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Research Article

Location of Customer Order Decoupling Point in the Supply Chain of Deteriorated Food Based on Dynamic Model

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Abstract: Considering the supply chain of deteriorated food, a two-stage dynamic model in the product life cycle is proposed. The optimal conditions and results are obtained by using the maximum principle. On this basis, the effect of location of CODP on the total cost is researched in the product life cycle. By the data simulation, the model constructs the relations of CODP, inventory policy and demand type. The results show that CODP locates in the downstream of the product life cycle which is a linear function of the product life cycle. The demand forecast is the main factors influencing the total cost. The model can reflect the relation between the total cost and inventory, demand of the supply chain for deteriorated food.

Keywords: CODP, deteriorated food, dynamic model, supply chain

INTRODUCTION

Deteriorated food has a short product life cycle, especially agricultural products are more prominent (Roy and Maiti, 2010). Because of the shorter product life cycle, the value of deteriorated food decreases linearly rapidly and the supply chain model has its characteristics. It decides the supply chain is suitable for mass customization (Wu and Wu, 2002; Zhang, 2012). The position of Customer Order Decoupling Point (CODP) in the supply chain effects production rate and inventory level and is determined by inventory forecast and demand jointly (Roy and Samanta, 2012; Roy *et al*., 2012).

A quadratic objective function was proposed to describe the total cost of two-stage product supply chain which is called HMMS model (Holt *et al*., 1960). On this basis, Olhager considered a P/D (Production lead time/Delivery lead time) ratio and the relative demand volatility for positioning of the customer order decoupling point (Olhager, 2003). Yadavalli *et al*. (2005) proposed a single perishing product inventory model in which items deteriorate in two phases and then perish. The re-ordering policy is an adjustable (S, s) policy with the lead-time following an arbitrary distribution. Skouri and Konstantaras (2009) researched the optimal replenishment policy under the different replenishment policies for seasonable/fashionable products subject to a period of increasing demand. The model was constructed by a period of level demand and then by a period of decreasing demand rate is considered. Yang (2011) proposed a generalized mathematical production inventory model for deterioration items with partial backlogging. The demand, production and backlogging rates are assumed to be continuous and varying with time. Imre (2011) discussed the bullwhip effect of supply chain based HMMS model and gave an extended HMMS model for decreasing the bullwhip effect. Tsai and Huang (2012) employed a two-step model to determine the optimal amount of replenishment and obtained the basic reorder quantity by considering three inventory management methods involving the consideration of the probability forecast of demand, hypothesis testing and the newsboy method. Wang and Lin (2009) proposed a new production model with the idea of different CODP for different products based on the analysis of the deficiency of single CODP in mass customization. Dan *et al*. (2009) assumed a supply chain which included one manufacturer and one supplier and constructed a cost optimization model of supply chain for implementing postponement strategies in mass customization. Li and Wang (2010) improved a CODP decision-making model based on the queuing theory model considered the cost optimization. In-Jae (2011) proposed a dynamic model to simultaneously determine the CODP and production-inventory plan in a supply chain considering that production rate was a constant based the HMMS model.

Researched above focus on places of inventory or production and CODP of the supply chain in general merchandise. Based on the results of these scholars, we simplify the structure of the supply chain for deteriorated food according to its characteristics, which be divided into the two-stage model of material product

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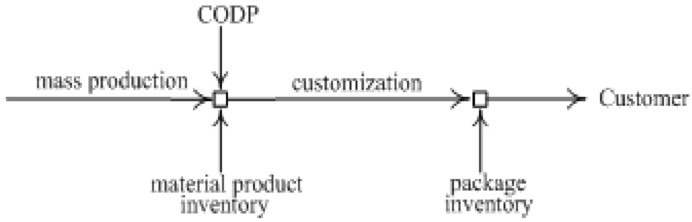

Fig. 1: The two-stage supply chain of deteriorated food

and package. Using the dynamic analysis method, we analyze the location of CODP based on optimal cost. Figure 1 shows the structure of the two-stage supply chain of deteriorated food.

METHOD AND HYPOTHESIS

It is assumed that there are two stages in the supply chain which are product and package. The inventory in the supply chain includes two points of material product and package. The two-stage supply chain model of deteriorated food is depicted in Fig. 2.
The following parameters are used in the model:

- T_0 : Position of CODP
- T : Product Life Cycle (PLC)
- $\bar{P}_1$: Target production rate during material product,
- $P_1(t)$: Production rate at time t during material product
- c_1 : Constant cost per unit deviation from the target production rate during material product
- $\bar{I}_1$: Target inventory level during material product
- $I_1(t)$: Inventory level at time t during material product
- c_2 : Constant cost per unit deviation from target inventory level during material product
- $\bar{P}_2$: Target production rate during package
- $P_2(t)$: Production rate at time t during package
- c_3 : Constant cost per unit deviation from the target production rate during the package
- $\bar{I}_2$: Target inventory level during package
- $I_2(t)$: Inventory level at time t during package
- c_4 : Constant cost per unit deviation from target inventory level during the package
- $Q_1(t)$: Demand for the package
- $Q_2(t)$: Demand from terminal customer

The objective function of minimizing total cost of the two-stage supply chain we can give according to Fig. 2 as follows:

$$f = \min \int_0^T \left\{ c_1\left(P_1(t) - \bar{P}_1\right)^2 + c_2\left(I_1(t) - \bar{I}_1\right)^2 \right\} dt + \min \int_0^T \left\{ c_3\left(P_2(t) - \bar{P}_2\right)^2 + c_4\left(I_2(t) - \bar{I}_2\right)^2 \right\} dt \quad (1)$$

$$s.t.: \begin{cases} \dot{I}_1(t) = P_1(t) - \dot{Q}_1(t) \\ \dot{I}_2(t) = P_2(t) - \dot{Q}_2(t) \\ I_1(t) \geq 0, I_2(t) \geq 0 \end{cases}$$

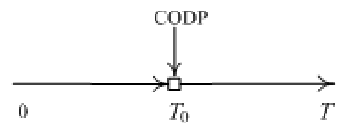

Fig. 2: Mathematical model

Model analysis: It is assumed that the demand for material product is a time-dependent constant. So formulas can be given: $Q_1(t) = q_1 t$, $Q_1(t) = q_2 t$.Applying the maximum principle of Pontryagin, we obtain the following objective function:

$$L = -c_1\left(P_1(t) - \bar{P}_1\right)^2 - c_2\left(I_1(t) - \bar{I}_1\right)^2 - c_3\left(P_2(t) - \bar{P}_2\right)^2 - c_4\left(I_2(t) - \bar{I}_2\right)^2 \quad (2)$$

The Hamilton function of the objective function can be formulized as follows:

$$H(P_1(t), I_1(t), P_2(t), I_2(t)) = -c_1\left(P_1(t) - \bar{P}_1\right)^2 - c_2\left(I_1(t) - \bar{I}_1\right)^2 - c_3\left(P_2(t) - \bar{P}_2\right)^2 - c_4\left(I_2(t) - \bar{I}_2\right) + \varphi_1(t)(P_1(t) - \dot{D}_1(t) + \varphi_2(t)(P_2(t) - \dot{D}_2(t)) \quad (3)$$

Assume optimal parameters as ($P_1^0(t)$, $I_1^0(t)$, $P_2^0(t)$, $I_2^0(t)$), where for $0 \leq t \leq T$, there are $\varphi_1(t) \neq 0$, $\varphi_2(t) \neq 0$. The first derivative of (3) on $I_1(t)$, $I_2(t)$, $P_1(t)$, $P_2(t)$ respectively is as follows:

$$\begin{cases} \dfrac{\partial H(P_1^0(t), I_1^0(t), P_2^0(t), I_2^0(t), t)}{\partial I_1(t)} = -2c_2(I_1^0(t) - \bar{I}_1) = -\dot{\varphi}_1(t) \\ \dfrac{\partial H(P_1^0(t), I_1^0(t), P_2^0(t), I_2^0(t), t)}{\partial I_2(t)} = -2c_4(I_2^0(t) - \bar{I}_2) = -\dot{\varphi}_2(t) \\ \dfrac{\partial H(P_1^0(t), I_1^0(t), P_2^0(t), I_2^0(t), t)}{\partial P_1(t)} = -2c_1(P_1^0(t) - \bar{P}_1) + \varphi_1(t) = 0 \\ \dfrac{\partial H(P_1^0(t), I_1^0(t), P_2^0(t), I_2^0(t), t)}{\partial P_2(t)} = -2c_3(P_2^0(t) - \bar{P}_2) + \varphi_2(t) = 0 \end{cases} \quad (4)$$

From the (4), we obtain the following equations:

$$\begin{cases} P_1(t) = \dfrac{1}{2c_1}\varphi_1(t) + \bar{P}_1 \\ P_2(t) = \dfrac{1}{2c_3}\varphi_2(t) + \bar{P}_2 \\ \varphi_1(t) = 2c_1(P_1(t) - \bar{P}_1) \\ \varphi_2(t) = 2c_3(P_2(t) - \bar{P}_2) \end{cases} \quad (5)$$

We can give the necessary and sufficient condition from Eq. (4)-(5) as follows:

$$\begin{pmatrix}\dot{I}_1^{\,0}\\ \dot{I}_2^{\,0}\\ \dot{P}_1^{\,0}\\ \dot{P}_2^{\,0}\end{pmatrix} = \begin{bmatrix}0&0&1&0\\0&0&0&1\\ \frac{c_2}{c_1}&0&0&0\\0&\frac{c_4}{c_3}&0&0\end{bmatrix}\begin{pmatrix}I_1^{\,0}\\ I_2^{\,0}\\ P_1^{\,0}\\ P_2^{\,0}\end{pmatrix} - \begin{pmatrix}q_1\\ q_2\\ \frac{1}{c_1}\bar{I}_1\\ \frac{1}{c_3}\bar{I}_2\end{pmatrix} \tag{6}$$

The boundary conditions are as follows:

$$\begin{cases}\begin{pmatrix}I_1^{\,0}(0)\\ I_2^{\,0}(0)\end{pmatrix} = \begin{pmatrix}\bar{I}_1\\ \bar{I}_2\end{pmatrix}\\ \begin{pmatrix}P_1^{\,0}(0)\\ P_2^{\,0}(0)\end{pmatrix} = \begin{pmatrix}\bar{P}_1\\ \bar{P}_2\end{pmatrix}\end{cases} \tag{7}$$

The optimal solution can be formalized as follows:

$$\begin{aligned} f &= \int_0^T \left\{(c_2-1)\bar{I}_1^2 t^2 + c_2(\bar{P}_1-q_1)^2 c_1 t^2 + (c_4-1)\bar{I}_2^2 t^2 + c_4(\bar{P}_2-q_2)^2 t^2\right\}dt \\ &= \frac{1}{3}T^3\left\{(c_2-1)^2\bar{I}_1^2 + c_2(\bar{P}_1-q_1)^2 + (c_4-1)\bar{I}_2^2 + c_4(\bar{P}_2-q_2)^2\right\}\end{aligned} \tag{8}$$

Let,

$$\begin{cases} C_1 = \frac{1}{3}T^3\left\{(c_2-1)^2\bar{I}_1^2 + c_2(\bar{P}_1-q_1)^2\right\}\\ C_2 = \frac{1}{3}T^3\left\{(c_4-1)\bar{I}_2^2 + c_4(\bar{P}_2-q_2)^2\right\}\end{cases}$$

C_1 and C_2 are cost of material product and cost of package, respectively. The optimization model result is irrelevant to C_1 and C_3 for a single CODP in the two-stage supply chain, which shows production rate is the target production rate in the process of material product and package when the total cost is minimum.

Theorem 1: When $\bar{I}_1 = 0$, the optimal solution is as follows:

$$f = \frac{1}{3}T^3\left\{c_2(\bar{P}_1-q_1)^2 + (c_4-1)\bar{I}_2^2 + c_4(\bar{P}_2-q_2)^2\right\} \tag{9}$$

Let $\bar{I}_2 = I_2(t)$, formula (9) can be formalized as follows:

$$\begin{cases} f = \frac{1}{3}T^3\left\{c_2(\bar{P}_1-q_1)^2 + (c_4-1)^2(\bar{P}_2-q_2)^2(T-T_0)^2 + c_4(\bar{P}_2-q_2)^2\right\}\\ I_2(t) = \int_{T_0}^{T}(\bar{P}_2-q_2)dt = (\bar{P}_2-q_2)(T-T_0)\end{cases} \tag{10}$$

Let $\frac{\partial f}{\partial T_0} = 0$, location of CODP is as follows:

$$T_0^{\,*} = T(c_4 \neq 1, \bar{P}_2 \neq q_2) \tag{11}$$

From the formula (11), the CODP locates at the downstream of the two-stage supply chain for obtaining the minimum of cost when the zero-inventory policy is adopted in the mass production stage, which can be regarded as an extreme institution in the deteriorated food supply chain.

Theorem 2: When $\bar{I}_2 = 0$, the optimal solution is as follows:

$$f = \frac{1}{3}T^3\left\{c_2(\bar{P}_1-q_1)^2 + (c_2-1)^2\bar{I}_1^2 + c_4(\bar{P}_2-q_2)^2\right\} \tag{12}$$

Let $\bar{I}_1 = I_1(t)$, the formula (12) can be formalized as follows:

$$\begin{cases} f = \frac{1}{3}T^3\left\{c_2(\bar{P}_1-q_1)^2 + (c_2-1)^2(\bar{P}_1-q_1)^2 T_0^2 + c_4(\bar{P}_2-q_2)^2\right\}\\ I_1(t) = \int_0^{T_0}(\bar{P}_1-q_1)dt = (\bar{P}_1-q_1)T_0\end{cases} \tag{13}$$

Let $\frac{\partial f}{\partial T_0} = 0$, location of CODP is as follows:

$$T_0^{\,*} = 0(c_2 \neq 1, \bar{P}_1 \neq q_1) \tag{14}$$

From the formula (14), the CODP is located at the starting point in the upstream of the two-stage supply chain when zero-inventory policy is adapted in the process of customization, which can be regarded as another extreme institution in the deteriorated food supply chain.

Theorem 3: When $\bar{I}_1 = 0$ and $\bar{I}_2 = 0$, the optimal solution is as follows:

$$f = \frac{1}{3}T^3\left\{c_2(\bar{P}_1-q_1)^2 c_4(\bar{P}_2-q_2)^2\right\} \tag{15}$$

From the formula (15), the optimal results are irrelevant to parameter T_0 when the whole process of the supply chain is under the zero-inventory policy. It implies that the CODP can be located in any position in the product life cycle and the whole supply chain reacts sensitively extremely, different demand in a different time and location of the supply chain can be matched to terminal customer.

Theorem 4: When $\bar{I}_1 \neq 0$ and $\bar{I}_2 \neq 0$, the optimal solution is as follows:

$$\begin{cases} f = \frac{1}{3}T^3\left\{c_2(\bar{P}_1-q_1)^2 + (c_2-1)(\bar{P}_1-q_1)^2 T_0^2 + (c_4-1)(\bar{P}_2-q_2)^2(T-T_0)^2 + c_4(\bar{P}_2-q_2)^2\right\}\\ I_1(t) = \int_0^{T_0}(\bar{P}_1-q_1)dt = (\bar{P}_1-q_1)T_0\\ I_2(t) = \int_{T_0}^{T}(\bar{P}_2-q_2)dt = (\bar{P}_2-q_2)(T-T_0)\end{cases} \tag{16}$$

Let $\frac{\partial f}{\partial T_0} = 0$, location of CODP is as follows:

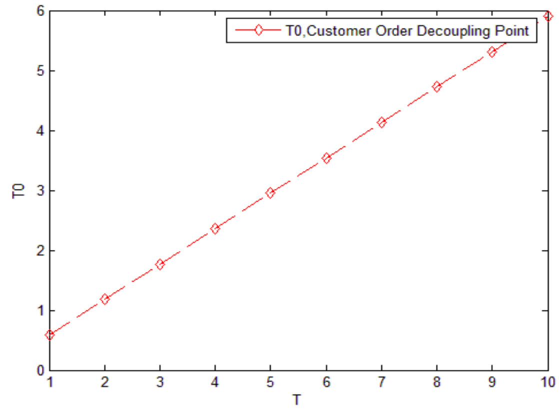

Fig. 3: The relation of PLC and CODP

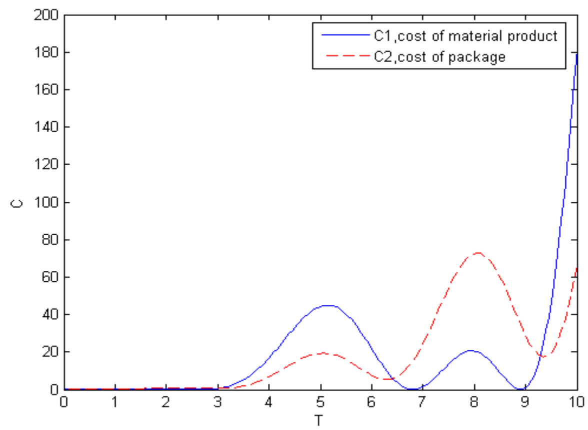

Fig. 4: The cost of the two-stage supply chain for deteriorated food

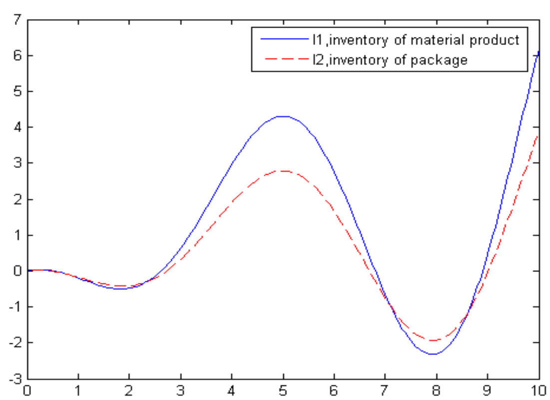

Fig. 5: The inventory level for deteriorated food

$$T_0^* = \frac{(c_4-1)^2(\overline{P}_2-q_2)^2 T}{(c_2-1)(\overline{P}_1-q_1)^2+(c_4-1)^2(\overline{P}_2-q_2)^2}, \qquad (17)$$

$$(c_2 \neq 1, c_4 \neq 1, \overline{P}_1 \neq q_1, \overline{P}_2 \neq q_2)$$

Let,

$$\begin{cases} \alpha = (c_2-1)(\overline{P}_1-q_1)^2 \\ \beta = (c_4-1)^2(\overline{P}_2-q_2)^2 \end{cases}$$

The formula (17) can be expressed as follows:

$$T_0^* = \frac{\beta}{\alpha+\beta} T \qquad (18)$$

From the formula (18), by the non-zero-inventory policy, the location of CODP in the product life cycle is proportional relevant to ratio of inventory cost of customization stage and total cost of the supply chain.

RESULTS AND DISCUSSION

It is assumed that target production rate and demand rate are constants based on non-zero-inventory policy and the data of parameters are $c_2 = 0.6$, $P_1 = 0.5$, $c_4 = 0.4$, $q_2 = 0.5$. Results of the model are shown in Fig. 3.

From the curve of the simulation results in the Fig. 3, the CODP of the two-stage supply chain when the total cost is optimized is located in the downstream position of the product life cycle, which satisfying the linear relation of formula (18). It implies the CODP should be close to the terminal customer in order to achieve the total optimal cost of the supply chain under non-zero-inventory.

Using the data of parameters in Fig. 3 and fitting the demand rate by the sin (t) function, we simulate two curves (Fig. 4) about the optimal costs of the material product and the package. It implies that both of cost are near to each other in the early stage of the product life cycle (the first period in the Fig. 4) and the former is higher with time of product life cycle gone. It shows that the cost of material product and package is an average distribution when the deteriorated food goes into the market early. As the product enters the mature period (the second period in the Fig. 4) with terminal customer demand increasing, the cost of material product stage has a faster increase than package stage due to its delayed prediction in the supply chain.

Using the data of parameters in Fig. 3 and fitting the demand rate by sin (t) function, we can obtain two curves (Fig. 5) about inventory level of the material product and the package. As can be seen, in the early stage of product life cycle, inventory levels of two stages are lower and which arise with the product entering mature period. But overall, inventory level of the package stage is lower than the material product. It also shows that package stage in the deteriorated food supply chain has a higher sensitivity to market demand and can reflect changes of customer demand rapidly.

CONCLUSION

The study proposes the CODP location model of the two-stage supply chain for deteriorated food based on dynamic cost optimization. We analysis numerical simulations under different inventory strategies and production conditions. The CODP of the supply chain for deteriorated food is located in the downstream of product life cycle. Location is more in front of the product life cycle, production and inventory level is more stable; on the contrary, they have larger fluctuation. But the overall cost is rising in the supply chain with time. It also shows that the mass customization production system requires more sensitive reflect to market demand and personalized customization especially for deteriorated food.

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