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Research Article

Finite Element Analysis Flexible Hinge of Food Material Supported Twin Parallel 4-bar Mechanism Based on Software Calculations

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Abstract: This study designed a flexible hinge of food material supported twin-parallel 4-bar guide mechanism based on the material mechanics principles, setting up the flexible hinge of food material engineering math model. Key flexible hinge of food material parameter-rigidity formula derivation is made to study the features of key flexible hinge of food material parameters with calculations in Matlab. The mechanism's key parameters: rigidity *K*, max stress $\sigma_{\max}$, max output displacement $\Delta y_{\max}$ and natural frequency *f* and their relations are studied and indicated through Matlab calculations. The static characteristics and stress simulation for the mechanism are presented by the finite element analysis in ANSYS.

Keywords: ANASYS, Flexible hinge of food material, Matlab, math model

INTRODUCTION

The flexible hinge of food material design is particularly import in flexible hinge of food material supported fast micro-feeding mechanism. The basic performance parameters of flexible hinge of food material includes: rigidity, precision and stress characteristics. The rigidity reflects load carrying ability of the flexible hinge of food material, represents the joints flexibilities. The mechanics analysis of rigidity is the key in flexible hinge of food material design. It is very difficult to establish the flexible hinge of food material's rigidity and stress formulas because of its shape complex. So the basic assumptions for flexible hinge of food material design are made:

- The material of flexible hinge of food material is isotropy.
- The flexible hinge of food material deformation only happens at the thin wall.
- When Y direction load applies, only bend occurs.

The basic requirements for flexible hinge of food material design parameters are:

- The flexible hinge of food material's interior stress is lower than the material's allowable stress.
- The flexible hinge of food material resilience is lower than the feeding mechanism's max drive force as feeding mechanism makes max output displacement.
- The feeding mechanism's rigidity and natural frequency shall be as high as possible (Chen, 2012; Yang and Luo, 2010; Sun, 2012).

Figure 1 is the flexible hinge of food material structure. The main flexible hinge of food material parameters are: width b, minimum thickness t, cutting radius R, height h, central angle θ m. The moment Mz applies on the left end, assuming the flexible hinge of food material right end fixed, the left end deformation is αz. The flexible hinge of food material deformations on X are tension and compression, on Y is bending. So the rotational rigidity and tension rigidity are most important parameters in flexible hinge of food material design.

Flexible hinge of food material's rotational rigidity: The formula derivation of flexible hinge of food material rotational rigidity around Z is obtained by engineering mechanics and differential calculus method (Shibuya, 2010; Yang, 2012).

Figure 2 shows a micro-unit of central angle θ. The micro-unit height is:

$$a = t + 2R - 2R\cos\theta$$

The micro-unit thickness:

$$de = d(R\sin\theta) = R\cos\theta d\theta$$

The micro-unit rotational deformation on Z under the moment is:

$$d\alpha_z = \frac{M_z}{EI_z}de = \frac{12M_z}{EbR^2}\frac{\cos\theta}{(t/R + 2 - 2\cos\theta)^3}d\theta$$

E is elastic modulus. The micro-unit inertia moment on Z is: $I_z = ba^3/12$ so:

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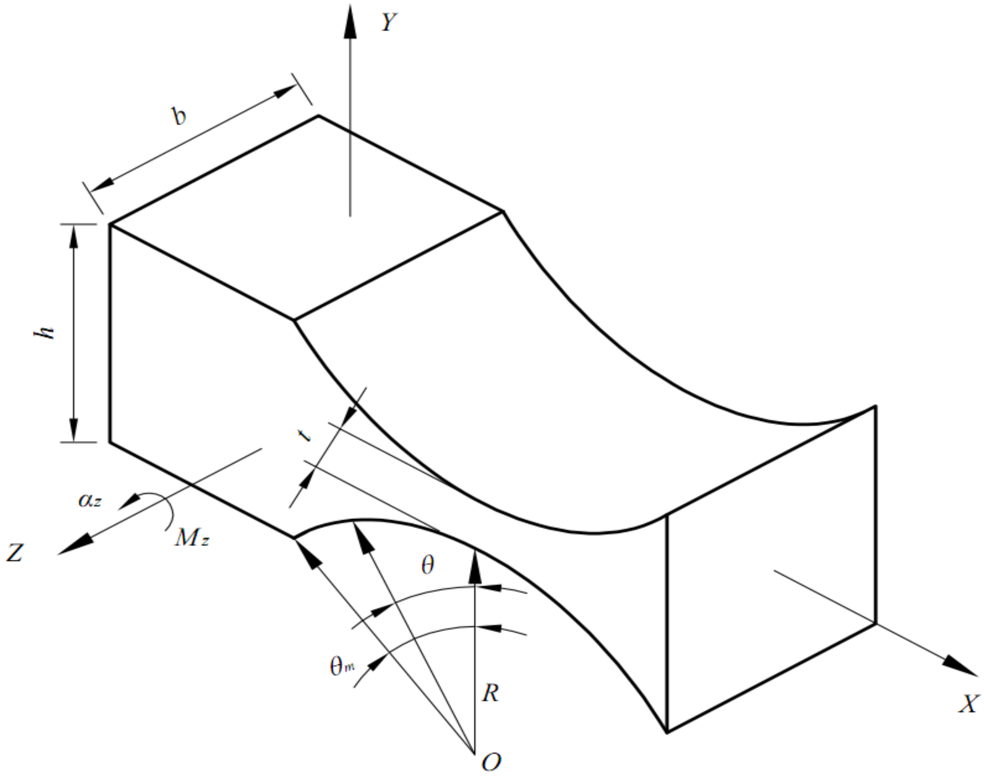

Fig. 1: The flexible hinge of food material structure

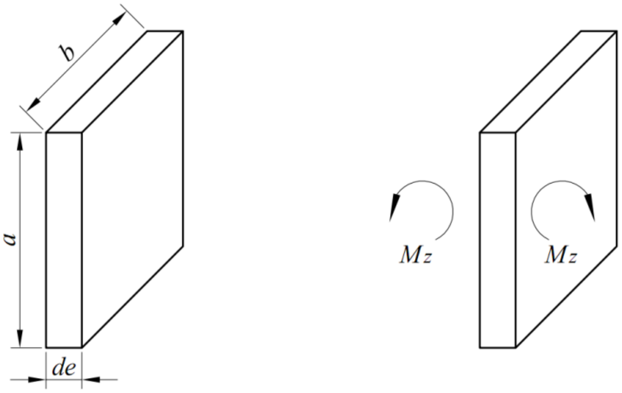

Fig. 2: A flexible hinge of food material micro-unit

$$\frac{\alpha_z}{M_z} = \frac{12}{EbR^2}\int_{-\theta_m}^{\theta} \frac{\cos\theta}{\left(t/R+2-2\cos\theta\right)^3}\,d\theta$$

$\int \frac{\cos\theta}{\left(t/R+2-2\cos\theta\right)^3}d\theta$ is the key integral formula in the deduction of the flexible hinge of food material's rotational rigidity. set $c = t/R+2$ so:

$$\int \frac{\cos\theta}{\left(t/R+2-2\cos\theta\right)^3}d\theta = \int \frac{\cos\theta}{\left(c-2\cos\theta\right)^3}d\theta$$

This indefinite integral's integrand is trigonometric rational form, which can be turned into rational function by trigonometric transformation to calculate the definite integral.
set $t = \tan(\theta/2)$, so:

$$d\theta = \frac{2}{1+T}dT$$

$$\sin\theta = \frac{2\tan\theta/2}{1+\tan^2\frac{\theta}{2}}dT = \frac{2T}{1+T^2}$$

$$\cos\theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}dT = \frac{1-T^2}{1+T^2}$$

And:

$$\int\frac{\cos\theta}{\left(c-2\cos\theta\right)^3}d\theta = -\frac{2}{\left(c+2\right)^2}\int\frac{1}{\left(c-2\right)+\left(c+2\right)T^2}dT +$$
$$\frac{4\left(c-2\right)}{\left(c+2\right)^2}\int\frac{1}{\left[\left(c-2\right)+\left(c+2\right)T^2\right]^2}dT +$$
$$\frac{16c}{\left(c+2\right)^2}\int\frac{1}{\left[\left(c-2\right)+\left(c+2\right)T^2\right]^3}dT$$

In above formula:

$$\int\frac{1}{\left[\left(c-2\right)+\left(c+2\right)T^2\right]^k}dT$$

Through the process of the mathematical induction, so:

$$\int\frac{1}{\left[\left(c-2\right)+\left(c+2\right)T^2\right]^{k+1}}dT$$
$$=\frac{2k-1}{2k(c-2)}\int\frac{1}{\left[\left(c-2\right)+\left(c+2\right)T^2\right]^k}dT +$$
$$=\frac{1}{2k(c-2)}\int\frac{1}{\left[\left(c-2\right)+\left(c+2\right)T^2\right]^k}dT$$

And:

$$\int\frac{\cos\theta}{\left(c-2\cos\theta\right)^3}d\theta =$$

$$\frac{4c}{(c-2)\left(c+2\right)^2}\frac{\tan\frac{\theta}{2}}{\left[\left(c-2\right)+\left(c+2\right)\tan^2\frac{\theta}{2}\right]^2}+$$

$$\frac{6c+2\left(c-2\right)^2}{(c-2)^2\left(c+2\right)^2}\frac{\tan(\theta/2)}{\left[\left(c-2\right)+\left(c+2\right)\tan^2(\theta/2)\right]}+$$

$$\frac{6c}{(c-2)^{\frac{5}{2}}\left(c+2\right)^{\frac{5}{2}}}\arctan\left[\sqrt{\frac{c+2}{c-2}}\tan\frac{\theta}{2}\right]+C_1$$

set $s=\frac{R}{t}$ and $c=\frac{1}{s}+2$.

The formula of flexible hinge of food material's rotational rigidity on Z is: $k=\frac{M_z}{\alpha_z}$ and:

$$\int_{-\theta_m}^{\theta}\frac{\cos\theta}{\left[\frac{t}{R}+2-2\cos\theta\right]^3}d\theta =$$

$$\frac{8s^4\left(2s+1\right)}{\left(4s+1\right)^2}\frac{\tan\frac{\theta_m}{2}}{\left[1+\left(4s+1\right)\tan^2\frac{\theta_m}{2}\right]}+$$

$$\frac{4s^3\left(6s^2+3s+1\right)}{\left(4s+1\right)^2}\frac{\tan\frac{\theta_m}{2}}{\left[1+\left(4s+1\right)\tan^2\frac{\theta_m}{2}\right]}+$$

$$\frac{12s^4\left(2s+1\right)}{\left(4s+1\right)^{\frac{5}{2}}}\arctan\left[\sqrt{4s+1}\tan\frac{\theta_m}{2}\right]$$

Straight circular flexible hinge of food material rigidity on *Z*: A straight circular flexible hinge of food material is selected for this study. The incisions of the straight circular flexible hinge of food material are two symmetrical half cylindrical surfaces which are vertical to the end face. The straight circular flexible hinge of food material's θ_m = 90°, which is substituted to the above equation and the straight circular flexible hinge of food material rigidity on Z is calculated:

$$\frac{\alpha_z}{M_z}=\frac{12}{EbR^2}\left[\frac{2s^3\left(6s^2+4s+1\right)}{\left(2s+1\right)\left(4s+1\right)^2}+\frac{12s^4\left(2s+1\right)}{\left(4s+1\right)^{\frac{5}{2}}}\arctan\sqrt{4s+1}\right]$$

Set:

$$\frac{\alpha_z}{M_z}=\frac{12}{EbR^2}\left[\frac{2s^3\left(6s^2+4s+1\right)}{\left(2s+1\right)\left(4s+1\right)^2}+\frac{12s^4\left(2s+1\right)}{\left(4s+1\right)^{\frac{5}{2}}}\arctan\sqrt{4s+1}\right]$$

The straight circular flexible hinge of food material rigidity on Z is:

$$k=\frac{M_z}{\alpha_z}=\frac{EbR^2}{12C} \tag{1}$$

The derivation of the straight circular flexible hinge of food material rigidity on Z is presented above.

Paros and Weisbord (1965) obtained the formula of the flexible hinge of food material rigidity on *Z*:

$$\frac{\alpha_z}{M_z} = \left(\frac{3}{2EbR^2(2\beta+\beta^2)}\right) \times \left\{\frac{1+\beta}{\gamma^2} + \frac{3+2\beta+\beta^2}{\gamma\left(2\beta+\beta^2\right)}\left[\sqrt{1-\left(1+\beta-\gamma\right)^2}\right] + \left[\frac{6\left(1+\beta\right)}{\left(2\beta+\beta^2\right)^{\frac{3}{2}}}\right]\arctan\left[\sqrt{\frac{2+\beta}{\beta}} \cdot \frac{\gamma-\beta}{\sqrt{1-\left(1+\beta-\gamma\right)^2}}\right]\right\}$$

$$\beta = t/2R\,;\ \gamma = h/2R \tag{2}$$

For most flexible hinge of food materials, the hinge's smallest thickness *t* is much smaller than the hinge's height *h* and cutting radius *R*. The above equation can be simplified as:

$$\frac{\alpha_z}{M_z} \approx \frac{9\pi}{2EbR^2\left(2\beta\right)^{\frac{5}{2}}} = \frac{9\pi R^{\frac{1}{2}}}{2Ebt^{\frac{5}{2}}}$$

$$k = \frac{M_z}{\alpha_z} = \frac{2Ebt^{\frac{5}{2}}}{9\pi R^{\frac{1}{2}}}$$

Straight circular flexible hinge of food material's deflection angle on *Z*: Driving force creates a moment on *Z* to the flexible hinge of food material which causes hinge's deflection angle α_z on *Z*. From Eq. (1):

$$\alpha_z = \frac{12CM_z}{EbR^2}$$

From Paros and Weisbord (1965) simplified formula:

$$\alpha_z' = \frac{9\pi M_z R^{\frac{1}{2}}}{2Ebt^{\frac{5}{2}}}$$

The straight circular flexible hinge of food material's *Y* displacement:

$$\Delta y = \alpha_z R = \frac{12CM_z}{EbR}$$

From Paros and Weisbord (1965) simplified formula:

$$\Delta y = \alpha_z R = \frac{9\pi M_z R^{\frac{3}{2}}}{2Ebt^{\frac{5}{2}}}$$

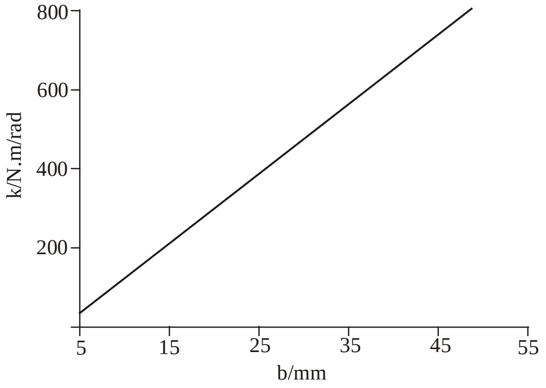

Fig. 3: The relation of *k* and *b*

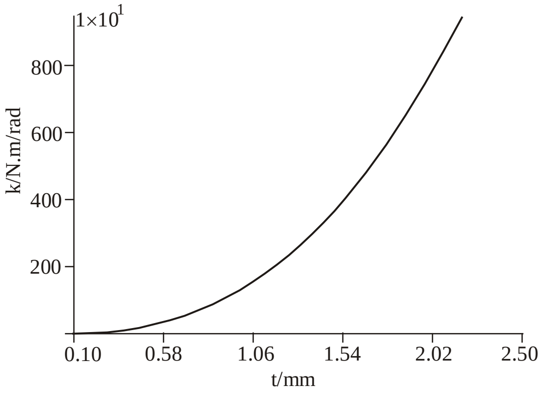

Fig. 4: The relation of *k* and *t*

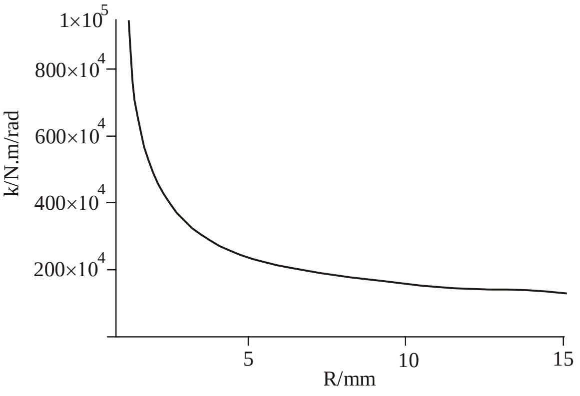

Fig. 5: The relation of *k* and *R*

Flexible hinge of food material rigidity calculations with Matlab: From above study, the flexible hinge of food material rotational rigidity is related to material elastic modulus *E*, hinge width *b*, cutting radius *R* and minimum thickness *t*. The relations of flexible hinge of food material rotational rigidity and hinge parameters are calculated in Matlab and shown in Fig. 3 to 5 (θ_m = 90°, *E* as constant).

As figures show:

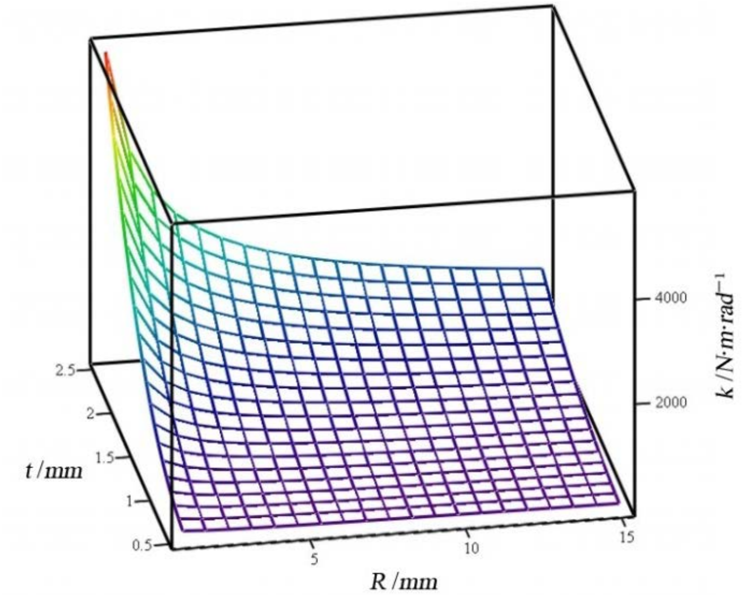

Fig. 6: The relation of *k* and *R*, *t*

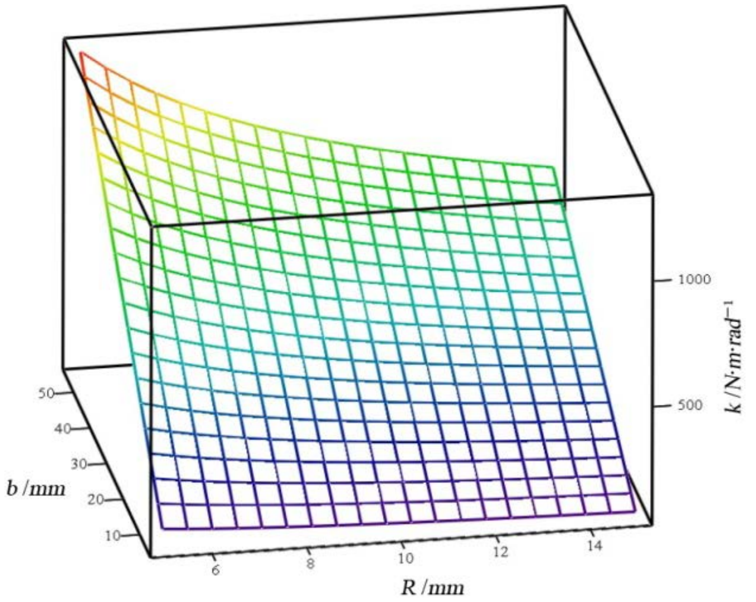

Fig. 7: The relation of *k* and *R*, *b*

- The flexible hinge of food material's rotational rigidity *k* is linearly increasing to the hinge width *b*
- The flexible hinge of food material's rotational rigidity *k* is incremental to the hinge's minimum thickness *t* and *k* increases faster as *t* increases.
- The flexible hinge of food material's rotational rigidity *k* decreases slower as hinge radius *R* increases

The main factors which influence the flexible hinge of food material rotational rigidity *k* are hinge's minimum thickness *t* and cutting radius *R*. Increasing the straight circular flexible hinge of food material's minimum thickness *t* will significantly increase the rotational rigidity while the cutting radius *R* remained the same.

From Fig. 3 and Eq. (2), the straight circular flexible hinge of food material rotational rigidity *k* is incremental to the material's elasticity modulus *E* and hinge width *b*. Increasing the material's elasticity modulus *E* and hinge width *b* will linearly increase rotational rigidity.

Figure 6 to 8 are the relations of rotational rigidity *k* with twin hinge parameters. From the figures, obviously the influence of minimum thickness *t* is much stronger than *R* and *b* (calculated in Matlab).

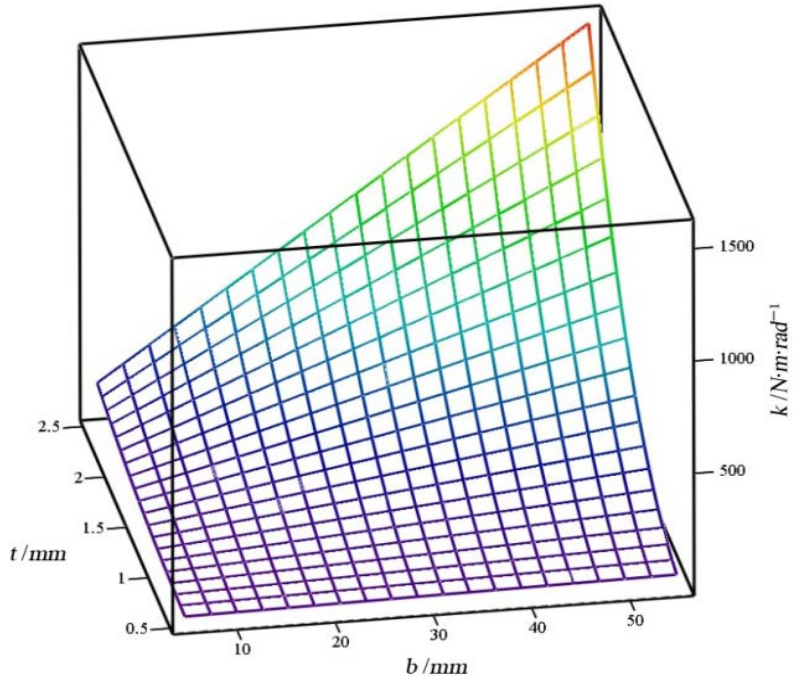

Fig. 8: The relation of *k* and *b*, *t*

When designing a flexible hinge of food material, the hinge material has been selected. And the changes in hinge's minimum thickness *t* are the most significant factor to the straight circular flexible hinge of food material rotational rigidity *k*. So first to decide the flexible hinge of food material's minimum thickness *t* which is supposed to be as small as possible to reach the high sensitivity and high definition of the mechanism and to effectively amplify the displacement. But if the *t* is too small, the hinge may not sustain larger or frequent alternate load which will cause malfunction or even fracture. Then to decide the hinge width *b* and the cutting radius *R*.

CONCLUSION

In this research, a flexible hinge of food material supported twin parallel 4-bar micro-feeding mechanism is designed. The conclusions of this study are as below:

- The influence of hinge minimum thickness *t* is much stronger than cutting radius *R* and hinge width *b*.
- The flexible hinge of food material mechanism rigidity *K* greatly increases as its minimum thickness *t* increases, decreases as its cutting radius *R* increases.
- The flexible hinge of food material's rigidity length *l* linearly correlated with ω under same max output displacement $\Delta y_{\max}$.
- The max out displacement $\Delta y_{\max}$ decreases as ω increases under same *l*. The flexible hinge of food material's rigidity length *l* places significant impact on mechanism's displacement and rigidity.
- The cutting radius *R* creates much less influence on hinge stress, but *t* places significant influence on the stress. The flexible hinge of food material's max stress $\sigma_{\max}$ increases as *l* increases but decreases as *b* increases.
- The mechanism' natural frequency *f* decreases as *R*, *l* increase and the hinge thickness *t* places stronger influence on the natural frequency.
- The max stress generated by the loading is much smaller than the allowable stress under ANASYS stress simulation.

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REFERENCES

Chen, H.J., 2012. Analysis of the Rigidity of Arc Flexible Hinge of Food Material with Different Geometrical Parameters. In: Li, D. and Y. Chen (Eds.), Computer and Computing Technologies in Agriculture V. Part III, IFIP Advances in Information and Communication Technology. Springer, Berlin, Heidelberg, 370: 195-200.

Paros, J.M. and L. Weisbord, 1965. How to design flexure hinge. Mach. Des., 37: 151-157.

Shibuya, T., 2010. Fast hinge detection algorithms for flexible protein structures. IEEE ACM T. Comput. Bi., 7(2): 333-341.

Sun, W., 2012. Simulation and optimization of flexible Hooke hinge. Appl. Mech. Mater., 201-202: 574-577.

Yang, C. and S. Luo, 2010. Analysis of the rigidity of an elliptical flexible hinge of food material with different geometrical parameters. Proceeding of the International Conference on System Science, Engineering Design and Manufacturing Informatization (ICSEM, 2010), 2: 205-208.

Yang, C., 2012. Calculation and analysis of parabolic flexure hinge. Proceeding of the International Conference on Measurement, Information and Control (MIC, 2012), 1: 5-8.