

## Research Article

# Force Transmission Performance Analysis of a Pure Translational Parallel Food Manipulators

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**Abstract:** The study discussed the force transmission performance of parallel food manipulator in order to design and confirm the power parameters of actuators. We take the Delta parallel food manipulator as example for analyzing force transmission of the food manipulator. In order to research on the load capability of parallel food manipulators, the concept of the force transmission capability is proposed. The force transmission capability of food manipulator can be used to confirm the power parameters of driving organs and describe the stability of force transmission in the work path or workspace of food manipulators. By analyzing force transmission capability of the Delta food manipulator, we obtain the relation of force transmission capability and condition number of the force Jacobian matrix and the distribution character of force transmission capability is analyzed primitively in whole workspace.

**Keywords:** Delta, force transmission, parallel food manipulators

## INTRODUCTION

In contrast to series food manipulators, the parallel food manipulators have high stiffness, high position precision, steady architecture, possess high status and application value in some industry fields. So force transmission performance of parallel food manipulators is important in engineering application, especially in force output performance. Studies on the character of velocity and force operation have been presented from the 1980 s in parallel food manipulator field. Salisbury and Craig (1982) define the robot's isotropy of velocity and force by Jacobi matrix condition number, however, the matrix condition number reveals only local character because the Jacobi matrix rely on machine position. Gosselin and Angeles (1991, 1998) defined GCI (Global Condition Index) by considering synthetically condition number of velocity Jacobi matrix and robot's workspace. The GCI emphasized basically on the robot's isotropy of velocity and force and can not show the character of force output. Asada (1983) and Asada and Granito (1985) presented GIE (Generalized Inertia Ellipsoid) concept in order to account for the anisotropy of force output character in robots, however, the GIE is still local performance and can not describe the force output of the robot in whole work path or workspace. Yoshikawa (1985) presented the concept about unit force vector ellipsoid, the force vector ellipsoid can indicate the force performance of robot in random direction, but can not reveal the most output force of food manipulator in work positions (Melchiorri *et al.*, 1993). In engineering application of

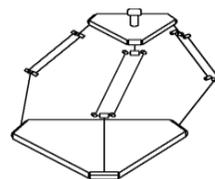


Fig. 1: Delta food manipulator sketch map manipulator

parallel food manipulators, the maximal output force should be confirmed in workspace because it shows the load capacity of the food manipulators. The maximal output force is determined by performance in force transmission process of food manipulators and parameters (torque, force) of the drive organ. So, analyzing of force transmission performance is very important in designing parallel food manipulators. The paper focus on analyzing in force transmission performance of parallel food manipulator by taking Delta food manipulator as a example. Supported by Foundation of Jiangsu University (11JDG044).

Delta is a 3-DOF (degree of freedom) parallel food manipulator and proposed by Reymond Clavel and one of the parallel robots with the most successful commercial application in industry. Figure 1 shown, Delta food manipulator consist of mobile platform, fixed platform and three machine chains. Every machine chain consists of the drive arm and the driven arm, the drive arm is linked in actuator of fixed flat by a turn joint, the joint of drive arm and driven arm are sphere joint and the driven arm is linked in mobile platform by a sphere joint, so Delta parallel food

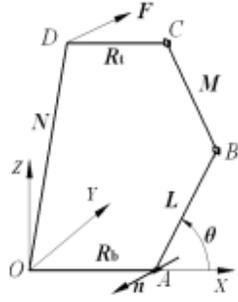


Fig. 2: Ramus link of delta food

manipulator has three pure translational DOF (Tsai *et al.*, 1996). At present, the problem of forward kinematics and inverse kinematics were solved about Delta food manipulator (Li and Xu, 2005; Stan *et al.*, 2011; Bi and Zong, 2003; Tsai and Joshi, 2002), some researches that Delta food manipulator was applied in robot and haptic devices was carried on (Renauda *et al.*, 2006; Tsumaki *et al.*, 1998; Miller, 2002; Szep *et al.*, 2011). Thereby, it is necessary to discuss the force output performance of the Delta food manipulator.

## MATERIALS AND METHODS

**Force transmission model of delta food manipulator:** Taking one of Delta food manipulator chains as shown in Fig. 2, point D is center of fixed platform, point A is turn joint, BC is driven arm, point B and point C are sphere joints. For the purpose of analysis, we assign a fixed Cartesian frame  $O\{X, Y, Z\}$  at the point O, which is centered of the fixed platform, assign a floating Cartesian frame  $D\{X_D, Y_D, Z_D\}$  at the point D, which is the centered of the mobile platform. Let  $V_D = [\dot{X}_D \ \dot{Y}_D \ \dot{Z}_D]^T$  be the velocity vector of the mobile platform and  $\dot{\theta} = [\dot{\theta}_1 \ \dot{\theta}_2 \ \dot{\theta}_3]^T$  be the vector of the actuated angular velocity in 2.1.

**Velocity analyse of delta food manipulator:** Referring to Fig. 2, let  $AB = L$ ,  $BC = M$ ,  $OA = R_b$ ,  $OD = N$ ,  $DC = R_t$ , a vector-loop equation can be written for the *i*th chain as follow:

$$N + R_t = R_b + L + M \quad (1)$$

Differentiating (1) with respect to time, leads to:

$$\dot{N} = \dot{L} + \dot{M} \quad (2)$$

Let  $\omega = [0 \ 1 \ 0]^T$  is the units vector of  $\dot{\theta}$ ,  $\omega_m$  is the angular velocity of *M*; (2) is leaded to:

$$V_D = \dot{L} + \dot{M} = \dot{\theta} \cdot \omega \times L + \omega_m \times M \quad (3)$$

To eliminate the passive variable  $\omega_m$ , we dot-multiply both sides of (3) by *M*, this give:

$$V_D \cdot M = \dot{\theta} \cdot \omega \times L \cdot M = M \times \omega \cdot L \cdot \dot{\theta} \quad (4)$$

So, kinematic equation of *i*th chains is given:

$$M_i \cdot V_D = M_i \times \omega_i \cdot L_i \cdot \dot{\theta}_i, \quad (i = 1, 2, 3) \quad (5)$$

where,

$$\omega_i = R_i \cdot \omega^T, \quad R_i = \begin{bmatrix} \cos \alpha_i & -\sin \alpha_i & 0 \\ \sin \alpha_i & \cos \alpha_i & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\alpha_i = (i-1) \cdot 2\pi/3, \quad (i = 1, 2, 3)$$

Writing (5) three times, once for *i* = 1, 2 and 3, respectively, yields three scalar equation, which can be written into a matrix form as following:

$$Q \cdot V_D = P \cdot \dot{\theta} \quad (6)$$

where,

$$Q = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix}, \quad P = \begin{bmatrix} M_1 \times \omega_1 \cdot L_1 & 0 & 0 \\ 0 & M_2 \times \omega_2 \cdot L_2 & 0 \\ 0 & 0 & M_3 \times \omega_3 \cdot L_3 \end{bmatrix}$$

When the food manipulator is away from singularities, in view of (6), we can obtain the velocity equation:

$$V_D = J \cdot \dot{\theta} \quad (7)$$

where,  $J = Q^{-1}P$  is defined as the  $3 \times 3$  Jacobian matrix of the food manipulator, which relates output velocity to the actuated angular velocity.

**Force transmission modeling:** The force transmission model shows that driving torques can be computed if the position and output force of mobile platform are given. The force transmission model is important in designing suitable strategies in order to control output force accurately.

Applying the principle of virtual work:

$$T^T \cdot \dot{\theta} = F^T \cdot V_D \quad (8)$$

where,

F = Load of the food manipulator

T = Driving torques of the food manipulator to overcome F

Substituting (7) into (8), yields:

$$T = J^T \cdot F \quad (9)$$

Eq. (9) is mathematical model of static force transmission for the Delta parallel mechanism.

**Analysis of force transmission performance:** If food manipulators have been applied in low speed state, the length of output force of food manipulator is the ratio between length of driving force vector and length of output force vector when the driving force vector is unit force vector. In view of the anisotropy of output force, the paper regards the length of the minimal output force of the work position in all orientations as the force transmission capacity in the work position. In the same way, the force transmission capacity of work path or workspace is defined the minimal force transmission capacity in all work position of work path or workspace. Therefore, after the force transmission capacity of work path or workspace is analyzed and the load of the food manipulator is given, the power parameter (force, torque) of driving organs can be computed by the force transmission capacity. On second thoughts, the range of force transmission capacity effects possibly stability of output force, so it is necessary to analyze the range of force transmission capacity in work path or workspace. On all accounts, discussion and analysis of the force transmission capacity is indispensable in work position, work path or workspace of food manipulator. In succession, denotation and calculating of force transmission capacity is discussed by the force transmission model of Delta food manipulator.

**RESULTS AND DISCUSSION**

**Analysis of the force transmission performance:** Relation between driving force and output force can be revealed by Jacobian matrix of force transmission, the proportion both output force and driving force can be reflected indirectly by condition number of the matrix. However, condition number of Jacobian matrix can not express the output force of parallel food manipulator, so it is necessary to discuss the index of output force. If dot-multiplication result of input force vector is constant, output force vector of food manipulator forms a force vector ellipsoid in work position. The length of the ellipsoid axis correlates with eigenvalue of Jacobian matrix, thus, the paper regards length of the shortest of ellipsoid axis as the force transmission capacity of food manipulator in work position, the certified process is expatiated as follow:

If driving force is a unit vector, the output force vector of food manipulator form a force vector ellipsoid and the size of ellipsoid axis is obtained by eigenvalue of Jacobian matrix. Thus, this study regards length of the shortest axis of the force ellipsoid as the force transmission capacity of work position. The force transmission capacity indicated the capacity that the food manipulator overcomes the maximum load in work position when driving force is a unit vector.

Let F, T and J are output force, driving force and Jacobian matrix respectively, from Eq. (9):

$$F = J \cdot T \tag{10}$$

Result of dot-multiplication driving force can be obtained:

$$|T|^2 = F^T (J^{-1})^T J^{-1} \cdot F \tag{11}$$

Eq. (11) can be written as the following by diagonalizing:

$$|T|^2 = (V^T F)^T \text{diag} \left( \frac{1}{\lambda_1^2}, \frac{1}{\lambda_2^2}, \frac{1}{\lambda_3^2} \right) (V^T F) \tag{12}$$

where,  $V \in R^{3 \times 3}$ , is  $3 \times 3$  orthogonal matrix,  $\lambda_i^2 (i = 1, 2, 3)$  is the eigenvalue of  $(J^{-1})^T \cdot J^{-1}$ .

From Eq. (12), if the driving force T is a unit vector sphere and shown as Fig. 3a, the output force ellipsoid can be described as Fig. 3b, so the axial length of the output force ellipsoid are  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$ , respectively and its orientation are corresponding eigenvector. Because driving force of every actuator of food manipulator is one component of the driving force vector, thus, the driving force vector forms a unit cube if the maximum of every actuator is unit force, at same time, the output force vector forms hexahedron. The driving force unit cube and output force hexahedron can be described as Fig. 4, the inscribed Sphere of the driving force cube is a unit vector sphere, the inscribed ellipsoid of hexahedron is a output force ellipsoid that unit driving force vector sphere generates. In Eq. (12), form F to VTF is a orthogonal transform, hence, length of output force is invariant from F to VTF. Thereby, the

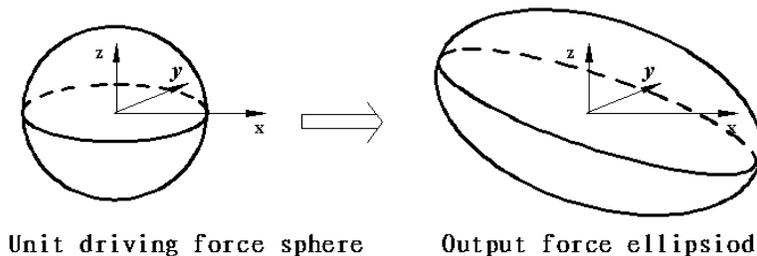


Fig. 3: Unit driving force sphere and output force ellipsoid

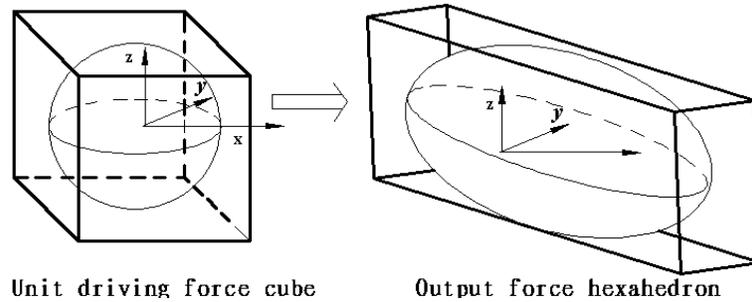


Fig. 4: Unit driving force cube and output force hexahedron

shortest axis of output force ellipsoid can be regarded as the load capacity of the food manipulator in a work position. On second thoughts, the minimal value of all the load capacity in work path or workspace can be regarded as the load capacity of the food manipulator in work path or workspace, named force transmission capacity of the parallel food manipulator in work path or workspace. In calculating process, the dimension of parameters choose international unit, so the unit of force transmission capacity is Newton.

From Eq. (12), if the length of driving force vector is  $d$ , the axis of the force output ellipsoid is  $\sqrt{d}$  times of the axis of unit output force ellipsoid that unit driving force vector generates in length. In other words, if the force transmission capacity of food manipulator need been raised  $d$  times, the length of driving force vector should be raised  $d^2$  time. It is useful for designer to confirm the power parameter of the food manipulator actuator.

**Simulation analysis of force transmission capacity and condition number of Jacobian:** Assuredly, the relation between force transmission capacity and condition number of Jacobian is exist, so a simulation analysis is carried on. As shown Fig. 2, the architectural parameters of Delta food manipulator are  $R_b, R_t, AB$  and  $BC$ . Then, let  $r_b, r_t, l$  and  $m$  are length of  $R_b, R_t, AB$  and  $BC$  respectively and shown as Table 1. A work path and a workspace of the Delta food manipulator are given as follows:

$$\begin{aligned} \text{Work path: } & x = 100 \cdot \cos \omega t ; y = 100 \cdot \sin \omega t ; \\ & z = 150 + 200 t \\ \text{Workspace: } & \text{Height, } 150\text{--}300 \text{ mm; diameter, } \varnothing 200 \text{ mm} \end{aligned}$$

In simulation process, the work path is divide into limited work positions equably. The workspace is divide into some section along height (170, 220, 270 mm, respectively) and every section is separated into finite work positions equably. The force transmission capacity of the work points is computed, the condition number of Jacobian matrix is computed accordingly in work positions. The results of simulation are shown in Fig. 5 and 6.

Table 1: Delta food manipulator parameters  
Architectural parameters/mm

$r_b$	$r_t$	$l$	$m$
60	130	170	250

The force transmission capacity and corresponding condition number of the given work path are shown in Fig. 5. The force capacity and corresponding condition number of the given workspace are shown in Fig. 6. Based on the simulation results and defining of force transmission capacity, the force transmission capacity of the Delta food manipulator are 7.1 N and 7.0 N in given work path and workspace respectively. Namely, the shortest axis of output force ellipsoid of the Delta food manipulator are 7.1 N and 7.0 N when driving force is a unit vector in the given work path and workspace. Thus, if the load of food manipulator is given, the power parameter of food manipulator actuators can be confirmed by the force transmission capacity. In addition, the simulation results shows that relation between force transmission capacity and condition number submits inverse proportion, the force transmission capacity of work position becomes greater as the condition number decreases. Actually, Eq. (9) revealed that force Jacobian matrix is transpose of velocity Jacobian matrix, thus, with decreasing of condition number of force Jacobian matrix, the stability of velocity performance and the force transmission capability be enhanced and vice versa. Finally, the change range of force transmission capability in work points is showed in Fig. 5 and 6, it reflects the stability of force transmission capability of the Delta food manipulator in given work path and workspace. In view of the above, the transmission capability and its change range can be helpful to optimize design of the parallel food manipulator.

**Distribution analysis of force transmission capability in workspace:** Considering the complication of force transmission capability of parallel food manipulator in workspace, distribution of force transmission capability is analyzed primitively in workspace. The curve of force transmission capability is showed as Fig. 7 in the sequence from bottom section to top section in workspace and it is observed that the

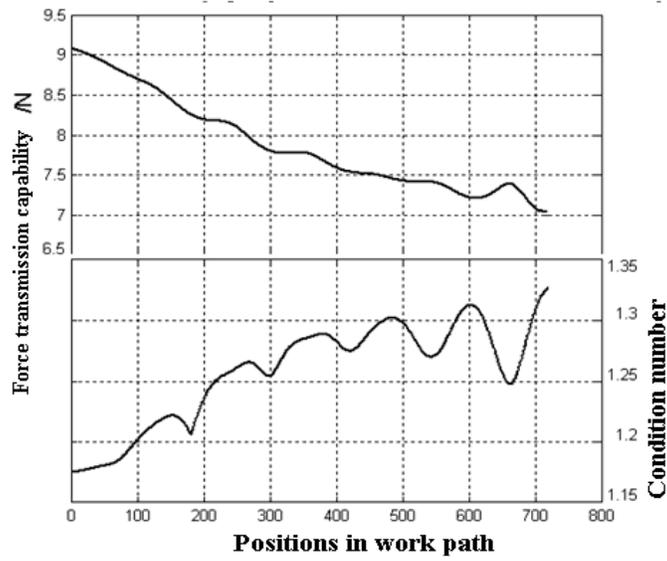


Fig. 5: Curve of force transmission capacity/condition number in work path

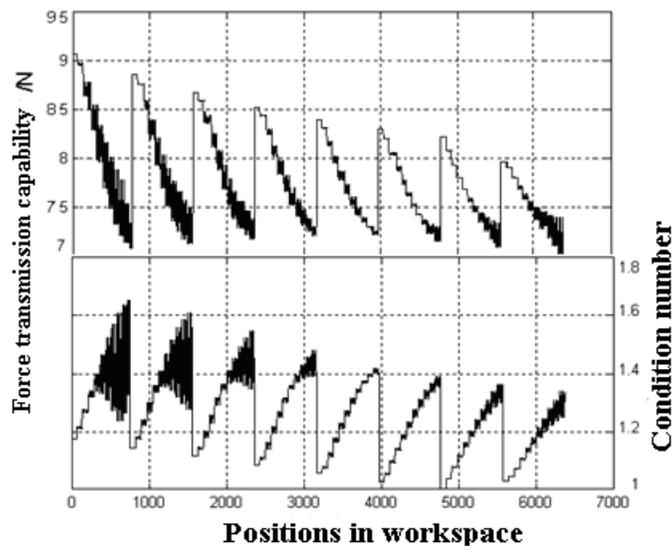
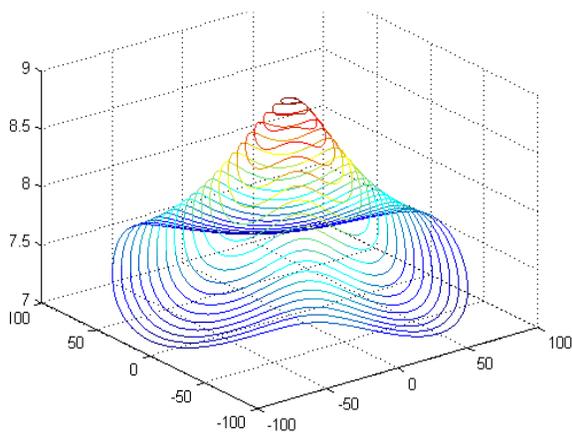
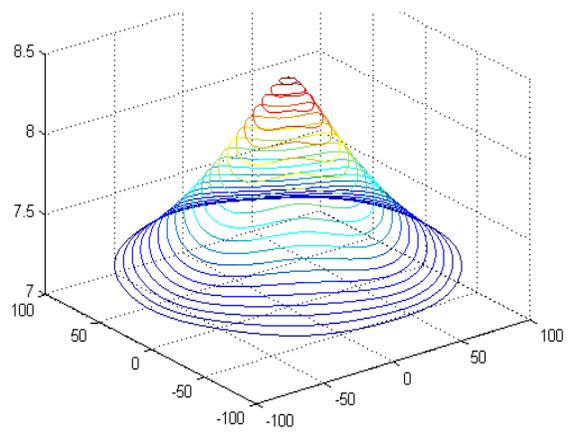


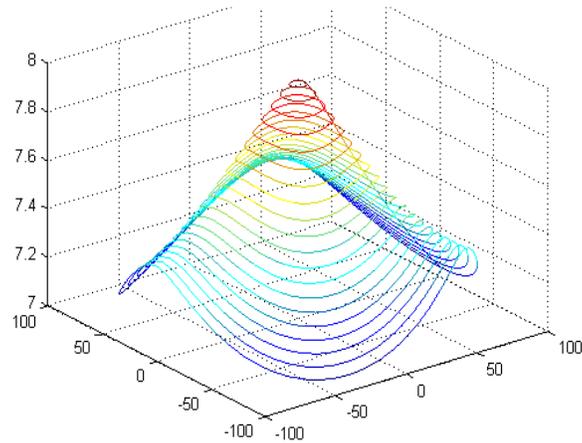
Fig. 6: Curve of force transmission capacity/condition number in workspace



(a) Cross-section in 170 mm



(b) Cross-section in 220 mm



(c) Cross-section in 270 mm

Fig. 7: Distribution of force transmission capacity on horizontal cross-section of workspace

force transmission capability is declining as the height of sections increases in workspace.

To describe and analyze the change of force transmission capability in the landscape orientation of workspace, the force transmission capability of landscape orientation sections is made chart as Fig. 7 shown; the height of landscape orientation sections are 170, 220 and 270 mm, respectively. In Fig. 7, the plane coordinate indicated the work position of each section, z-coordinate indicate force transfer capability values of the food manipulator in the work position. Just as Fig. 7 shown, force transmission capacity value of sections center is the highest and force transmission capacity is decreasing by distance from center of the sections; Near the middle section of the working space, as shown in Fig. 7b shows, force transfer capability is evenly decreasing along the radial orientation; Near the bottom and top workspace, as shown in Fig.7a and c, the force transmission capacity is decreasing by distance from center of the sections, but the change is uneven.

### CONCLUSION

The paper discussed the force transmission performance of parallel food manipulator in order to design and confirm the power parameters of actuators. Taking Delta parallel food manipulator as a example, the mathematic model of force transmission is established and provides the theory model for analyzing force transmission of the food manipulator. We propose the concept of the force transmission capability in work position of the food manipulator by the shortest axis of output force ellipsoid, which is used to indicate the load capability of parallel food manipulators. The force transmission capability of food manipulator can be used to confirm the power parameters of driving organs. The change range of force transmission capability can be used to describe the stability of force transmission in

the work path or workspace of food manipulators, which can applied in optimizing design of the food manipulator. By analyzing force transmission capability of the Delta food manipulator, we obtain the relation of force transmission capability and condition number of the force Jacobian matrix that submits inverse proportion. By the simulation results of the Delta food manipulator, the distribution character of force transmission capability is analyzed primitively in whole workspace.

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