

Research Article

On Prediction and Simulation of Wheat Yield in Bengbu City by Using G (1, 1) Model Based on Sine Function Transformation Type

Huaxi Chen

Department of Mathematics and Physics, Bengbu College, Bengbu, Anhui 233030, P.R. China,
Tel.: 15056365266

Abstract: In order to predict future years' wheat yield in Bengbu city as accurately as possible, the paper is based on the wheat yield data of this area between 1998 and 2010 which presents the change trend close to periodical oscillation, in addition, by adopting sine function transformation GM(1, 1) model, the paper predicts yearly wheat yield from 2011 to 2015 in Bengbu city, at the same time, it compares prediction result with that based on grey GM(1, 1) model as well as true value, whose result shows the prediction accuracy based on sine function transformation GM(1, 1) model is obviously higher than that based on GM(1, 1). This finding is of great significance for ensuring the smooth development of national economy in Bengbu city.

Keywords: GM (1, 1) model, prediction, sine function, wheat yield

INTRODUCTION

Grain problem is of vital importance to the nation's economy and people's livelihood and grain yield has a direct impact on development level of national economy and other sectors. As China's second important grain crop, wheat plays the most important role in all the grain crops in Bengbu city. However, the wheat yield varies within wide limits because it is easily affected by some comprehensive factors including policy, price, means of production, climate, disaster, plant diseases and insect pests, agricultural acreage and meteorological phenomena. Therefore, it has always been a hot topic to seek various effective and quick prediction methods and continuously improve the accuracy of wheat yield prediction (Lian, 2005). Tradition prediction methods include stepwise linear regression, exponential smoothing and autoregressive moving average ones (Zhang *et al.*, 2008). Although the above methods are computationally simple, quick in prediction and appropriate for linear data prediction, they are unsuited to wheat yield prediction, for wheat yield has the nonlinear trend (Wu *et al.*, 2002).

Combined with mathematics methods, a sort of prediction method, grey prediction theory was put forward and developed in the early 1880s by Chinese scholar professor Deng (1987) and it applies such views and methods as general systems theory, information theory and cybernetics to social, economic, ecological and other abstract systems. In this theory, GM (1, 1) model is regarded as the most important and practical application one, which is a first order linear differential equation (the former "1" indicates the order; the latter

"1" stands for variables). This method works better in linear distribution data prediction better, however, it can be seen from the discrete response equation that sequence calculated based on grey prediction model bears monotonicity, which makes reduction sequence also monotonic, indicating that predicted data will have monotonically increasing (or decreasing) characteristics. In fact, most of the wheat yield tends to exhibit nearly periodic shocks, therefore, in order to improve the prediction accuracy of wheat yield in Bengbu city, this study presents a method that after the original data $x^{(0)}$ is mapped into a sine function ($y^{(0)} = \sin x^{(0)}$ $y^{(0)} = \arcsin x^{(0)}$), original data $y^{(0)}$ can be applied into GM (1, 1) model for wheat yield prediction in Bengbu city (This method above is called GM (1, 1) model based on sine function transformation type), compares the prediction results with the prediction results of the grey GM (1, 1) model as well as real value and draws the corresponding conclusions.

GREY SYSTEM PREDICTION MODEL-GM (1, 1) MODEL

The general form of GM (1, 1): Suppose variable $x^{(0)} = \{x^{(0)}(i), i = 1, 2, \dots, M\}$ is a non-negative monotonic raw data column of one predictive object. For establishing grey prediction model, $x^{(0)}$ should be firstly given order accumulation (AGO) to generate a cumulative sequence:

$$x^{(1)} = \{x^{(1)}(k), k = 1, 2, \dots, M\}$$

In the meantime:

Table 1: Grey prediction accuracy test grade standards

Test indicators/Precision grade	p	c
Fine	>0.95	>0.35
Qualified	>0.80	>0.50
Barely qualified	>0.70	>0.65
Unqualified	≤0.70	≤0.65

$$x^{(1)}(k) = \sum_{i=1}^k x^{(0)}(i) = x^{(1)}(k-1) + x^{(0)}(k)$$

As for $x^{(1)}(k)$, its trend can be described approximately in the following differential equation:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = u \tag{1}$$

a and u in (1) can be obtained in adopting the following method of least squares fitting:

$$\begin{bmatrix} a \\ u \end{bmatrix} = (B^T B)^{-1} B^T Y_M \tag{2}$$

Y_M in Eq. (2) is column vector $Y_M = [x^{(0)}(2), x^{(0)}(3), \dots, x^{(0)}(M)]^T$ and B is a structural data matrix:

$$\begin{bmatrix} -\frac{1}{2}[x^{(1)}(1) + x^{(1)}(2)] & 1 \\ -\frac{1}{2}[x^{(1)}(2) + x^{(1)}(3)] & 1 \\ \vdots & \vdots \\ -\frac{1}{2}[x^{(1)}(M-1) + x^{(1)}(M)] & 1 \end{bmatrix}$$

The time response function corresponding to differential Eq. (1) is:

$$x^{(1)}(t+1) = \left[x^{(0)}(1) - \frac{u}{a} \right] e^{-at} + \frac{u}{a} \tag{3}$$

Equality (3) is the basis formula for sequence prediction. Based on equality (3) and prediction value $\hat{x}^{(1)}(t)$ by means of order accumulation generation sequence, reduction value of the original data can be obtained:

$$x^{(0)}(k) = x^{(1)}(k) - x^{(1)}(k-1) \tag{4}$$

In the above equality $k = 1, 2, \dots, M$, with the regulation $\hat{x}^{(0)}(0) = 0$.

Prediction accuracy test of GM (1, 1) model: Residual value $\varepsilon^{(0)}(k)$ and relative error value $q(k)$ between reduction value of original data and actual observed value are shown as follows:

$$\begin{cases} \varepsilon^{(0)}(k) = x^{(0)}(k) - \hat{x}^{(0)}(k) \\ q(k) = \frac{\varepsilon^{(0)}(k)}{x^{(0)}(k)} \times 100\% \end{cases} \tag{5}$$

The following method can test whether prediction accuracy equality (3) will meet the accuracy requirement.

Calculation is firstly conducted as follows:

$$\bar{x}^{-(0)} = \frac{1}{M} \sum_{i=1}^M x^{(0)}(k)$$

$$s_1^2 = \frac{1}{M} \sum_{i=1}^M [x^{(0)}(k) - \bar{x}^{-(0)}]^2$$

$$\bar{\varepsilon}^{-(0)} = \frac{1}{M-1} \sum_{i=2}^M \varepsilon^{(0)}(k)$$

$$s_2^2 = \frac{1}{M-1} \sum_{i=2}^M [\varepsilon^{(0)}(k) - \bar{\varepsilon}^{-(0)}]^2$$

Secondly, variance and a small error probability

$$p = \left\{ \left| \varepsilon^{(0)}(k) - \bar{\varepsilon}^{-(0)} \right| < 0.6745s_1 \right\} \text{ are calculated.}$$

Generally, the accuracy test prediction equality (3) may be given in Table 1. If P and c are within the allowable range, the predicted values can be calculated, otherwise, it is required to amend equality (3) by means of analyzing the residual series $\{\varepsilon^{(0)}(k)\}_{k=2}^M$. The common correction methods for grey prediction are residual series and cycle analysis (Deng, 1987).

Application scope of GM (1, 1) model: Although GM (1, 1) model has been applied well in many fields, the sequence calculated based on it bears monotonicity and reduction sequence is monotonic. Therefore GM (1, 1) model works better in simulation prediction for smooth monotonic sequence and it functions not well in simulation effect of non-monotonic sequence with a certain number of oscillations.

GM (1, 1) MODEL BASED ON SINE FUNCTION TRANSFORMATION TYPE

The original sequence with periodic shocks in nature can be transformed by applying sine function (Shen *et al.*, 2001).

Suppose $x^{(0)} = \sin y^{(0)}$ and because this transformation is not conducted one by one, it is not equivalent to $y^{(0)} = \arcsin x^{(0)}$. For the purpose of more satisfactory simulation results of GM (1, 1), it is required to map value of $x^{(0)}$ into monotonically increasing sequence $y^{(0)}$.

Considering that domain definition of inverse sine function $y^{(0)} = \arcsin x^{(0)}$ is (-1,1) as well as original sequence $x_k^{(0)} > 0 (k = 1, 2, \dots, M)$, it is required to apply initialization in $x^{(0)}$.

The specific approaches are as follows: firstly, use the formula $\hat{x}_k^{(0)} = x_k^{(0)} - C$ to make translational change to get $\hat{x}_k^{(0)}$, improving the smoothness of the sequence $y^{(0)}$ and $k = 1, 2, \dots, M$, C is a constant more than 0; then suppose $\hat{x}_k^{(1)} = \hat{x}_k^{(0)}/M$, where $M = \max_k |\hat{x}_k^{(0)}|$ ($k = 1, 2, \dots, M$), the obtained sequence based on this transformation $\hat{x}^{(1)} = \{\hat{x}_k^{(1)} | k = 1, 2, \dots, M\}$ meets the domain definition of inverse sine function; next, by transforming $\hat{x}^{(1)} = \sin y^{(0)}$, a new sequence $y^{(0)} = \{y_k^{(0)} | k = 1, 2, \dots, M\}$ will be obtained; based on diversity of the value of $y_k^{(0)}$ and equation $y_k^{(0)} = \arcsin \hat{x}_k^{(1)} + 2n\pi$, $y^{(0)}$ has monotone increasing prosperity (where $k = 1, 2, \dots, M$, $n = 1, 2, \dots$ and the specific value of n is on the basis of monotone increasing sequence of positive sequence $y^{(0)}$).

Finally, as the original data, $y^{(0)}$ is applied into GM (1, 1) model and the estimated value $\hat{y}_{k+1}^{(0)}$ can be obtained, at the same time, after inverse transformation and initialization inverse transformation of inverse sine function, observed value of sequence $x^{(0)}$ will be obtained (in each observation time): $\hat{x}_{k+1}^{(0)}$ ($k = 1, 2, \dots, M$).

SIMULATION EXPERIMENT OF WHEAT YIELD PREDICTION IN BENGBU CITY

Data source: By means of collecting data of wheat yield in Bengbu city over Anhui Provincial Bureau of Statistics Website from 1998 to 2015, the obtained number of data is 18, in which the former 13 are used as the training sample of model and the latter 5 as the test comparison objects. Table 2 shows the specific data:

Table 2: Wheat yield in Bengbu city from 1998 to 2015

Year	Wheat yield (ten thousand tons)	Year	Wheat yield (ten thousand tons)
1998	60.18	2007	121.07
1999	93.94	2008	138.10
2000	73.45	2009	140.54
2001	65.47	2010	145.97
2002	76.17	2011	146.95
2003	48.04	2012	140.76
2004	96.62	2013	145.74
2005	102.97	2014	152.73
2006	122.05	2015	154.74

Table 3: Prediction result for the wheat yield in Bengbu city from 2011 to 2015

Year	Wheat yield (Ten thousand tons)	GM (1, 1) model			GM(1, 1) model based on sine function transformation type		
		Prediction value (Ten thousand)	Residual value (Ten thousand)	Relative error (percent)	Prediction value (Ten thousand)	Residual value (Ten thousand)	Relative error (percent)
2011	146.95	148.35	-1.40	0.0095	147.63	-0.68	0.0046
2012	140.76	138.57	2.19	0.0156	139.34	1.42	0.0101
2013	145.74	147.23	-1.49	0.0102	144.89	0.85	0.0058
2014	152.73	156.72	-3.99	0.0261	154.56	-1.83	0.0120
2015	154.74	160.38	-5.64	0.0364	155.67	-0.95	0.0060

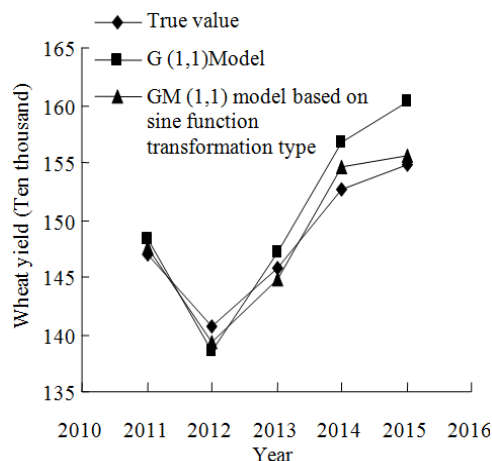


Fig. 1: Comparison of the two prediction results of the wheat yield in Bengbu city from 2013 to 2015 on the basis of two different models

Calculation and analysis of model: By using GM (1, 1) model based on sine function transformation type put forward in this study, the wheat yield in Bengbu city from 2011 to 2015 is predicted. The results are shown in Table 3 and Fig. 1:

As can be seen from Table 3, the error of prediction result by using GM (1, 1) model is relatively large; what's more, the error tends to be increasing year by year. Meanwhile, the error of prediction result by using GM (1, 1) based on sine function transformation type is relatively small, with the slightly large errors in only individual days ($k = 2014$). As can be seen from diagram 1, prediction result by using GM (1, 1) model tends to be monotonically decreasing, gradually departing from the true value; prediction result by using GM(1, 1) based on sine function transformation type shows a trend cycle shocks, closer to the true value.

CONCLUSION

Aiming at the trend that majority of the wheat yield in Bengbu city appears close to cyclical changes, this study presents a network traffic prediction method GM (1, 1) model based on sine function transformation type. Simulation results show that this method is superior owing to GM (1, 1) model's advantages of nonlinear data prediction, furthermore, it improves GM (1, 1) model, avoiding the shortcoming of monotonic prediction data and greatly improving prediction accuracy of the wheat yield in Bengbu city. In this sense, it is of some practical value.

ACKNOWLEDGMENT

Funded projects: Colleges and universities in Anhui province provincial outstanding young talent funded project (2013SQRW072ZD); Project from Huaihe cultural research center of Bengbu College (bbxyhhwh201303); Project on Supporting Outstanding Youth Talents at Colleges and Universities in Anhui Province in the Year of 2014.

REFERENCES

- Deng, J.L., 1987. Grey Forecasting Decision Making [M]. Huazhong University Publishing House, Wuhan.
- Lian, L.S., 2005. Impact of climate change and natural disasters on grain yields in Shandong province in past 40 years [J]. Meteorol. Sci. Technol., 33(1): 73-76.
- Shen, J.H., S.T. Shang and X.R. Zhao, 2001. Functional transformation GM(1,1) model constructed for ship pitching [J]. J. Harbin Inst. Technol., 33(3): 291-294.
- Wu, C.X., Y. He and J.P. Cai, 2002. Combined forecasting model and its application in grain yield forecasting [J]. Syst. Sci. Comprehen. Stud. Agric., 18(1): 17-19.
- Zhang, W.Y. and H.C. Peng, Y.Y. Li, A.G. Li and Y.Q. Li, 2008. Analysis and forecast of wheat yield of Hebei province with grey system [J]. J. Hebei Agric. Sci., 12(3): 22-24.