

## Research Article

# Reliability and Sensitivity Analysis of Cast Iron Water Pipes for Agricultural Food Irrigation

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**Abstract:** This study aims to investigate the reliability and sensitivity of cast iron water pipes for agricultural food irrigation. The Monte Carlo simulation method is used for fracture assessment and reliability analysis of cast iron pipes for agricultural food irrigation. Fracture toughness is considered as a limit state function for corrosion affected cast iron pipes. Then the influence of failure mode on the probability of pipe failure has been discussed. Sensitivity analysis also is carried out to show the effect of changing basic parameters on the reliability and life time of the pipe. The analysis results show that the applied methodology can consider different random variables for estimating of life time of the pipe and it can also provide scientific guidance for rehabilitation and maintenance plans for agricultural food irrigation. In addition, the results of the failure and reliability analysis in this study can be useful for designing of more reliable new pipeline systems for agricultural food irrigation.

**Keywords:** Agricultural food irrigation, cast iron pipes, life time prediction, reliability analysis

## INTRODUCTION

Cast iron has been widely used as a construction material in general and as pipe material in water industry, in particular before the 1960s. In the United Kingdom, although cast iron pipes are being phased out of the water pipeline network, a significant portion of current networks are comprised of cast iron pipes and some of them can be up to 150 years old (Mohebbi and Li, 2011). There are about 335,000 km of water mains in the UK and more than 60% is estimated to be cast iron pipes (Mahmoodian and Li, 2011a). Pipes used in trunk mains networks, which take water from reservoirs to major settlements, are of large diameter, usually greater than 300 mm, whilst pipes used in distribution mains networks are typically of a diameter between 75 to 300 mm.

Same as other infrastructure, age is one of predominant causes of pipe failure. Thus, it is predictable for infrastructure managers that the number of incidents in agricultural food irrigation systems is likely to be increasing in the future as systems grow older. To have an optimum strategy for maintenance and rehabilitation plans in for agricultural food irrigation systems, accurate prediction of service life is essential. Predictive models can be classified into deterministic and probabilistic categories. Deterministic models do not consider variation in deriving breakage patterns, based on pipe age and breakage history. Probabilistic models consider some or all parameters as random variables that affect the system. A comprehensive review of models for service life

prediction of water mains can be found in Rajani and Kleiner (2001).

In the case of failure assessment of cast iron water pipes for agricultural food irrigation, the uncertain nature of the problem necessitates infrastructure managers to use probabilistic approaches rather than deterministic ones. Material corrosion is the most common form of pipeline deterioration and is a matter of concern for both the safety and serviceability of the pipes. Corrosion affects strength of the pipe and it is of practical importance to know how to incorporate the effect of corrosion in the structural analysis of a pipeline. There are several parameters which may affect corrosion rate and hence the reliability of pipelines. In conventional methods for service life prediction of pipelines, these parameters are considered to be deterministic. However, in reality there are uncertainties associated to these parameters. In a time-dependent reliability problem all or some of uncertain variables are modelled as stochastic processes. The evaluation of the contributions of these uncertain parameters is carried out by using sensitivity analysis techniques.

Ahamed and Melchers (1997) proposed a reliability method to predict the probability of failure of underground pipes. In their study models for both the residual capacity and stresses in the pipe were proposed. A nonlinear corrosion model was used to represent the loss of pipe wall thickness over time. The probability of failure of the pipes was calculated by first order second moment method. Camarinopoulos *et al.* (1999) presented a method to assess the reliability of water pipes, considering both the corrosion pit depth

and the load that was likely to cause failure. A power law model was developed for corrosion pits which leads to the loss of wall thickness. Monte-Carlo simulation technique was employed for reliability calculation of the pipes. Yves and Patrick (2000) also presented a method to calculate the reliability of the buried water mains, using the maintenance records and the Weibull distribution for underlying variables. The method appears to rely entirely on the historical data, which in most cases is unknown. Rajani *et al.* (2000) proposed a method to estimate the remaining service life of grey cast iron mains by considering that the corrosion pits reduce the structural capacity of the pipes. The residual capacity of the pipes was calculated by a reiterative model based on corrosion pit measurement and the anticipated corrosion rate, which may not always be available due mainly to operational conditions. To consider the uncertainties involved in all factors contributing to the corrosion and subsequent failures, various researches on probabilistic assessment of cast iron pipes have been undertaken. Sadiq *et al.* (2004) developed a probabilistic method to predict the remaining service life of in-service pipes based on Monte Carlo simulation. A model of residual capacity of the pipes was proposed based on corrosion induced deterioration of the pipe. The failure of pipes was determined when the factor of safety of an individual pipe segment falls below a minimum acceptable value set by the utility owner. Lee *et al.* (2010) also used First Order Reliability Method (FORM) to evaluate time-dependent reliability index for a fully deteriorated piping component rehabilitated with FRP considering the demand of internal fluid pressure, external soil pressure and traffic loading. Mahmoodian and Li (2011a) developed an analytical time dependent reliability method to predict the probability of failure for concrete buried pipes. The analytical results, obtained from first passage theory, then verified by Monte Carlo simulation method. Mahmoodian and Li (2011b) also used a numerical method for system reliability analysis of cast iron pipes. They considered two limit state functions as failure modes of cast iron pipes and by using system reliability analysis they predicted service life of corrosion affected cast iron mains.

Inspection of service failures of trunk mains in service reveals that most cast iron water mains failures are of fracture type, i.e., the failure is caused by the growth of a crack and subsequent collapse of the pipe (Marshall, 2001). Cast iron pipes can fail as a result of loss of toughness due to the stress concentration at the tips of cracks or in general defects in the pipes. The review of the research literature also shows that the time effect of the deterioration process of cast iron pipes has not been explicitly considered in the assessment of pipe collapse. It is well known that both the deterioration of pipes and the external actions on the pipes are not only time-variant but also highly

uncertain. As such a method based on the theory of stochastic processes is more appropriate in assessing the pipe collapse and its remaining service life. However, little work has been done to address the reliability and sensitivity analysis of cast iron water pipes for agricultural food irrigation. It is imperative to develop useful techniques to deal with this issue.

In order to investigate the reliability and sensitivity of cast iron water pipes for agricultural food irrigation, this study proposed the Monte Carlo simulation based method for fracture assessment and reliability analysis of a cast iron pipe in an agricultural food irrigation system. Sensitivity analysis also is carried out to show the effect of each basic variable on the reliability and service life of the pipe system.

## MATERIALS AND METHODS

The predominant deterioration mechanism on the exterior of cast iron pipes is electro-chemical corrosion with the damage occurring in the form of corrosion pits. The damage to cast iron is often identified by the presence of graphitisation, a result of iron being leached away by corrosion. Each form of metal loss represents a corrosion pit that grows with time and reduces the thickness and mechanical resistance of the pipe wall. This process eventually leads to the breakage of the pipe.

Corrosion pits have a variety of shapes with characteristic depths, diameters or widths and lengths. They can develop randomly along any segment of water pipe and tend to grow with time at a rate that depends on environmental conditions in the immediate vicinity of the water main (Rajani and Makar, 2000). Models for estimation of the depth of corrosion pit are usually presented the form of the following equation:

$$a = kt^n \quad (1)$$

where,  $t$  is exposure time and  $k$  and  $n$  are empirical constants largely determined from experiments and/or field data.

Rajani *et al.* (2000) proposed a two-phase corrosion model to accommodate this self-inhibiting process:

$$a = \alpha t + \beta(1 - e^{-\lambda t}) \quad (2)$$

where  $\alpha$ ,  $\beta$  and  $\lambda$  are constant parameters.

In the first phase of the above equation there is a rapid exponential pit growth and in the second phase there is a slow linear growth. This model was developed based on the data set that lacked sufficient points in the early exposure times. Therefore prediction of pit depth in the first 15-20 years of pipe life should be considered highly uncertain when (2) is used.

**Formulation of fracture assessment and reliability analysis:** Failure can be defined in relation to different

possible mechanisms and can be describe by a limit state function. In the theory of structural reliability these criteria can be expressed in the form of limit state functions as follows:

$$G(S, R, t) = S(t) - R(t) \tag{3}$$

where,

- $G(S, R, t)$  = Termed the limit state function
- $S(t)$  = The action (load effect) at time  $t$
- $L(t)$  = The critical limit (resistance) for the action or its effect

The reliability of a structure or a component is defined as its probability of survival as follows:

$$\text{Reliability} = P_s = 1 - P_f \tag{4}$$

where,

- $P_s$  = The probability of survival
- $P_f$  = The probability of failure

There are two approaches for calculation of the probability of failure in a time dependent reliability analysis problem: Analytical approaches and numerical approaches. In this study numerical method (Monte Carlo simulation) is used for calculation of the probability of failure.

Considering the safety definition, the structure will fail if its resistance  $R$  is less than the stress resultant  $S$  action on it. Therefore the probability of failure can be expressed as follows:

$$P_f(t) = P[R(t) - S(t) \leq 0] = P[G(R(t), S(t)) \leq 0] \tag{5}$$

In the presence of corrosion, failure can be defined as when the relevant stress intensity factors exceed fracture toughness.

**Fracture toughness limit state:** To consider these failure criteria, fracture toughness limit state function is established as follows.

Stress Intensity Factor,  $K$ , is used in fracture mechanics to more accurately predict the stress state near the tip of a crack caused by a remote load or residual stresses. It is a parameter that amplifies the magnitude of the applied stress that includes the geometrical parameter. Al Laham (1999) presents the formulation for stress intensity factor as the following:

$$K_I = \sqrt{\pi a} \sum_{i=0}^3 \sigma_i f_i \left( \frac{a}{d}, \frac{2c}{a}, \frac{R}{d} \right) \tag{6}$$

where,

- $K_I$  = The stress intensity factor for crack mode  $I$
- $a$  = The depth of the corrosion pit
- $c$  = The half-length of crack
- $R$  = The internal diameter of pipe
- $\sigma_i$  = The stress components normal to the prospective crack plane
- $f_i$  = The geometry functions, depending on  $a$ ,  $c$  and  $R$

When  $a$  changes with time, the time dependent form of stress intensity factor would be:

$$K_I(t) = \sqrt{\pi a(t)} \sum_{i=0}^3 \sigma_i f_i(t) \tag{7}$$

If  $K_{IC}$  is the critical stress intensity factor, known as fracture toughness, beyond which the pipe cannot sustain the pit crack, therefore the limit state function would be:

$$G(K_{IC}, K_I, t) = K_{IC} - K_I(t) \tag{8}$$

And the failure criterion for the pipe is:

$$K_{IC} < K_I(t) \tag{9}$$

**Stresses in a buried cast iron pipe:** Rajani *et al.* (2000) developed a formulation for total external

Table 1: Stresses on buried pipes considered in this study

Stress type	Model	Reference
$\sigma_F$ , hoop stress due to internal fluid pressure	$\frac{pD}{2d}$	Rajani <i>et al.</i> (2000)
$\sigma_s$ , soil pressure	$\frac{3K_m \gamma B_d^2 C_d E_p t D}{E_p d^3 + 3K_d p D^3}$	Ahamed and Melchers (1994)
$\sigma_L$ , frost pressure	$f_{frost} \cdot \sigma_s$	Rajani <i>et al.</i> (2000)
$\sigma_V$ , Traffic/vehicular stress	$\frac{3K_m I_c C_i F E_p d D}{A(E_p d^3 + 3K_d p D^3)}$	Ahamed and Melchers (1994)
$\sigma_{Te}$ , Thermal stress	$-E_p \alpha_p \Delta T_e$	Rajani <i>et al.</i> (2000)
$\sigma_{Fv}$ , axial stress due to internal fluid pressure	$\frac{p}{2} \left( \frac{D}{d} - 1 \right) \nu_p$	Rajani <i>et al.</i> (2000)

stresses including all circumferential and axial stresses.  $\sigma_\theta$  is the hoop or circumferential stress, which is equal to:

$$\sigma_F + \sigma_S + \sigma_L + \sigma_V$$

where,

- $\sigma_F$  = The hoop stress due to internal fluid pressure
- $\sigma_S$  = The soil pressure
- $\sigma_L$  = The frost pressure
- $\sigma_V$  = The traffic stress

Similarly axial stress,  $\sigma_x$ , would be equal to:

$$\sigma_{Te} + (\sigma_{F'} + \sigma_S + \sigma_L + \sigma_V) \nu_P$$

where,

- $\sigma_{Te}$  = Stress related to temperature difference
- $\sigma_{F'}$  = Axial stress due to internal fluid pressure
- $\nu_P$  = Pipe material Poisson's ratio and other parameters have already mentioned

Equations and references used for above mentioned stresses have been presented in Table 1.

## RESULTS AND DISCUSSION

The proposed Monte Carlo method for system reliability analysis is applied on a worked example. The input data for a cast iron pipeline system is presented in Table 2. The study presented here included 1000 iterations in each Monte Carlo simulation, to develop relationship between the age of the pipe and the probability of the pipe failure. The common corrosion model presented in (1) was used reliability analysis in this study.

A MATLAB code was written to perform Monte Carlo simulation using inverse transform method for

random number generation (Melchers, 1999). The results in Fig. 1 show the probability of failure changing with time. This result provides realistic information on the prediction of service life to infrastructure.

For repair and rehabilitation planning analysis of the system, the time of the fracture of pipe, i.e.,  $T_c$ , can be determined for a given acceptable risk  $P_a$ . For example, using the graph in Fig. 1, it can be obtained that  $T_c = 23$  years for  $P_a = 0.2$ . If there is no intervention during the service period of (0, 23) years for the structure of concern, such as maintenance and repairs,  $T_c$  represents the time for the failure of the pipe, based on the reliability analysis. The information of  $T_c$ , i.e., time for interventions, is of practical importance to structural engineers and infrastructure managers with regard to planning for repairs and/or rehabilitation of the pipeline. An optimum funding allocation for the pipeline can be concluded by doing cost analysis for the repair and replacement of corroded pipes with higher risk of failure.

To analyze the effect of variables on the reliability of the pipeline, a parametric study was carried out. The effect of changing in pipe wall thickness on the probability of failure has been shown in Fig. 2. The graphs show that the thicker the pipe wall, the longer the service life of the pipe. Although this correlation can be obvious without any calculations, the amount of increase in service life is of practical interest for design engineers in design of new pipeline systems. The designer can analyze how using a thicker pipe can improve the service life of the system. A cost benefit analysis can clearly show either more capital investment for having a long life pipeline system (thicker pipe wall) is economical or not. For instance, assuming  $P_a = 0.2$  as acceptable risk, the result presented in Fig. 2 shows that by increasing the wall thickness from 14mm to 16mm the service life of the

Table 2: Values of basic random variables for reliability analysis

Symbol	Parameter	Units	Min.	Mean	S.D.	Max.
$S$	Toughness exponent	-	0.5	1	0.1	1.2
$b_l$	Geometric constant for Strength of pipe	-	-0.3	-0.25	0.03	-0.2
$p$	Internal Pressure	MPa	0.2	0.64	0.17	1.3
$D$	Internal diameter	mm	240	254	14.28	260
$d$	Wall thickness	mm	-	16	0.7	-
$\alpha$	Final pitting rate constant	Mm/yr	0.001	0.009	0.0005	0.015
$\beta$	Pitting depth scaling constant	mm	2.5	6.27	1.5	7.5
$\lambda$	Corrosion rate inhibition factor	Yr <sup>-1</sup>	0.01	0.1	0.04	0.18
$K_m$	Bending moment coefficient	-	-	0.235	0.04	-
$C_d$	Calculation coefficient	-	-	1.32	0.25	-
$B_d$	Width of ditch	mm	-	625	125	-
$E_p$	Modulus of elasticity of pipe	MPa	-	105000	15000	-
$K_d$	Defection coefficient	-	-	0.108	0.02	-
$I_c$	Impact factor	-	-	1.5	0.35	-
$C_t$	Surface load coefficient	-	-	0.12	0.025	-
$F$	Wheel load of traffic	N	30000	50000	20000	100000
$A$	Pipe effective length	mm	-	5800	200	-
$\gamma$	Unit weight of soil	N/mm <sup>3</sup>	-	$18.5 \times 10^{-6}$	$18.5 \times 10^{-7}$	-

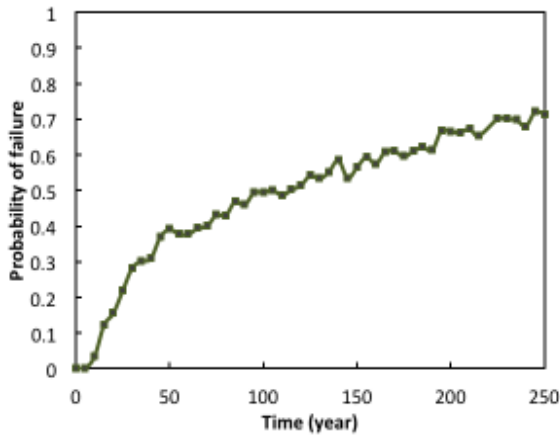


Fig. 1: Failure probability of fracture limit state

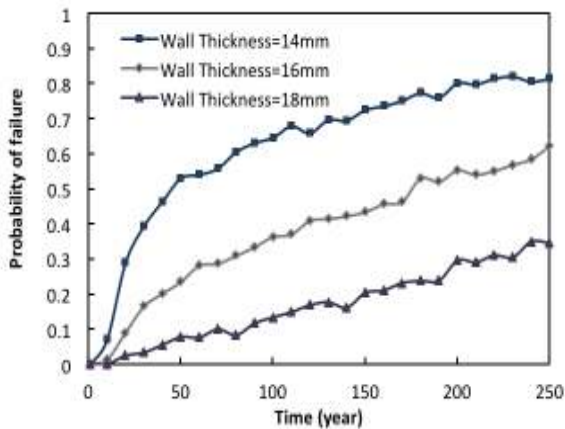


Fig. 2: Effect of wall thickness on probability of failure

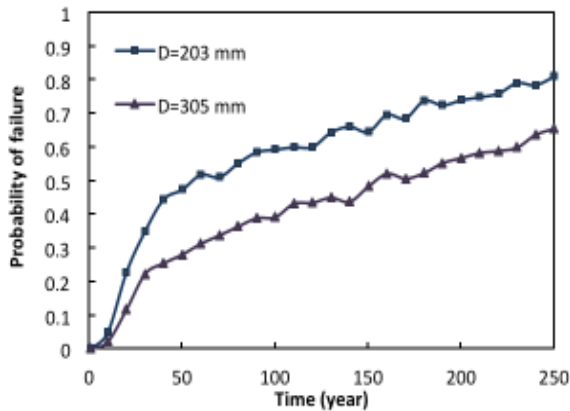


Fig. 3: Effect of pipe diameter on probability of failure

system increases from 15 years to 39 years. Interestingly, changing the wall thickness from 14 to 18 mm will dramatically increase the service life of the system from 15 years to 150 years. This considerable difference in service life may encourage the asset managers for more capital investment to have thicker pipes with significant longer service life.

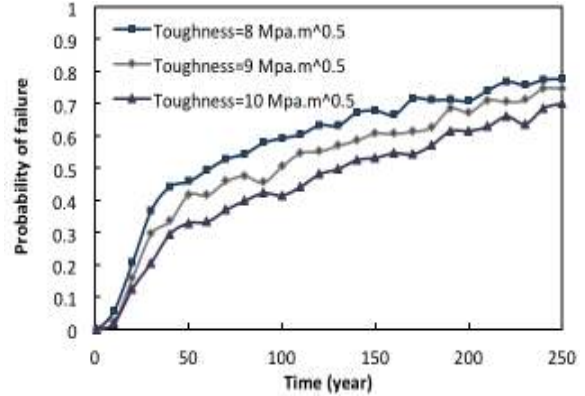


Fig. 4: Effect of fracture toughness on probability of failure

The effect of changing of pipe diameter on probability of failure was also examined by using different typical sizes of cast iron water mains. The results for three various diameters are presented in Fig. 3. In this analysis, except the wall thickness (which was increased for greater diameters correspondingly), all other variables were kept the same. As the diameter increases, the difference between service lives of pipes increases for higher degree of risk.

The graphs in Fig. 3 show that, for an acceptable risk of 20% ( $P_a = 0.2$ ), a pipe with diameter of 203 mm will have less than 18 years service life, while at the same condition, a pipe with diameter of 305 mm will have a service life more than 28 years.

Fracture toughness of iron base material can be change by changing the amount of Carbon used in the production process. To study how change in properties of pipe material can change the reliability of the pipe, different fracture toughness was examined. Figure 4 shows the effect of fracture toughness of pipe material on probability of failure. As it can be concluded from illustrated results, by increasing ductility of pipe material (e.g., increase in fracture toughness) service life of the pipe increases.

## CONCLUSION

Fracture assessment of corrosion affected cast iron pipes for agricultural food irrigation was developed in this study using reliability analysis method. Prediction of service life of the pipeline can be achieved by reliability analysis based on considering fracture failure. A parametric study has also been undertaken to identify factors that affect the probability of pipe failure due to corrosion. The results show how changes in the pipeline material property and pipeline geometry affect the service life of the pipe. It has been found that the thicker the pipe wall, the longer the service life. These results can be used by design engineers for designing of more economical new pipeline and it can help infrastructure managers to develop a risk-informed and cost-effective strategy in the management of corrosion

affected pipelines for agricultural food irrigation. It can be concluded that reliability analysis methods are rational tools for comprehensive fracture assessment of corrosion affected pipelines. Accurate prediction of the service life of existing pipes has the potential to achieve risk-cost optimization management of the system.

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