

## Research Article

### Food Vehicle Frame Topology Optimization Based on Bi-directional Interpolation Model

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**Abstract:** Bi-Directional Interpolation Model (BDIM) decided by a threshold is presented in this study to treat intermediate food vehicle frame density elements. Based on BDIM a mathematical model of food vehicle frame topology optimizing is established by integrating the analysis of the traditional interpolation model, only acting on sensitivity with BDIM and acting on sensitivity and rigidity with BDIM under the Gradient Projection Method. And it is exposed that the only acting on sensitivity with BDIM can get a global convergence and boundaries clear optimization results, which provides a new method to resolve the difficult problem of intermediate density elements. This study discusses its application in the field of food transportation vehicle frame.

**Keywords:** BDIM, food vehicle frame, topological optimization, traditional interpolation model

## INTRODUCTION

In the field of food vehicle frame food vehicle frame topology optimization, it's always a key research point for researchers to seek a better and faster way to model and solve. It can acquire the best distribution of material while containing or improving the performances (Chen and Liu, 2012). It makes the complicated structures being neatly and reasoningly selected in concept design stage. Since 1988, the food vehicle frame food vehicle frame topology optimization design has been presented, the study of food vehicle frame food vehicle frame topology optimization has developed further through the tireless effort of Chinese and foreign researchers. During this period, researchers put forward serious modeling method of food vehicle frame structure food vehicle frame topology optimization which including Homogenization method (Guedes and Kikuchin, 1990), Variable density method (Yang and Chuang, 1994), ICM method, Level set method (Sui, 1996). Above these methods, the Variable Density method is the most popular one, its advantages includes less design variables, better optimization effect, high general applicability and so on. In this study, this strategy is called Bi-Directional Interpolation Model (BDIM) (Mlejnek and Schirrmacher, 1993; Zhou and Rozvany, 2001).

## MATERIALS AND METHODS

**Bi-directional material interpolation model:** Based on the above bi-directional mechanism of intermediate

density, the bi-directional interpolation function is built, which is shown as Eq. (1):

$$\varphi(x_i) = \begin{cases} \frac{1}{2} \left( 1 - [\cos \pi x_i]^{\frac{1}{q}} \right) & x_{min} \leq x_i \leq 0.5 \\ \frac{1}{2} \left( 1 + [-\cos \pi x_i]^{\frac{1}{q}} \right) & 0.5 < x_i \leq 1 \end{cases} \quad (1)$$

where,  $\varphi(x_i)$  is the bi-directional function  $x_{min}$  is the minimum relative density of the elements ( $x_{min}$  is taken as 0.0015 to avoid that total stiffness matrix is singular),  $q$  is a bi-directional factor and its function is to penalize the elements which meet  $x_{min} \leq x_i \leq 0.5$  and encourage which meet  $0.5 < x_i \leq 1$ .

Introducing sign function, formula (1) can be expressed:

$$\varphi(x_i) = \frac{1}{2} (1 - \text{sign}(0.5 - x_i) [\text{sign}(0.5 - x) \cos(\pi x_i)]^{\frac{1}{q}}) \quad (2)$$

where,  $x_{min} \leq x_i \leq 1$ .

In finite element analysis, it can always achieve the purpose of modifying stiffness matrix by modifying the elastic modulus of the elements materials. According to the functional relationship between bi-directional interpolation function and the elastic modulus of the elements materials, the element stiffness matrix can be get:

$$\begin{cases} E_i = \varphi(x_i) \cdot E_0 \\ \mathbf{K}_i = \varphi(x_i) \cdot \mathbf{K}_0 \end{cases} \quad (3)$$

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where,  $E_i, K_0, \varphi(x_i), x_i$  are optimizing elastic modulus of the elements materials, optimizing element stiffness, bi-directional function and element density;  $E_0, K_0$  are elastic modulus of the elements materials and element stiffness before optimization. Considering the process of traditional SIMP penalty function,  $E_0$  is always treated as the elastic modulus of structure entity. In fact, the purpose of introducing penalty function is merely to penalize those elastic modulus or stiffness of which possess intermediate density elements materials in the process of optimizing. In the iteration process of food vehicle frame topology optimization, the designed element densities are variables and the corresponding elastic modulus of the elements materials are dynamic. More exact method should, therefore, be to penalize those results which possess intermediate density element (elastic modulus or stiffness) in previous iteration, rather than invariably treat the elastic modulus and stiffness of entity materials as standard, so function (3) should be:

$$\begin{cases} E_i^k = \varphi(x_i^{k-1}) \cdot E_i^{k-1} \\ \mathbf{K}_i^k = \varphi(x_i^{k-1}) \cdot \mathbf{K}_i^{k-1} \end{cases} \quad (4)$$

where,  $K$  is the current iteration step,  $E_i^k, E_i^{k-1}$  are the elastic modulus of element  $i$  of the current  $k$  and the former  $k-1$  iteration steps.  $K_i^k, K_i^{k-1}$  are the stiffness of element  $i$  of the current  $k$  and the former  $k-1$  iteration steps.  $x_i^{k-1}$  is the iteration result of step  $k-1$  of density of element  $i$ ,  $x_i^k$  is the optimizing variables of element  $i$  of the current iteration step.

It can directly adopt the following formula in traditional SIMP food vehicle frame topology optimization to get the sensitivity of design variables influenced by the element stiffness:

$$\frac{\partial \mathbf{K}_i}{\partial x_i} = (x_i^k)' \mathbf{K}_0 = k \cdot x_i^{k-1} \cdot \mathbf{K}_0 \quad (5)$$

Nevertheless the optimization calculation is based on the structure finite element analysis, so the mandatory effect of penalty function mainly reflects on it, namely, the optimization calculation is based on the penalty of the finite element analysis results every time (Guo *et al.*, 2006). In essence, the design sensitivity calculating needed by optimization calculating should satisfy the constitutive relations of materials, namely the relation between element density and stiffness should be linear (Gao and Yu, 1992). So the element stiffness sensitivity of design variables should be calculated by linear relation, then deal the sensitivity with bi-directional interpolation according to bi-directional interpolation model. In the optimizing process, it is equal to adjust continually the iterative direction according to the value of element density. Combined with the above formulae (1) to (5), the stiffness sensitivity function based on bi-directional interpolation model can be:

$$\left( \frac{\partial \mathbf{K}_i}{\partial x_i} \right)^k = \varphi(x_i^{k-1})(x_i \mathbf{K}_i^0)' = \varphi(x_i^{k-1}) \mathbf{K}_i^0 \quad (6)$$

Listed foods must comply with the requirements of China food safety standards, including all kinds of harmful residues must be lower than the maximum of the residue limits. Therefore, as for the listed foods, the amount of the residue hazard must be strictly limited to the scope of Chinese residue hazards within the standard of detection. If some hazards in foods are not detected, it must exist risk, thus it must release the early warning reports, moreover, it must list out the undetected items correspondingly.

**Optimizing model based on bi-directional interpolation model:**

For the problem that under volume constraint the structure overall flexibility is minimum, the corresponding food vehicle frame topology optimization model based on BDIM after the design variables of slack element density is:

$$\begin{aligned} \text{find } \mathbf{x} &= (x_1, x_2, \dots, x_n) \\ \text{Min } f &= \mathbf{u}^T \mathbf{K} \mathbf{u} \\ \text{s.t. } \mathbf{F} &= \mathbf{K} \mathbf{u} \\ &\sum_{i=1}^n x_i V_i^0 \leq V_0 \\ &0 \leq x_i \leq 1, i = 1, 2, \dots, n \end{aligned} \quad (7)$$

In the above formulae, it adopts two methods to calculate the stiffness, the first one is the linear relation between element stiffness and density:  $K = \sum_{i=1}^n x_i K_i^0$ , using the bi-directional interpolation method on sensitivity simply, which is called method BDIM01; another one is using the bi-directional interpolation method on both stiffness and sensitivity, which is called method BDIM02;  $n$  is the number of design variables element;  $\mathbf{u}$  is the structural displacement,  $f$  is the flexibility function,  $V_i^0$  is the initial volume of element  $i$ ,  $V^0$  is the volume limit constraint, combined with formula (6), it's obvious to obtain the objective function sensitivity of bi-directional interpolation:

$$\begin{aligned} \frac{\partial f}{\partial x_i} &= -\mathbf{u}_i^T \left( \frac{\partial \mathbf{K}_i}{\partial x_i} \right)^k \mathbf{u}_i^T = -\varphi(x_i^{k-1}) \mathbf{u}_i^T (x_i \mathbf{K}_i^0)' \mathbf{u}_i \\ &= -\varphi(x_i^{k-1}) \mathbf{u}_i^T \mathbf{K}_i^0 \mathbf{u}_i \end{aligned} \quad (8)$$

**Gradient projection algorithm of food vehicle frame topology optimization:**

The iteration strategy of design variables based on bi-directional interpolation model combined with the gradient projection algorithm of food vehicle frame topology optimization is as follows:

$$x^{k+1} = x^k + \beta d = x^k + \beta(-\nabla f + \sum_{i \in S} \lambda_i \nabla g_i) \quad (9)$$

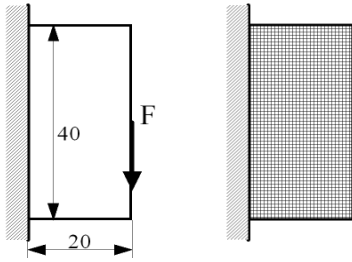


Fig. 1: The base structure and the finite element optimization model

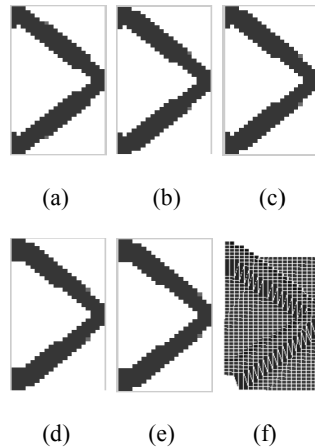


Fig. 2: The optimizing result and sketch map of different bi-directional factors q; (a): q = 1; (b): q = 2; (c): q = 3; (d): q = 4; (e): q = 5; (f): Sketch map of optimizing food vehicle frame topology

where,  $\beta$  is the step length coefficient,  $d$  is The steepest descent direction,  $s$  is the functional constraint index set of constraint function  $g(x) \leq 0$ ,  $l$  is the functional constraint index set of constraint  $(x_j - x_i) \leq 0$ ,  $\lambda_l$  represents for Lagrange multiplier of constraint functions.

**Numerical validation of bi-directional interpolation model:** Example 1: As shown in Fig. 1, the initial design domain is a 20×40 plane, thickness is 1, the material elastic modulus  $E = 1$ , the discrete design domain is divided into 20×40 rectangular finite element, a concentrated load  $F$  is acting on the midpoint of right boundary, the left border all adopt fixed constraint. In order to simplify the calculation, the design parameters which treated as dimensionless, will not affect the effectiveness of the optimization. The most optimal food vehicle frame topology structure is calculated when the volume constraint score is 0.3.

From Fig. 2, it can be concluded that bi-directional interpolation optimization can get structural food vehicle frame topology whose boundary is clear under the circumstance of not adopting acuity filtering and the structural food vehicle frame topology keeps roughly consistent when bi-directional factors get different

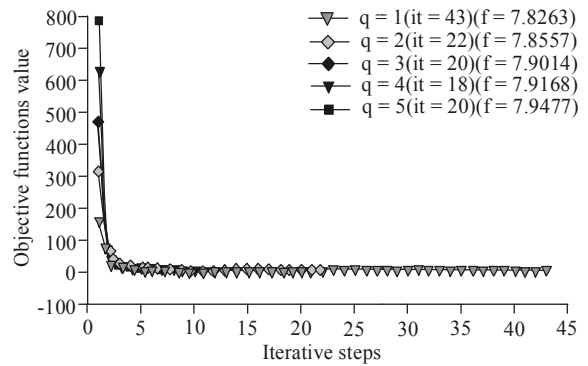


Fig. 3: Compare of the optimizing iterative process of different bi-directional factors

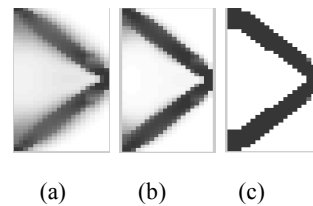


Fig. 4: The sketch map of optimizing result in different iterative steps when the unfiltered bi-directional factors q = 5; (a): It = 4; (b): It = 5; (c): It = 20

values, namely different bi-directional factors' values have little influence on optimizing result; from Fig. 3, the final objective function value of convergence tend to be consistent, in addition to the iteration step is longer when the bi-directional factor q = 1, the convergence speed is roughly consistent when q = 2-5 and it will be in steady convergence condition after the iteration step 5. Figure 4 shows the optimizing result of different iterative steps of bi-directional interpolation food vehicle frame topology optimization when q = 5.

When adopting spatial sensitivity filter in the process of BDIM and the filtering radius is 1.2, bi-directional factors are 1 and 3, the optimizing result and iterative process are showed as Fig. 5, it is clear that the optimizing food vehicle frame topology results' boundaries are vague and indistinct and there are distinctions between optimizing food vehicle frame topology and unfiltered methods: the optimizing food vehicle frame topology components have different size in Fig. 2, but the two in Fig. 5 look roughly consistent. Investigating the reason, it dues to the sensitivity balanced in the filtering radius for adopting spatial sensitivity filtering method.

## RESULTS AND DISCUSSION

Examples indicate that compared with model BDIM02 and traditional model SIMP which uses bi-directional interpolation on both stiffness and sensitivity, method BDIM01 which only interpolate on sensitivity is comprehensive to optimize problem, it can get a food vehicle frame topology optimizing result whose boundaries are clear without any filter.

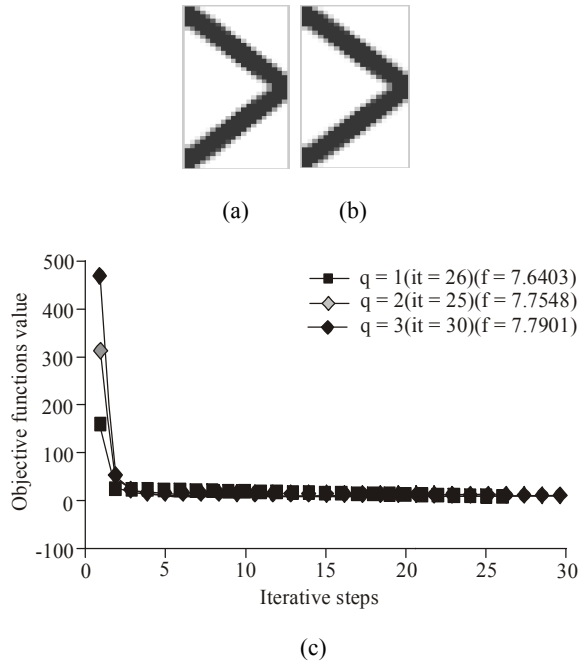


Fig. 5: The result of BDIM spatial filtering optimization; (a):  $q = 1$ ; (b):  $q = 3$ ; (c): The iterative process of sensitivity filtering optimization

Meanwhile the aims of a smaller target design value and less iteration steps are achieved.

### CONCLUSION

Aim at the unidirectional penalty treating of intermediate density elements in traditional food vehicle frame topology optimization, this study presents a Bi-Directional Interpolation Model (BDIM), its feasibility is proved by the gradient projection algorithm combined with examples. It provides a new and more reasonable way for the food vehicle frame topology optimization researches and to solves the difficult problem of intermediate density element.

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