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Research Article

Optimal Control of Decoupling Point for Deteriorating Food with Time-Varying Demand

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Abstract: The position of decoupling point denotes the penetration degree of customer demand into supply chain. To optimize the performance of deteriorating food supply chain, we consider the decoupling point control in conjunction with food production and inventory management under time-varying demand over a finite time horizon. Using dynamic models, optimal position of decoupling point and production-inventory plan are simultaneously determined. The results show that the optimal decoupling point is not related to the change of food demand under zero-inventory policy and it is a monotonically ascending function of demand rate under production smoothing policy. The simulation illustrates a diagram depicting that the optimal decoupling point shifts to the upstream suppliers along with the increase of food deteriorating rate while shifts downstream to the end customers with the growth of the time elasticity of food demand.

Keywords: Decoupling point, deteriorating food, production-inventory management, time-varying demand

INTRODUCTION

In the last decade, the market for fresh foods has continued to expand (Taylor *et al*., 2012; Qin *et al*., 2014). The demand for seasonal and exotic fruits and vegetables has increased rapidly and trade with fresh produce is more and more international (Kirezieva *et al*., 2015). On the other hand, the foods can easily spoil or deteriorate, which often results in product loss as well as economic loss. It is estimated that as much as 25-30% of perishable food production is wasted (Coulomb, 2008; Stonehouse and Evans, 2015). In the food supply chain, manufacturer and retailer have to manage their productions and inventories with full consideration of deterioration (Zhang *et al*., 2015; Chebolu-Subramanian and Gaukler, 2015). Thus, how to design a production and inventory system for fresh foods to decrease cost and meet customer requirements are a current managerial concern as well as an important research issue.

In order to get such an optimal strategy for deteriorating food supply chain, the concept of decoupling point is necessary. A Decoupling Point (DP) is a push-pull boundary to separates the forecast-driven activities from order-driven the activities (Jeong, 2011). From the upstream of the supply chain to the decoupling point, the production is scheduled based on the demand forecast which is a push strategy; meanwhile, from the decoupling point to the end customers, the supply chain operations are driven by the customer orders rather than forecasts which is a pull strategy (Olhager, 2003). So the position of decoupling point indicates how deeply the customer order penetrates into the goods flow (Van Donk, 2001). As shown in Fig. 1, the feasible positions of the decoupling point exist on any stage of the food supply chain.

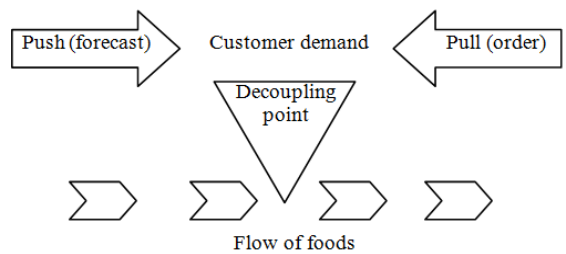

Fig. 1: Feasible position of a decoupling point in food supply chain

Recently, many researches especially mathematic approaches are focused on decoupling point control as well as deteriorating supply chain management. Viswanadham and Raghavan (2000) proposed a Petri nets model for decoupling point design and performed a simulation based on generalized stochastic Petri nets to minimize the sum of inventory carrying cost and the delayed delivery cost. Soman *et al*. (2004) proposed a comprehensive hierarchical planning framework that covers the important production management decisions to serve as a starting point for evaluation and further research on the planning system for MTO-MTS situations. Gupta and Benjaafa (2004) developed models to compute the costs and benefits of delaying differentiation in series production systems when the

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order lead times are load dependent. Sun *et al*. (2008) proposed a mathematical model which subject to satisfying customer delivery time. Since customers are usually sensitive to quality changes of fresh produce and foods, the time-varying demand rate is more realistic (Hsieh and Dye, 2013). Bai and Kendall (2008) formulated an inventory model for fresh produce, where demand rate is assumed to be dependent on the displayed inventory and the freshness of an item. Choi *et al*. (2005) proposed a novel optimization algorithmic framework based on stochastic dynamic programming to solve the decoupling point under demand uncertainty. Jolai *et al*. (2006) presented an optimization framework for a perishable item that follows a two-parameter Weibull distribution to derive optimal production over a fixed planning horizon with a stock-dependent demand rate. Chang (2014) addressed the integrated production and inventory problem for deteriorating items in a two-echelon supply chain with a goal to minimize the total cost of the entire supply chain. Qin *et al*. (2014) considered the production and lot-sizing problem for fresh foods with quality and physical quantity deteriorating simultaneously, in which the demand rate is assumed to be deterministic and dependent on the quality of an item, the selling price per unit and the on-display stock level.

Most of the previous researches assume that the decoupling point is the unique decision variable, or hinge on considering a static or steady state equilibrium of deterioration supply chains. However, the optimization of food supply chain should take account of production and inventory police and other operation activities simultaneously and the nature of the problem is dynamic. In this study, we attack the optimal decoupling point and production-inventory management simultaneously for fresh foods with deterioration rate and time-varying demand to minimize the total cost. Using the optimal control theory, we draw a closed form of the optimal solution based on the proposed dynamic model. In addition, a detailed application is discussed under zero-inventory policy and production smoothing policy.

MATERIALS AND METHODS

Notations: The notations shown in Table 1 are used for the mathematical formulation of the proposed model.

Table 1: Notations and parameters used in this paper

Notation	Description
T	Decoupling point in food supply chain
T^*	The optimal decoupling point in food supply chain
P_0	Estimated demand rate (i.e., production rate) during the pre-decoupling point
$P(t)$	A food production rate at time t during the post-decoupling point
$\hat{P}(t)$	The target food production rate at time t during post-decoupling point
t_f	Length of the life cycle of fresh food, also represent the planning horizon
$D(t)$	Demand rate at time t
$I(t)$	The inventory level of a food at time t during post-decoupling point
$\hat{I}(t)$	The target inventory level of food at time t during post-decoupling point
θ	Constant deterioration rate of the fresh food
k	Constant cost per unit deviation from target production rate
h	Constant cost per unit deviation from target inventory level

Assumptions: Consider a single fresh produce and the food supply chain can be multiple stage. The mathematical model of the optimization of decoupling point problem is based on the following assumptions:

The planning horizon as well as the life cycle of fresh food is assumed to be finite and is taken as t_f time units. The fresh foods are subject to deterioration at a constant rate θ, where $0<\theta<1$.

We assume that the initial production rate of a food P_0 is the demand rate estimated by the production planner. From $t = 0$ to the decoupling point $t = T$, the production rate keeps as P_0. When the production during the pre-decoupling point is finished, the inventory level would be $I(L) = P_0T$, where L is the throughout time of the food supply chain. Without loss of generosity, we can shift the time axis so that $t = L$ becomes $t = 0$. Then the problem transforms to the optimization during the planning horizon $t \in [0, t_f]$ with the initial inventory condition $I(0) = P_0T$.

The model: The production at rate $P(t)$ increases the inventory level; meanwhile the demand at rate $D(t)$ and deterioration at rate $\theta>0$ decreases the inventory level. The change in the level of inventory is therefore given by the following state equations:

$$\dot{I}(t) = -\theta I(t) + P(t) - D(t),\ \ I(0) = I_0 = P_0T. \qquad (1)$$

In the terminology of optimal control theory, $I(t)$ represent the state variables while the control variables are $P(t)$. In other words, we seek to find the optimal production rate, which is the rate that minimizes some performance index. Now, in order to build this performance index, we assume that the firm has set the following goals and penalties which are incurred for each variable to deviate from its corresponding goal. Denoting these penalties by h and k, the General Dynamic Model (GEN) can be described as follows:

$$\begin{aligned} &\min J = \frac{1}{2}\int_0^{t_f} \left\{ h[I(t)-\hat{I}(t)]^2 + k[P(t)-\hat{P}(t)]^2 \right\} dt, \\ &s.t. \qquad \dot{I}(t) = -\theta I(t) + P(t) - D(t), I(0) = I_0 = P_0T. \end{aligned} \qquad (2)$$

Optimal solution of the model: Denote the Hamiltonian by:

$$H = \frac{1}{2}h[I-\hat{I}]^2 + \frac{1}{2}k[P-\hat{P}]^2 + \lambda[-\theta I + P - D],$$

where λ is the adjoint variable. From the Pontryagin Maximum Principle, we have:

$$\dot{\lambda} = -h(I - \hat{I}) + \theta\lambda, \quad \lambda(t_f) = 0, \tag{3}$$

And:

$$0 = \frac{\partial H}{\partial P} = k(P - \hat{P}) + \lambda \cdot \tag{4}$$

From (4), when $k \neq 0$, we have $P = -\frac{1}{k}\lambda + \hat{P}$ and $\dot{I} = -\theta I - \frac{1}{k}\lambda + \hat{P} - D \cdot$

Finding the optimal solutions to minimize the objective (2) is equivalent to solving the following differential equations:

$$\begin{cases} \dot{I} = -\theta I - \frac{1}{k}\lambda + \hat{P} - D, I(0) = I_0 \\ \dot{\lambda} = -hI + \theta\lambda + h\hat{I}, \lambda(t_f) = 0. \end{cases} \tag{5}$$

Based on the first equation in (5), we have:

$$\frac{\lambda}{k} = -\dot{I} - \theta I + \hat{P} - D \tag{6}$$

Also, differentiating the first equation in (5), we get:

$$\ddot{I} = -\dot{\theta} I - \theta \dot{I} - \frac{1}{k}\dot{\lambda} + \dot{\hat{P}} - \dot{D} \tag{7}$$

From (5), (6) and (7), the optimal inventory rate (optimal path) is a solution of the following Riccati equation:

$$\ddot{I} + (\theta^2 + \frac{h}{k})I = \theta(D - \hat{P}) - \frac{h}{k}\hat{I} + \dot{\hat{P}} - \dot{D}, \; I(0) = I_0, \tag{8}$$

The optimal co-state variables satisfy the following equation:

$$\ddot{\lambda} = (\theta^2 + \frac{h}{k})\lambda + h(\dot{\hat{I}} + \theta\hat{I} - \hat{P} + D), \lambda(t_f) = 0, \tag{9}$$

And the optimal production rate (optimal control) satisfies the following Riccati equation:

$$\frac{d^2(P - \hat{P})}{dt^2} = (\theta^2 + \frac{h}{k})(P - \hat{P}) - \frac{h}{k}(\dot{\hat{I}} + \theta\hat{I} - \hat{P} + D), \tag{10}$$

With:

$$P(0) = P_0, \; \dot{P}(0) = \theta(P_0 - \hat{P}(0)) + \frac{h}{k}(I_0 - \hat{I}(0)) + \dot{\hat{P}}(0). \tag{11}$$

For the Eq. (10) and (11) with initial value, the optimal production rate can be expressed as:

$$\begin{aligned} P = {} & k_1 e^{\sqrt{at}} + k_2 e^{-\sqrt{at}} + \\ & c_1(t)e^{\sqrt{at}} + c_2(t)e^{-\sqrt{at}} + \hat{P}, \end{aligned} \tag{12}$$

And the optimal inventory rate is:

$$\begin{aligned} I(t) &= I_0 e^{-\theta t} + e^{-\theta t}\int_0^t P(\tau)e^{\theta\tau}d\tau - e^{-\theta t}\int_0^t D(\tau)e^{\theta\tau}d\tau \\ &= I_0 e^{-\theta t} + \frac{k_1}{\theta + \sqrt{a}}(e^{\sqrt{a}t} - e^{-\theta t}) + \frac{k_2}{\theta - \sqrt{a}}(e^{-\sqrt{a}t} - e^{-\theta t}) \\ &\quad + e^{-\theta t}\int_0^t [c_1(\tau)e^{\sqrt{a}\tau} + c_2(\tau)e^{-\sqrt{a}\tau} + \hat{P}(\tau) - D(\tau)e^{\theta\tau}]d\tau, \end{aligned} \tag{13}$$

where $a = \theta^2 + \frac{h}{k}$

$$\begin{cases} k_1 = \frac{1}{2}(1 + \frac{\theta}{\sqrt{a}})(P_0 - \hat{P}(0)) + \frac{h}{2k\sqrt{a}}(I_0 - \hat{I}(0)) - c_1(0), \\ k_2 = \frac{1}{2}(1 - \frac{\theta}{\sqrt{a}})(P_0 - \hat{P}(0)) - \frac{h}{2k\sqrt{a}}(I_0 - \hat{I}(0)) - c_2(0), \end{cases} \tag{14}$$

$$\begin{cases} c_1 = \int (-\frac{h}{2k\sqrt{a}}e^{-\sqrt{a}t}(\dot{\hat{I}} + \theta\hat{I} - \hat{P} + D))dt, \\ c_2 = \int (\frac{h}{2k\sqrt{a}}e^{\sqrt{a}t}(\dot{\hat{I}} + \theta\hat{I} - \hat{P} + D))dt, \end{cases} \tag{15}$$

Let $J = J(T)$ be the optimal objective function for the control problem that minimizes the objective J of (2) with constant deterioration rate and then $J(T)$ can be differentiated as follows:

$$\frac{\partial J}{\partial T} = \int_0^{t_f} \{h[I - \hat{I}]\frac{\partial I}{\partial T} + k[P - \hat{P}]\frac{\partial P}{\partial T}\}dt. \tag{16}$$

Notice that:

$$\frac{\partial I}{\partial T} = \frac{hP_0}{2k\sqrt{a}}(\frac{e^{\sqrt{a}t}}{\theta + \sqrt{a}} - \frac{e^{-\sqrt{a}t}}{\theta - \sqrt{a}}), \tag{17}$$

$$\frac{\partial P}{\partial T} = \frac{hP_0}{2k\sqrt{a}}(e^{\sqrt{a}t} - e^{-\sqrt{a}t}). \tag{18}$$

From $\frac{\partial J}{\partial T} = 0$, we get the optimal decoupling point:

$$T^* = \frac{\dot{I}(0)}{P_0}\left(\begin{array}{c} 1 + \dfrac{2\sqrt{a}}{\begin{array}{c}(\sqrt{a} - \theta)e^{2\sqrt{a}t_f} - \\ (\sqrt{a} + \theta)e^{-2\sqrt{a}t_f} + 2\theta \end{array}} \\ (e^{-(\sqrt{a}+\theta)t_f} - e^{(\sqrt{a}-\theta)t_f}) \end{array} \right)$$

$$-\frac{2\sqrt{a}}{P_0\left((\sqrt{a} - \theta)e^{2\sqrt{a}t_f} - (\sqrt{a} + \theta)e^{-2\sqrt{a}t_f} + 2\theta\right)}$$

$$\left\{ \frac{P_0 - \hat{P}(0)}{2\sqrt{a}}((e^{\sqrt{a}t_f} - e^{-\sqrt{a}t_f})^2 \right.$$

$$+ \frac{\theta}{2\sqrt{a}}(e^{-2\sqrt{a}t_f} - e^{2\sqrt{a}t_f})) - \frac{c_1(0)}{\theta + \sqrt{a}}$$

$$(e^{-(\sqrt{a}+\theta)t_f} + e^{2\sqrt{a}t_f} - e^{-(\sqrt{a}-\theta)t_f} - 1)$$

$$- \frac{c_2(0)}{\theta - \sqrt{a}}(e^{-(\sqrt{a}+\theta)t_f} + e^{-2\sqrt{a}t_f} - e^{(\sqrt{a}-\theta)t_f} - 1)$$

$$+ \int_0^{t_f} \hat{I}(t)\left((\theta - \sqrt{a})e^{\sqrt{a}t} - (\theta + \sqrt{a})e^{-\sqrt{a}t}\right)dt$$

$$+ (e^{(\sqrt{a}-\theta)t_f} - e^{-(\sqrt{a}+\theta)t_f})$$

$$\int_0^{t_f} (c_1(t)e^{(\theta+\sqrt{a})t} + c_2(t)e^{(\theta-\sqrt{a})t})dt$$

$$\left. + \int_0^{t_f} (\hat{P}(t) - D(t))((e^{(\sqrt{a}-\theta)t_f} - e^{-(\sqrt{a}+\theta)t_f}) e^{\theta t} + e^{-\sqrt{a}t} - e^{\sqrt{a}t})dt \right\}.$$

RESULTS AND DISCUSSION

In spite of obtaining the analytical solution for the optimal decoupling point and the target settings of production and inventory rate, the expression is too complicated to obtain directly implication values. With a focus on the application of the optimal solutions, in the following section, we further analyze two different production policies sampled by zero-inventory policy and production smoothing policy.

Optimal solution for GEN under zero-inventory policy: An important issue in the proposed GEN model is the determination of the target production rate and the target inventory level which are highly dependent on the planning policy. In this section, we derive the optimal time path of the production rate, inventory level and the optimal position of the decoupling point in closed forms under zero-inventory strategy.

While applying zero-inventory policy, in order not to incur unnecessary inventory, the production does not start until the initial inventory is exhausted which is produced during the pre-decoupling stage. When the inventory level drops to zero, the production rate follows the exact form of the demand rate which will cause no inventory. One possible setting for zero-inventory policy is as follows:

$\hat{I} = 0$ and $\hat{P} = D$

The ideal production rate is the demand rate so that it cannot produce unnecessary inventory. The target inventory level could be zero which is the ideal inventory level.

Proposition 1: From (15) we can obtain that when t = 0, there conducted:

$$c_1(0) = c_2(0) = 0$$

With $\hat{I} = 0$ and $\hat{P} = D$, the optimal production and inventory rate can be derived from (12) and (13) as follows:

$$P = k_1 e^{\sqrt{a}t} + k_2 e^{-\sqrt{a}t} + D(t), \tag{19}$$

$$I(t) = I_0 e^{-\theta t} + \frac{k_1}{\theta + \sqrt{a}}(e^{\sqrt{a}t} - e^{-\theta t}) + \frac{k_2}{\theta - \sqrt{a}}(e^{-\sqrt{a}t} - e^{-\theta t}), \tag{20}$$

where,

$$\begin{cases} k_1 = \frac{1}{2}(1 + \frac{\theta}{\sqrt{a}})(P_0 - D(0)) + \frac{h}{2k\sqrt{a}} I_0, \\ k_2 = \frac{1}{2}(1 - \frac{\theta}{\sqrt{a}})(P_0 - D(0)) - \frac{h}{2k\sqrt{a}} I_0. \end{cases} \tag{21}$$

Hence, the first derivate of J on T can be simplified and the optimal decoupling point can be calculated by setting $\frac{\partial J}{\partial T} = 0$, that is:

$$T^* = \frac{D(0) - P_0}{P_0} \frac{(2\sqrt{a} - \theta)e^{2\sqrt{a}t_f} + (2\sqrt{a} + \theta)e^{-2\sqrt{a}t_f} - 4\sqrt{a}}{2\sqrt{a}((\sqrt{a} - \theta)e^{2\sqrt{a}t_f} - (\sqrt{a} + \theta)e^{-2\sqrt{a}t_f} + 2\theta)} \tag{22}$$

From the above equations, it indicate that the optimal decoupling point and optimal inventory level are only depend on initial value of demand, whereas the optimal production rate is closely related to the changes of demand.

Corollary 1: When θ = 0 and $\hat{I}, \hat{P}, D$ are assumed to be constant value, from Eq. (22), the optimal decoupling point can be determined as follows:

$$T^* = \frac{1}{2} \frac{(D - P_0)(e^{2\sqrt{h/k}t_f} + e^{-2\sqrt{h/k}t_f})}{P_0\sqrt{\frac{k}{h}}(e^{2\sqrt{h/k}t_f} - e^{-2\sqrt{h/k}t_f})}$$

This implies that when ignoring the deterioration rate, the optimal decoupling point exists under zero-inventory policy when the demand rate is underestimated during the pre-decoupling stage that is $P_0<D(0)$.

Optimal solution for GEN under the production smoothing policy: Production smoothing refers to the phenomena in which if firms face a cost to drastically change production levels and when those costs exceed inventory holding cost, inventory should be used to smooth production. Stated differently, production smoothing is the decrease in standard deviation of orders as one moves-up the supply chain.

One possible setting of the target production rate and the target inventory level for production smoothing policy is as follows:

$$P(t)=\hat{P}=P_0 \text{ and } \hat{I}>0$$

Then the penalty term for the deviation from the target production rate is eliminated from the GEN model and a new dynamic model can be developed as follows:

$$\begin{aligned} &\min J=\frac{1}{2}\int_0^{t_f} h[I(t)-\hat{I}(t)]^2 dt, \\ &s.t. \quad \dot{I}(t)=-\theta I(t)+P-D(t). \end{aligned} \tag{23}$$

Using the same logic, the optimal inventory level can be determined as follows:

$$\begin{aligned} I(t) &= I_0 e^{-\theta t}+e^{-\theta t}\int_0^t (P-D(\tau))e^{\theta\tau}d\tau \\ &= PTe^{-\theta t}+e^{-\theta t}\int_0^t (P-D(\tau))e^{\theta\tau}d\tau. \end{aligned} \tag{24}$$

Proposition 2: When $\theta \neq 0$, the first derivate of J on T can be expressed as:

$$\begin{aligned} \frac{\partial J}{\partial T} &= \int_0^{t_f} h(I-\hat{I})\frac{\partial I}{\partial T}dt \\ &= \frac{Ph}{2\theta}(PT(1-e^{-2\theta t_f})+\frac{P}{\theta}(1+e^{-2\theta t_f}-2e^{-\theta t_f}) \\ &\quad +\int_0^{t_f} D(t)e^{\theta t}(e^{-2\theta t_f}-e^{-2\theta t})dt-2\theta\int_0^{t_f}\hat{I}(t)e^{-\theta t}dt). \end{aligned} \tag{25}$$

Under the condition of $\frac{\partial J}{\partial T}=0$, the optimal decoupling point is as follows:

$$T^*=\frac{\frac{P}{\theta}\left(1+e^{-2\theta t_f}-2e^{-\theta t_f}\right)+\int_0^{t_f} D(t)e^{\theta t}(e^{-2\theta t_f}-e^{-2\theta t})dt-2\theta\int_0^{t_f}\hat{I}(t)e^{-\theta t}dt}{P\left(e^{-2\theta t_f}-1\right)} \tag{26}$$

While $\theta = 0$:

$$\frac{\partial J}{\partial T}=Ph(PTt_f+\frac{1}{2}Pt_f^2-\int_0^{t_f} D(t)(t_f-t)dt-\int_0^{t_f}\hat{I}(t)dt), \tag{27}$$

with $\frac{\partial J}{\partial T}=0$, the optimal decoupling point can be expressed as follows:

$$T^*=\frac{2\int_0^{t_f} D(t)(t_f-t)dt+2\int_0^{t_f}\hat{I}(t)dt-Pt_f^2}{2Pt_f} \tag{28}$$

Proposition 2 shows that the consideration of deterioration rate of foods is critical to the optimal position of decoupling point. If the deterioration rate was ignored and assumed to be zero in foods supply chain management, the decision is not optimal to minimize the entire cost.

Corollary 2: When $\theta = 0$ and $\hat{I}, \hat{P}, D$ are set as constant values, under the production smoothing policy, Eq. (28) can be expressed as follows:

$$T^*=\frac{2\hat{I}-(P_0-D)t_f}{2P_0} \tag{29}$$

It is obvious that $(\partial^2 J/\partial T^2)=2P_0 t_f(P_0-D)>0$ if and only if $P_0>D$.

Corollary 2 implies that the decoupling point satisfying Eq. (29) is global optimal if and only if $P_0>D$. It also indicates that under these circumstances, the overestimation of the demand rate P_0 during the pre-decoupling stage guarantees the existence of the optimal decoupling point under the production smoothing policy.

Proposition 3: When $D(t) = D$, which means the demand rate D is varying without time, the Eq. (26) can be formalized as follows:

$$T^*=\frac{\frac{P}{\theta}\left(1+e^{-2\theta t_f}-2e^{-\theta t_f}\right)+D\int_0^{t_f} e^{\theta t}(e^{-2\theta t_f}-e^{-2\theta t})dt-2\theta\int_0^{t_f}\hat{I}(t)e^{-\theta t}dt}{P\left(e^{-2\theta t_f}-1\right)} \tag{30}$$

The first derivative of T^* on D can be calculated as:

$$\frac{\partial T^*}{\partial D}=\frac{1-e^{-\theta t_f}}{P\theta\left(e^{-\theta t_f}+1\right)}>0$$

Proposition 3 implies that when demand rate is constant, the optimal position of decoupling point is monotonically an ascending function of demand. The increase of demand rate shifts the optimal decoupling point forward to the end customers with the purpose to find a balance between the inventory costs and the profits of customer order fulfillment. When confronting a huge demand, the forward shifting contribute to decrease the customer satisfaction lead time.

When the demand rate varies with time, we set $D(t)=(\alpha-\beta\xi)e^{\eta t}$ in (26) with $\eta\neq 0$ as a time-varying demand, where α, β are constant and α>0, β>0. α-βξ>0 is a rational assumption since the demand rate is non-negative. The basic demand rate $D(t)=(\alpha-\beta\xi)e^{\eta t}$ is a linearly decreasing function of the price ξ and decreases (increases) exponentially with time when η>0 (η<0). Given a different η, which can be either positive or negative, this form can represent most cases where demand is time-varying.

Proposition 4: When $D(t)=(\alpha-\beta\xi)e^{\eta t}$, by solving Eq. (26) we can carry out that:

$$T^*=\frac{\frac{P}{\theta}\left(1+e^{-2\theta t_f}-2^{-\theta t_f}\right)+(\alpha-\beta\xi)\int_0^{tf}e^{\eta t}e^{\theta t}\left(e^{-2\theta t}\right)dt-2\theta\int_0^{tf}\hat{I}\left(e^{-\theta t}dt\right)}{P\left(e^{-2\theta t_f}-1\right)}$$

Hence:

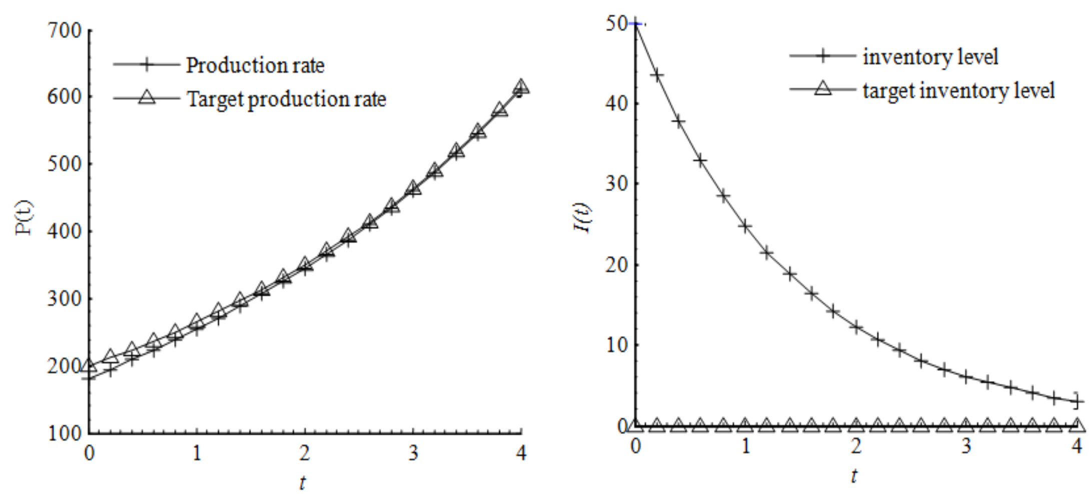
$$\frac{\partial T^*}{\partial\eta}=\frac{\alpha-\beta\xi}{P(e^{-2\theta_f}-1)}\frac{\partial}{\partial\eta}\int_0^{t_f}e^{\eta}e^{\theta}\left(e^{-2\theta_f}-e^{-2\theta}\right)dt$$
$$=\frac{\alpha-\beta\xi}{P(e^{-2\theta t_f}-1)}\int_0^{t_f}te^{\eta t}e^{\theta t}\left(e^{-2\theta t_f}-e^{-2\theta t}\right)dt<0.$$

Proposition 4 implies that under this condition, the optimal position of decoupling point is monotonically increasing with the growth of the time elasticity of demand η. It indicates that when the changes of demand rate increase rapidly (decrease tardily) with time, the decoupling point moves downstream to the end customers. In detail, when η>0, the growth of η illustrates a sharply increase of demand with time, a large amount of inventory should be stocked to meet the next customer demand which pushes the decoupling point forward shifting. Otherwise if η<0, the increase of η means the weak of attenuation, especially when η→-0, the demand doesn't decay with time, under certain conditions, the demand is known and fixed, the decoupling point should move as close as possible to end customers to improve the response speed.

Numerical experiment: It is always worth investigating the sensitivity of the optimal solution changes in the system parameters. In our case, the deterioration rate of foods is our main concern and the impact of the time elasticity of demand η is under consideration. We are also interested in the optimal path of inventory and production rate. This section presents two cases where the results are illustrated under zero-inventory policy and production smoothing policy. The simulation is run in MATLAB to illustrate a diagram depicting for the optimal solutions.

Example 1: Employed the zero-inventory policy, the values of the following parameters are set as: h = 1 ($/unit/year), k = 2.5 ($/unit/year), P_0 = 180 (units/year), I_0 = 50 (units/year), θ = 0.3, t_f = 4 (years), D(0) = 200 (units/year), the demand rate $D(t) = 200e^{\eta t}$, η = 0.28.

Figure 2 illustrates the optimal time path of food production rate and inventory level as well as the variations from target settings under zero-inventory



(a) Path of production rate (b) Path of inventory level

Fig. 2: The optimal production rate and inventory level for food with constant deterioration rate under the zero-inventory policy

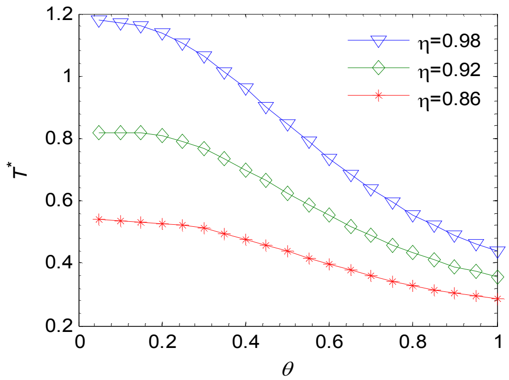


Fig. 3: The relationship between T^* and θ with the consideration of η under production smoothing policy

policy. Figure 2a shows the convergence of the optimal production rate toward the target production rate. A similar convergence is depicted in Fig. 2b for the optimal inventory towards the inventory goal level.

Example 2: Adopting the production smoothing policy, to depict the impact of food deteriorating rate θ on the optimal decoupling point T* while considering the time elasticity of demand η, the following parameters are taken in appropriate units: $D(t) = (\alpha - \beta\xi)e^{\eta t}$, $\alpha = 200$, $\beta = 4$, $\xi = 20$ (\$/unit), $\eta = 0.86, 0.92, 0.98$, $t_f = 4$ (years), $P(t) = P = 500$ (units/year), $\hat{I}(t) = \hat{I} = 150$ (units/year).

Figure 3 denotes that under the production smoothing policy, the optimal decoupling point decreases with the growth of food deterioration rate while increases with the time elasticity of market demand. The derived tendencies in Fig. 3 are in accordance with practical operational manners. The growth of deteriorating rate is always led to an ascendance in food inventory loss. The optimal decoupling point shifts backward to the upstream to condense the length of inventory and reduce the deteriorating cost. Additionally, it also demonstrates that when demand varies with time, the growth of η pushes the optimal decoupling point forward to end customers so that the speed of customer responding can be improved.

CONCLUSION

The study propose a dynamic model for deteriorating food supply chain to simultaneously determine the optimal position of decoupling point and production-inventory plan under time-varying demand. The analytical results show that their existing the unique optimal decoupling point. And analyses indicate that under zero-inventory policy, the optimal decoupling point of fresh food supply chain and the inventory level are independent on the changes of demand rate while the optimal production rate is influenced timely by food demand variation. The corresponding numerical simulations indicate a convergence of optimal path of food production rate and inventory level toward target values. Meanwhile under the production smoothing policy, the optimal decoupling point is monotonically an ascending function of market demand. The increase (decrease) of food demand rate shifts the optimal decoupling point forward (backward) to the end customers. The simulation also illustrates that the optimal decoupling point shifts upstream with the increase of deterioration rate while shifts downstream with the growth of the time elasticity of demand.

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