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**Research Article**

# A Study on Loading and Unloading Criteria and Strength Criterion of Unloading Food Mass

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**Abstract:** Based on basic deformation mechanism of jointed food mass, combining loading and unloading concept of classical mechanics, we explored the practicability of classical loading and unloading criterion and proposed the principle is not suitable for deformation mechanism of jointed food mass, initially put forward the loading and unloading criterion for jointed food mass. We conducted the model test of unloading food mass. In order to reflect the jointed food mass properties, embedded-film technique is applied in model test. Food mass under unloading condition showed compression-shear failure. When the angle between jointed plane and unloading direction is 40° or 60°, the failure will be along the jointed plane. Finally, we proposed the unloading strength criterion.

**Keywords:** Food mass, model test, strength criterion

## INTRODUCTION

At present, unloading concept has been widely used in the field of food technology engineering, such as "unloading cracks", "excavation unloading" etc. In 1995, Professor Qiulin Ha first put forward the basic thought of unloading food mass mechanics (Qiulin and Jianlin, 1998). In the previous study, loading and unloading are been discriminated according to different food technology engineering, which may cause the false concept that loading is loading and excavation is unloading and is difficult to application in the theoretical analysis and the numerical calculation. Therefore, food mass loading and unloading mechanical concepts should be starting from basic mechanics, determine the food mass loading and unloading criteria according to the study of stress and deformation mechanism of the unit body. The mechanical properties of food mass is greatly different with mechanics properties of classic materials, it is necessary to put forward the theory of mechanics of food mass on the loading, unloading criterion.

Many scholars have studied the failure criterion of food mass, many famous failure criteria were obtained including theoretical and empirical ones. Coulomb theory is applied to the friction material, which has little or no cohesive force and it has many problems to the brittle material (Coulomb, 1998). The criterion Hoke and Brown (1980) proposed is the most representative, it has become most receptive to the fractured food strength criterion in the food engineering and engineering geology, but it is based on the loading conditions (Hoke and Brown, 1980). Griffith failure theory can reflect the initial state of the brittle failure of food, but it cannot reflect the whole process of food failure (Griffith, 1990).

A large number of engineering practice shows that the unloading failure of food mass is very common in the excavation of food mass. However, the classical failure criterion is based on the loading conditions. Because of the different stress paths, the compressive strength of food materials is much greater than that of the tensile properties. The damage effect of food mass is mainly reflected in tension aspect. Therefore, it is necessary to establish a new strength criterion for food mass unloading stress state.

## MATERIALS AND METHODS

**Classic loading and unloading criteria:** In the classical elastic-plastic theory, when the stress is in the plastic yield surface and loading increment increasing or decreasing, stress and strain obey different laws. As the stress increases, continue to produce the plastic strain, while reducing stress, material back to the elastic stress state, stress increment only produce elastic strain

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and does not produce plastic strain. The stress increment whether to generate a new plastic strain increment, which is an important feature of the plastic stage (Jianlin, 2003). So it is necessary to have a guidelines to distinguish loading and unloading of material, which is called loading and unloading criterion. Loading is the abbreviation of the plastic loading, while the "unloading" refers to the situation from the plastic to return to the case of elastic state. Thus the loading and unloading criterion of classical elastic-plastic material is the signs whether to produce plastic deformation. Elasticity constitutive model is adopted at unloading, elastic-plastic constitutive model is adopted at loading. Therefore, we must determine whether the stress $\sigma_{ij}$ is on the loading surface and relative relationship between stress increment $d\sigma_{ij}$ and $\phi = 0$.

For the ideal material, loading plane (subsequent yield surface) is the same with initial yield surface, that is $\phi = f$, due to the yield surface can not be expanded, when the stress reaches the yield surface, the stress increment vector $d\vec{\sigma}$ can not point to the yield surface outside, the plastic loading is only stress along the yield surfaces moving. The ideal elastic-plastic material loading, unloading criteria is:

$$f(\sigma_{ij}) < 0 \qquad \textit{elastic state}$$

$$\left.\begin{aligned} & f(\sigma_{ij}) = 0 \\ & df = \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} < 0 (d\vec{\sigma} \cdot n = 0) \end{aligned}\right\} \textit{loading}$$

$$\left.\begin{aligned} & f(\sigma_{ij}) = 0 \\ & df = \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} < 0 (d\vec{\sigma} \cdot n < 0) \end{aligned}\right\} \textit{unloading}$$

For strengthening material, stress space $\phi = 0$ can be expanded or mobile, so $d\vec{\sigma}$ can point to the surface $\phi = 0$ outside. Mathematical expression for the loading, unloading criteria:

$$\phi = 0 \quad \frac{\partial \phi}{\partial \sigma_{ij}} d\sigma_{ij} > 0 (d\vec{\sigma} \cdot n > 0) \quad \textit{loading};$$

$$\phi = 0 \quad \frac{\partial \phi}{\partial \sigma_{ij}} d\sigma_{ij} = 0 (d\vec{\sigma} \cdot n = 0) \quad \text{neutral loading};$$

$$\phi = 0 \quad \frac{\partial \phi}{\partial \sigma_{ij}} d\sigma_{ij} < 0 (d\vec{\sigma} \cdot n < 0) \quad \textit{unloading};$$

The above classic elastic-plastic loading and unloading criteria solve an important mechanical problem, which is the elastic stress stage and plastic stress stage, based on the divide of the stress stage, mechanical constitutive model can be created under different stress environment, providing the necessary theoretical basis for mechanics mechanism and numerical analysis. Therefore, the loading and unloading criteria of the food mass also must solve the problem.

**Objective analysis on mechanical properties of food mass:** There are great differences between mechanical properties of food mass and mechanics properties of the ideal elastic-plastic materials, mainly in the following points:

The food mass is a non-homogeneous, non-continuous, anisotropic materials, the change of mechanical properties of the structure plane have a very important impact on the mechanical properties of the food mass.

On the condition of stress state and different stress path, stress-strain relationship is different, when the structure plane normal stress is reduced, the mechanical strength and deformation parameters of food mass weakened. Changes in the nature of the structure plane cause changes in mechanical properties of the food mass.

In the classical elastic-plastic mechanics, deformation is mainly divided into elastic deformation and plastic deformation. In food mass, in addition to elastic deformation and plastic deformation, the aperture deformation and damage and creep deformation of material are contained, two kinds of material constitutive model is not exactly the same. Loading and unloading criteria reflect the mechanical concepts that stress-strain relationship is different under different stress environment, thus classic loading and unloading criteria can not be entirely suitable for food mechanics criteria.

The loading and unloading criterion only when a stress is applied in the yield surface, to reflect the different stress increment under stress and strain change, but does not take into account the change of the mechanical properties when different stress increment in any stress state in the unit body, loading and unloading criterion for food mass should not only consider the stress generated by the mechanical changes of stress increment in yield surface, but also reflected mechanical properties under different stress increment in any state of stress.

Food mass permeability properties obey different laws when fractured surface pressure and pressure relief. Because the pressure relief, open of structural planes, the deterioration of groundwater seepage condition, the seepage field corresponding to change significantly, seepage flow increases.

Apparently the loading and unloading of the classic criteria can not be completely clear demarcation between food mass constitutive relations of the various stages.

**Loading and unloading criteria for food mass:** The normal stress and shear stress change of the structure plane is the major factor which affect the mechanical properties of the joint plane, the loading and unloading of the food mass criteria should be combined with the normal stress and shear stress of structure plane. Food mass stress-strain constitutive model can be divided into the following eight kinds:

- In compression-shear stress state, the state of stress is within the yield surface, the normal stress of structure plane increasing
- In compression-shear stress state, the state of stress is within the yield surface, the normal stress of structure plane reduction
- In compression-shear stress state, the state of stress is on the yield surface, the normal stress of structure plane increasing
- In compression-shear stress state, the state of stress is on the yield surface, the normal stress of structure plane reduction
- In tensile-shear stress state, the state of stress is within the yield surface, the normal stress of structure plane increasing
- In tensile-shear stress state, the state of stress is within the yield surface, the normal stress of structure plane reduction
- In tensile-shear stress state, the state of stress is on the yield surface, the normal stress of structure plane increasing
- In tensile-shear stress state, the state of stress is on the yield surface, the normal stress of structure plane reduction.

Take the normal stress and shear stress of structural plane as the strength parameters, the set joint plane strength function as:

$$f=\phi(\sigma_n,\tau_n)=0$$

If compression-shear failure for sturcture plane, Mohr-Coulomb strength criterion can be used, yield function is:

$$f=\phi(\sigma_n,\tau_n)=\tau_n-c-\sigma_n tg\varphi=0$$

If tensile-shear failure for structure plane, tensile-stress strength criterion can be used, yield function is:

$$f=\phi(\sigma_n,\tau_n)=\sigma_n-\sigma_l=0$$

Known stress state of a point, the stress of the joint surface:

$$\left.\begin{array}{l} p_1=l_1\sigma_{11}+l_2\sigma_{12}+l_3\sigma_{13} \\ p_2=l_1\sigma_{12}+l_2\sigma_{22}+l_3\sigma_{23} \\ p_3=l_1\sigma_{13}+l_2\sigma_{23}+l_3\sigma_{33} \end{array}\right\} \text{ or } p_i=\sigma_{ij}l_j\ (j=1,2,3)$$

$$\sigma_n=l_1p_1+l_2p_2+l_3p_3 \text{ or } \sigma_n=p_jl_j\ (j=1,2,3)$$

$$\tau_n^2=p_1^2+p_2^2+p_3^2-\sigma_n^2$$
$$\text{or } \tau^2=p_i^2-\sigma_n^2\ (j=1,2,3)$$

$l_i$ is structural plane direction cosine.
Three principal stress is expressed as:

$$\sigma_n=l_1^2\sigma_1+l_2^2\sigma_2+l_3^2\sigma_3 \quad (1)$$

$$d\sigma_n=l_1^2d\sigma_1+l_2^2d\sigma_2+l_3^2d\sigma_3 \quad (2)$$

$$\tau_n^2=l_1^2\sigma_1^2+l_2^2\sigma_2^2+l_3^2\sigma_3^2-\sigma_n^2 \quad (3)$$

When the compressive-stress state, namely $\sigma_n > 0$, stress state is within the yield surface, namely $\phi < 0$, loading and unloading criterion can be judged based on the increase and decrease of the stress increment of structural plane:

$$\left.\begin{array}{ll} \phi<0 \quad d\sigma_n>0 \ \ or \ \ l_1^2d\sigma_1+l_2^2d\sigma_2+l_3^2d\sigma>0 & loading \\ \phi<0 \quad d\sigma_n<0 \ \ or \ \ l_1^2d\sigma_1+l_2^2d\sigma_2+l_3^2d\sigma<0 & unloading \end{array}\right\}$$

When the compressive-stress state, namely $\sigma_n > 0$, stress state is on the yield surface, namely $\phi = 0$, loading and unloading criteria can be judged according to the following formula:

$$\left.\begin{array}{ll} \phi=0 \quad \frac{\partial\phi}{\partial\sigma_n}d\sigma_n>0(d\sigma_n\cdot n>0) & loading \\ \phi=0 \quad \frac{\partial\phi}{\partial\sigma_n}d\sigma_n<0(d\sigma_n\cdot n<0) & unloading \end{array}\right\}$$

## RESULTS AND DISCUSSION

**Model making:** This experient uses RMT-150C food mechanics test system. Cement mortar is selected as the model material. Joint is simulated by embedded thin film. Standard specimen for D (diameter) = 50 mm, H (height) = 100 mm of the cylinder is used, the mortar is simulated material. Mortar mix ratio, mechanical parameters are shown in Table 1.

**Experiment scheme:** The stress path of constant axial pressure and unloading confining pressure is used in the strength test. Uniaxial compressive strength test was done before three axis strength test and the uniaxial compressive strength of the specimen was obtained.

The stress path for Triaxial strength test: Plus the amount of axial pressure and confining pressure up to a certain value which is slightly higher than the uniaxial compressive strength, then constant axial pressure, gradually unloading confining pressure, the sample can reach the failure in the unloading process.

**Analysis on the test result:** The shear of the joints along the crack surface is compared with the shear strength of the joints in the strength model test, as shown in Table 2.

The normal stress and shear stress of the structural plane in the table are calculated according to the stress formula of the oblique section of the material mechanics. The shear strength of fractured surface is calculated by the following formula:

Table 1: Mortar mix ratio and related parameter

| Sample | Ratio Cement | Water | Sand | Unit weight (kN/m$^3$) | Elastic modulus (GPa) | Compressive strength (MPa) | Poisson ratio |
|---|---|---|---|---|---|---|---|
| Hc | 1.25 | 1 | 4.68 | 2.08 | 6.48 | 16 | 0.22 |

Table 2: Shear force and shear strength on fracture

| Angle between structure plane and unloading direction | $\sigma_1$ /MPa | $\sigma_3$ /MPa | Normal stress of structure plane /MPa | Shear stress of structure plane/MPa | Shear strength of structure plane/MPa | Shear strength/ Shear stress |
|---|---|---|---|---|---|---|
| 20° | 23.7 | 1.70 | 21.2 | 7.09 | 11.47 | 1.62 |
| 40° | 18.18 | 2.01 | 11.5 | 7.96 | 8.12 | 1.02 |
| 60° | 17.11 | 2.73 | 6.23 | 6.23 | 5.92 | 0.95 |

$$\tau = (\sigma_n tg\varphi + c)A_1 / A + f\sigma_n A_2 / A$$

$\tau$ is shear strength of fractured surface; A, $A_1$, $A_2$, respectively, the total area in the direction of structure plane, the area of the food and the area of the structure plane; $\sigma_n$ is normal stress of structural plane; $\varphi$, $c$ is the internal friction angle and cohesion of food and $f$ is the friction coefficient of the structural plane.

Under the compression-shear stress state of food mass, the unloading failure of food mass is brittle shear failure. With increasing of angle between structural plane direction and unloading direction, the normal stress decreases of structural plane, the shear stress increases of structural plane, the shear strength of the structural plane is accordingly decreased. When the angle is small as 20°, the normal stress value of the structural plane is larger and the shear stress is relatively small. The shear capacity provided by the normal stress is also larger. The average shear strength on the structural plane is 1.62 times that of.
The proposition of intensity criterion:

**The shear stress on the structural plane:** The shear failure is not destroyed along the structural plane. But when the angle between structural plane direction and unloading direction up to 40°and 60°, the normal stress value of the structural plane is relatively small and the shear stress is relatively larger. Then the shear capacity provided by the normal stress is accordingly small. The average shear strength on the structural plane is 1.02 times and 0.95 times that of the shear stress on the structural plane. That is to say shear stress is basically equal to shear strength. Thus, in the compression-shear stress state, the failure of food mass is mainly in the form of shear failure along the structural plane. It is necessary to establish a unified function to represent the failure conditions in the complex stress state. For the general case, this function may be:

$$\Phi(\sigma_{ij}, \varepsilon_{ij}, \dot{\varepsilon}_{ij}, t, T, w) = 0$$

If the time effect $t$, groundwater $w$ and the strain rate of unloading $\varepsilon_{ij}$ are not considered, it is close to the normal temperature $T$, Function can be simplified to:

$$\Phi(\sigma_{ij}) = 0$$

The failure phenomenon and result analysis of food mass model test show that the failure of food mass is shear failure along the structural plane and the failure conditions can be expressed by the following formula:

$$\tau_n - [ntg\varphi + (1-n)f]\sigma_n - cn = 0 \quad (4)$$

where,
$\tau_n$ = The shear strength along the structural plane
$\sigma_n$ = Normal stress along the structural plane
$n$ = Structure surface connectivity
$\phi$ = The internal friction angle of food
$c$ = The cohesive force of food
$f$ = The friction coefficient of the structural plane

Take (1) and (2) into (4):

$$\sqrt{(p_1^2 + p_2^2 + p_3^2) - (l_1 p_1 + l_2 p_2 + l_3 p_3)^2}$$
$$-[ntg\varphi + (1-n)f](l_1 p_1 + l_2 p_2 + l_3 p_3) - cn = 0$$

令A=$[ntg\varphi + (1-n)f]$, $B = cn$, then:

$$(p_1^2 + p_2^2 + p_3^2) - (1 + A^2)(l_1 p_1 + l_2 p_2 + l_3 p_3)^2 -$$
$$2AB(l_1 p_1 + l_2 p_2 + l_3 p_3) - B^2 = 0$$

Expression of shear failure of food mass along the structural plane:

$$p_1^2 + p_2^2 + p_3^2 - (l_1 p_1 + l_2 p_2 + l_3 p_3)^2$$
$$-[A(l_1 p_1 + l_2 p_2 + l_3 p_3) + B]^2 = 0 \quad (5)$$

In the principal stress space, the expression of the damage conditions is:

$$l_1^2\sigma_1^2 + l_2^2\sigma_2^2 + l_3^3\sigma_3^2 -$$
$$(l_1^2\sigma_1 + l_2^2\sigma_2 + l_3^2\sigma_3)^2 -$$
$$[A(l_1^2\sigma_1 + l_2^2\sigma_2 + l_3^2\sigma_3) + B]^2 = 0 \quad (6)$$

## CONCLUSION

Classical loading and unloading criterion is not suitable for deformation mechanism of jointed food mass, the loading and unloading criterion for jointed food mass was been proposed.

The failure of unloading food mass is mainly manifested by the tension-shear composite failure. When the structural plane is in a certain direction (such as 20°), under certain stress conditions, damage is not occurring along the structural plane, but still in the food material.

When unloading damage, the failure surface vary with different joint angle. The specimens of the joints of 40° and 60° are expressed as the failure of the structural surface.

The failure phenomenon and result analysis of the food mass model test show that failure along the structure plane is shear failure.

A new unloading failure criterion of food mass is established.

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