\bar{R} + \bar{K}$  is of mixed type. The rest cases of wave sinter action are called as interference ( $\bar{D} + \bar{D}$ ,  $\bar{R} + \bar{R}$ ,  $\bar{D} + \bar{R}$ ).

**Problem dimensionality and generic shock-wave structure:** From the point of view of dimensionality, time  $t$  is a coordinate like space coordinates. In this context, one-dimensional transient centered Riemann wave is totally equivalent to Prandtl-Meyerplane stationary wave (Fig. 1a). Oblique shock is equivalent to progressive one-dimensional D-wave. Curved shock wave is equivalent to D-wave travelling with acceleration.

There are different approaches to shock-wave structure classification and their interaction. In offered classification based on the integrated SWC (Uskov, 1979). Integrated SWC consists of all possible discontinuities and waves: three incoming, one main, one tangential and one reflected discontinuities (Fig. 4). The last one may be both a compression shock and a

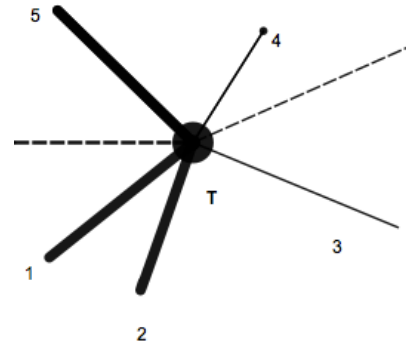


Fig. 4: Generic shock-wave structure; 1-2 incoming compression shocks of the same direction; 3: Reflected discontinuity (jump or shock); 4: Main outgoing discontinuity; 5: Incoming counter compression shock. - - - tangential discontinuities

depression wave. This classification is in complete, as it includes no simple waves. Also, the centered compression wave center, which is also a discontinuity, is missed.

Nevertheless, for an individual case of dimensionality 2, the integrated SWC allows introduction of comprehensive classification of interference problems. Relative to the interference point T (Fig. 4), gas-dynamic discontinuities in the generated SWC are divided into incoming ( $R_a$ ) and outgoing ( $R_p$ ). For incoming discontinuities, a velocity vector component for discontinuity direction goes to point T and for outgoing discontinuities-back from it.

Taking in to consideration the direction of interacting waves (wake ( $\bar{W}$ ) and contra directional ( $\bar{W}$ ) waves relative to the original streamline), we obtain two classes of interaction problems: incoming waves of the same ( $\bar{W} + \bar{W}$  or  $\bar{W} + \bar{W}$ ) or different ( $\bar{W} + \bar{W}$ ) directions. Both classes of problems occur for

Table 2: Classification of the interactions

No	Interference formula	Incoming discontinuity characteristics								Desired intensities	Notes
		$\bar{\sigma}_1$		$\bar{\sigma}_2$		$\bar{\sigma}_3$		$\tau$			
		$J_1$	$\chi_1$	$J_2$	$\chi_2$	$J_3$	$\chi_3$	$J_\tau$	$\chi_\tau$		
1	$\sigma_1 + W \Rightarrow \begin{cases} \bar{\sigma}_3 \\ \bar{\sigma}_4 + \hat{\tau}_{3-4} + \bar{\sigma}_3 \end{cases}$	$J_1$	+I	I	0	I	0	I	0	$J_3$	TC-2
		$J_1$	+I	I	0	I	0	I	0	$J_3, J_4$	
2	$\bar{\sigma}_1 + \tau \Rightarrow \begin{cases} \bar{\sigma}_4 + \hat{\tau}_{3-4} + \bar{\sigma}_3 \\ \bar{\sigma}_4 + \hat{\tau}_{3-4} + \bar{\omega}_3 \end{cases}$	$J_1$	+I	I	0	I	0	$J_\tau$	-	$J_3, J_4$	
		$J_1$	+I	I	0	I	0	$J_\tau$	-	$J_3, J_4$	
3	$\bar{\sigma}_1 + \bar{\sigma}_2 \Rightarrow \begin{cases} \bar{\sigma}_4 + \hat{\tau}_{2-4} \\ \bar{\sigma}_4 + \hat{\tau}_{3-4} + \bar{\sigma}_3 \\ \bar{\sigma}_4 + \hat{\tau}_{3-4} + \bar{\omega}_3 \end{cases}$	$J_1$	+I	-	+I	I	0	I	0	$J_2, J_4$	TC-3
		$J_1$	+I	$J_2$	+I	I	0	I	0	$J_3, J_4$	
		$J_1$	+I	$J_2$	+I	I	0	I	0	$J_3, J_4$	
4	$\bar{\sigma}_1 + \bar{\sigma}_3 \Rightarrow \begin{cases} \bar{\sigma}_3 + \hat{\tau}_{3-5} \\ \bar{\sigma}_3 + \hat{\tau}_{3-4} + \bar{\sigma}_4 \end{cases}$	$J_1$	+I	I	0	-	-I	I	0	$J_3, J_5$	TC-1
		$J_1$	+I	I	0	$J_3$	-I	I	0	$J_3, J_4$	

wave interference and for wave refraction. It is obviously that different direction waves cross constantly and possibility of the same direction waves interaction needs supportive analysis.

The interference formula can be generally written as:

$$\sum R_a(k) \rightarrow \sum R_p(k) \tag{8}$$

For example, interaction of follow compression shocks (the same direction discontinuity) may be presents as:

$$\overline{\sigma_1} + \overline{\sigma_2} \rightarrow \overline{r_3} + \hat{r} + \overline{\sigma_4} \tag{9}$$

Tangential discontinuity separates two streamlines going through discontinuities 1 and 2 (follow incoming compression shocks) and reflected discontinuity 3. For contra directional compression shock we have (Table 2):

$$\overline{\sigma_1} + \overline{\sigma_5} \rightarrow \overline{\sigma_3} + \hat{r} + \overline{\sigma_4} \tag{10}$$

**Topological classification of transformations shock-wave structures:** Article by Bogaevsky (2002) covers the discontinuity reorganization classification in potential solutions of Burgers equation and in viscous solutions of Hamilton-Jacob equation with convex Hamiltonian. It turns out that all these classifications are the same.

A symptotics of potential solutions ( $v = \nabla_x S$ ) of Burgers equation  $\partial v / \partial t + v \partial v / \partial x = \varepsilon \Delta v$  when the viscosity  $\varepsilon$  tends to zero expresses in function, where  $F(\lambda) = \min f_\lambda(y)$ , where  $\lambda = (t, x)$  is a space-time point,  $f$ -smooth function family. Even the  $f$  family is smooth; function  $F$  has its own features. At every point of time these features represent the shock-wave system in  $x$ -space. At change of time  $t$ , the shock-wave system reorganizes. Any space coordinate can work as  $t$ .

We are interested in only features and reorganization stable relative to any small enough disturbances of the smooth initial conditions. Other

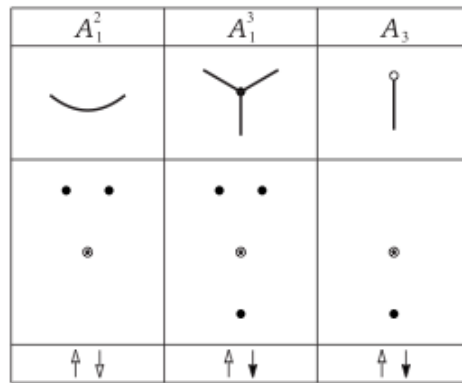


Fig. 5: Typical shock-wave reorganization in one-dimensional ( $d = 1$ ) case

features and reorganization are structurally unstable and cannot be used in practice. It turns out that at typical points of time generic shock wave has features from the finite list (Arnold *et al.*, 1991) and are subject to reorganization at individual points of time. To describe them, we consider so called world shock wave lying in space-time. Instantaneous shock waves are combination of world shock-wave crossing by isochrones  $t = \text{const}$ .

Features of the functions of general position families minimum depending on a few parameters, are examined in Bryzgalova (1977, 1978). First, Gurbatov and Saichev (1984) and Gurbatov *et al.* (1984) paid their attention to existence of reorganization of minimum functions unrealizable by shock waves. Baryshnikov (1990) that at any initial conditions homotopy types of instantaneous shock wave addition sat the reorganization moment and just after it coincide. Or, what is the same, around the point of reorganization in space-time, instantaneous shock waves at close points of time, directly following the reorganization, are the point homotopic.

Let us consider one-dimensional case (dimensionality  $d = 1$ ). Instantaneous shock wave consists of isolated points and world shock wave is a curve on the surface with regular, triple and end points (Fig. 5).

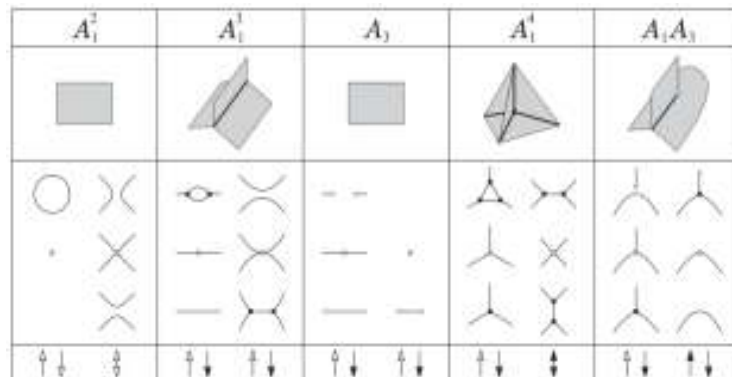


Fig. 6: Shock-wave typical reorganizations in two-dimensional ( $d = 2$ ) case

The first line covers the designation of the world shock wave features. The wave can arise at some moment (endpoint), propagate in space (regular points) and decay with formation of three waves (triple points). At typical moments of time, an instantaneous shock wave can be reorganized according to black arrows in Fig. 5. Particularly, any triple point of the world shock wave gives a couple of instantaneous shock wave points (shown in the second line) and the world shock wave end point generates a new point of instantaneous shock wave. The set of reorganizations totally include all reorganizations of the general position (Bogaevsky, 2002).

Figure 6 illustrates a flat case ( $d = 2$ ). World shock wave is a surface with features; all these features are shown in the second line. Instantaneous shock wave is a curve which can have triple points and end points (the same features which world shock waves have in case of dimensionality  $d = 1$ ). Instantaneous shock wave can be reorganized according to black arrows in Fig. 6.

All reorganizations of wave fronts and shock-wave given in Fig. 5 and 6 totally cover possible types of interference of one-dimensional transient waves and two-dimensional stationary and transient waves and discontinuities. In the same wave, the classification of triple stationary and transient waves may be introduced. In this case, the world wave will be a hyper surface in four-dimensional space-time. And its isochron cross-sections will be instantaneous triple SWCs. This case is not covered by the work.

## CONCLUSION

The given classification is the most common. It consists of formulae following the aggregate SWC and shows the direction of wave possible interaction and reorganization. We have considered different classification signs: thermodynamic, cinematic, transiency, discontinuity direction, incoming and outgoing discontinuities, aggregate SWC which configuration is specified by parameters of incoming discontinuities and DCC at the outgoing in the interference point tangential discontinuity. On the other hand, DCC execution at the tangential discontinuity after SWC is necessary, but insufficient condition of shock-wave structure existence. For SWC existence the following structural stability conditions given in *i. 2.2* must be executed. In relation to dimensionality, time is just a coordinate; cinematically transient problems are equal to stationary problems with one more dimensionality. It is necessary to take into consideration the important fact that even dimensionality and odd dimensionality is described by different geometrical types (simplistic and contact), accordingly and the SWC problem classification is separated into two sets: even and odd space-time.

The problems cinematic equivalence does not mean the dynamic equivalence existence. At transient

discontinuities, the total enthalpy changes, but at stationary compression shocks (standing wave) it remains constant.

## ACKNOWLEDGMENT

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