Research Article
Height-diameter Models of Chinese Fir (*Cunninghamia lanceolata*) Based on Nonlinear Mixed Effects Models in Southeast China

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Abstract: Tree height and diameter at breast height are two important forest factors. The best model from 23 height-diameter equations was selected as the basic model to fit the height-diameter relationships of Chinese fir with one level (sites or plots effects) and nested two levels (nested effects of sites and plots) Nonlinear Mixed Effects (NLME) models. The best model was chosen by smaller Bias, RMSE and larger R^2_{adj}. Then the best random-effects combinations for the NLME models were determined by AIC, BIC and -2LL. The results showed that the basic model with three random effects parameters ϕ_1, ϕ_2 and ϕ_3 was considered the best mixed model. The nested two levels NLME model considering heteroscedasticity structure (power function) possessed with higher predictable accuracy and significantly improved model performance (LRT = 469.43, p<0.0001). The NLME model would be allowed for estimating accuracy the height-diameter relationships of Chinese fir and provided better height predictions than the models using only fixed-effects parameters.

Keywords: *Cunninghamia lanceolata*, height-diameter models, heteroscedasticity, nonlinear mixed effects models

INTRODUCTION

Chinese fir (*Cunninghamia lanceolata* (Lamb.) Hook) is the most important afforestation tree species, which has fast-growth and good wood qualities in southeast China. According to the National Continuous Forest Inventory, about 11.26 million ha and 734.09 million m[3] of Chinese fir were distributed more than 10 provinces in China in 2010, making it the major species in terms of volume harvested. However, as a result of great variability exists among stands in terms of silvicultural and ecological conditions, it needs proper management, especially management tools.

Knowledge of the individual tree height (*h*) and diameter at breast height outside bark (*dbh*, measured 1.30 m above ground level) is fundamental for developing growth and yield models in forest stands. However, measuring diameter is more accurate, quicker and cheaper than measuring tree height (Colbert *et al.*, 2002). As a result, in forest inventories, diameter is measured for all the sampled trees, but height is measured only for a subsample of trees (Calama and Montero, 2004) and as *h* and *dbh* are correlated, it is common practice to fit height-diameter (*h-d*) models to predict *h* from measured *dbh* (Crecente-Campo *et al.*, 2010b).

The relationship between *h* and *dbh* varies from stand to stand (Curtis, 1967), which is usually determined by measurements of both variables from trees growing in different stands or site class of the sample plot. This hierarchical structure (i.e., plots within a site class) makes the measurements dependent (Calama and Montero, 2004; West *et al.*, 1984). As a result, there is spatial correlation among measurements in the sampling unit (Fox *et al.*, 2001). However, the stochastic structure is often ignored and independence between measurements is assumed (Biging, 1985; Fitzmaurice *et al.*, 2004; Gutzwiller and Riffell, 2007; Keselman *et al.*, 1999). Therefore, the Ordinary Non-linear Least Squares (ONLS) regression has biased estimates of the standard error of parameter estimation (Schabenberger and Gregoire, 1995). Many researchers take the Nonlinear Mixed-Effects (NLME) models into account to develop *h-d* models (Adame *et al.*, 2008a; Calama and Montero, 2004; Crecente-Campo *et al.*, 2010a; Sharma and Parton, 2007).

The fixed-effects parameters in NLME models are associated with an entire population, as in traditional regression, while the random-effects parameters are associated with individual experimental units (Pinheiro and Bates, 2000). Because of their versatility, the mixed-effect modeling approach is more useful than ONLS fitting. Use of NLME models allows *h-d* models to be calibrated by predicting random components from plot- or site-level covariates when a new subject is available (one not used in the fitting of the model), using the Empirical Best Linear Unbiased Predictors

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(EBLUPs) (Adame *et al*., 2008b; Calama and Montero, 2005; Nigh, 2012).

The research aimed to develop an *h-d* model for *C. lanceolata* (Lamb.) Hook in Fujian province, southeast China. A nested two levels nonlinear mixed modeling approach which included both fixed and random components was applied to the hierarchical structure of the dataset to diminish the level of variance among sampling units were included as covariates. In this study, we considered one level and nested two levels to build NLME *h-d* models, the first level is site and the second level is plot (nested within site). Our preliminary analysis showed that NLME models with random effects effectively removed the heteroscedasticity could provide important tools for sustainable management for this species in the study area. The predictive ability of the developed model and the applicability of NLME model technique were demonstrated on a separate validation dataset.

MATERIALS AND METHODS

Data: The Chinese fir stands located in Jiangle state-owned forest farm (117°05′-117°40′E, 26°26′-27° 04′ N) in southeast China. The soil type is red soil. Average annual precipitation is about 1699 mm. Annual mean frost-free season is 287 days. Annual mean temperature is 18.7°C.

Data used in this study were collected from 29 sample plots of Chinese fir even trees from 4 sites

which located in 4 regions, Qiantan, Shuinan, Yuhua and Yuandang work areas, which were numbered I, II, III and IV, respectively (Fig. 1). These sample plots were established from 2010 to 2013 and square in sharp but vary in size from 400 to 600 m^2. All standing live trees (height>1.3 m) on the plots were measured for breast-height diameter over bark (at 1.3 m above ground), total tree height. These data were randomly divided into two datasets with 75% used for model fitting and 25% used for model validation. Summary statistics for both fitting and validation datasets are shown in Table 1.

Methods:
Nested two levels NLME model: Available data were based on a sample of multiple measurements (diameter at breast height-height) taken from different plots located in different regions. As a consequence of this nested structure, we detected high correlation among measurements taken from the same plot. To alleviate this, a mixed-model approach has been proposed by other authors (Crecente-Campo *et al*., 2010b; Adame *et al*., 2008a, b; Sharma and Parton, 2007). A general expression for a NLME model can be defined as (Lindstrom and Bates, 1990; Vonesh and Chinchilli, 1997):

$$h_{ijk} = f\left(\phi_{ijk}, d_{ijk}\right) + \varepsilon_{ijk}, \ i = 1,...,M$$
$$j = 1,...,M_i, \ k = 1,...,n_{ij} \qquad (1)$$
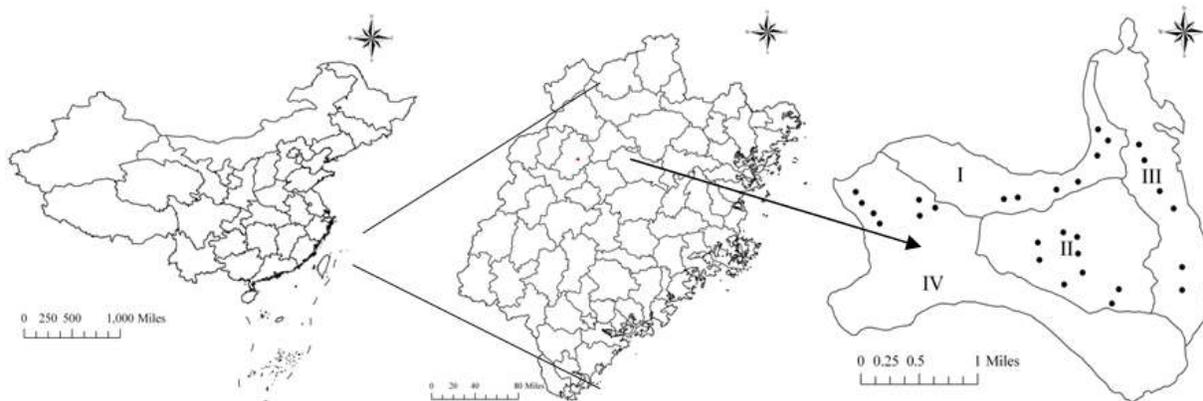
Fig. 1: Twenty nine sample plots located in four regions in Fujian province, southeast China

Table 1: Summary statistics for both fitting and validation datasets

Dataset	Region	Plot amount	dbh (cm)	h (m)	SAL (m)	SS (°)	SCD (%)	SAG (year)
Fitting data	I	5	4.7-34.5 (6.1)	4.7-27.7 (4.2)	234	28	0.7	37
	II	7	2.9-42.7 (6.2)	3.8-28.0 (4.5)	249	30	0.7	25
	III	5	5.0-34.2 (7.0)	3.5-30.6 (5.3)	245	34	0.8	29
	IV	5	2.0-25.3 (4.8)	1.8-22.9 (4.2)	220	32	0.7	10
Validation data	I	2	5.3-35.1 (6.8)	5.1-29.1 (5.5)	253	27	0.8	34
	II	2	3.6-31.0 (5.5)	4.4-28.1 (3.9)	266	33	0.8	27
	III	1	5.3-35.6 (6.8)	4.9-24.4 (3.8)	244	32	0.8	26
	IV	2	2.1-14.9 (2.3)	2.3-11.6 (1.6)	210	34	0.9	10
Total		29 (7)	2.0-42.7 (6.7)	1.8-30.6 (5.7)	240	31	0.8	25

SAL: Stand altitude; SS: Stand slope; SCD: Stand canopy density; SAG: Stand age; The numbers in the brackets of plots are sample plots and the numbers in the brackets of *dbh* and *h* are standard deviations

Table 2: Mathematical expressions of the height-diameter equations

Function No.	Function	Function No.	Function	Function No.	Function
1	$h = 1.3 + \phi_1 d^{\phi_2}$	9	$h = 1.3 + \phi_1 \left[d/(d+1) \right]^{\phi_2}$	17	$h = 1.3 + \phi_1 \big/ \left[1 + 1/\left(\phi_2 d^{\phi_3} \right) \right]$
2	$h = 1.3 + e^{\phi_1 + \phi_2/(d+1)}$	10	$h = 1.3 + \phi_1 \big/ \left(1 + \phi_2 e^{-\phi_3 d} \right)$	18	$h = 1.3 + \phi_1 e^{\phi_2/(d+\phi_3)}$
3	$h = 1.3 + \phi_1 d/(\phi_2 + d)$	11	$h = 1.3 + \phi_1 \left(1 - e^{-\phi_2 d} \right)^{\phi_3}$	19	$h = 1.3 + e^{\phi_1} d^{\phi_2}$
4	$h = 1.3 + \phi_1 \left(1 - e^{-\phi_2 d} \right)$	12	$h = 1.3 + \phi_1 \left(1 - e^{-\phi_2 d^{\phi_3}} \right)$	20	$h = 1.3 + \phi_1 d e^{-\phi_2 d}$
5	$h = 1.3 + d^2 \big/ \left(\phi_1 + \phi_2 d \right)^2$	13	$h = 1.3 + \phi_1 e^{-\phi_2 e^{-\phi_3 d}}$	21	$h = 1.3 + \phi_1 d^{\phi_2} e^{-\phi_3 d}$
6	$h = 1.3 + \phi_1 e^{\phi_2/d}$	14	$h = 1.3 + \phi_1 e^{-\phi_2 d^{-\phi_3}}$	22	$h = 1.3 + \phi_1 d^{\phi_2} e^{-\phi_3 d^2}$
7	$h = 1.3 + \phi_1 e^{\phi_2/d}$	15	$h = 1.3 + d^2 \big/ \left(\phi_1 + \phi_2 d + \phi_3 d^2 \right)$	23	$h = 1.3 + \phi_1 d + \phi_2 d^2$
8	$h = 1.3 + \phi_1 d/(d+1) + \phi_2 d$	16	$h = 1.3 + \phi_1 d^{\phi_2 d^{-\phi_3}}$		

ϕ_1, ϕ_2 and ϕ_3: The parameters of models

where,

M = The number of sites

M_i = The number of plots within the ith site

n_{ij} = The number of measurements

h_{ijk} = The height of the kth tree in the jth plot within the ith site

d_{ijk} = The diameter at breast height

ϕ_{ijk} = A parameter vector

$r \times 1$ = (where r is the number of parameters in the model)

f = A nonlinear function of the predictor variables and the parameter vector

ε_{ijk} = The within-group error including the within-group variance (Davidian and Giltinan, 1995) which is assumed normally distributed with zero expectation and a positive-definite variance-covariance structure R_{ijk}, generally expressed as a function of the parameter vector δ (Meng and Huang, 2009):

$$\varepsilon_{ijk} \sim \mathrm{N}\left(0, R_{ijk} \right)$$

Moreover, ϕ_{ijk} can be expressed as:

$$\phi_{ijk} = A_{ijk}\lambda + B_{i,jk}\mu_i + B_{ijk}\mu_{ij} \qquad (2)$$

$$\mu_i \sim \mathrm{N}\left(0, \psi_i \right), \quad \mu_{ij} \sim \mathrm{N}\left(0, \psi_{ij} \right)$$

where, λ is the p×1 vector of fixed population parameters (where p is the number of fixed parameters in the model), μ_i and μ_{ij} are the $q_1 \times 1$ and $q_2 \times 1$ vectors of random effects associated with the first level and the second level, respectively (where q_1 and q_2 are the numbers of random parameters of two levels in the model), which are assumed to be normal (or Gaussian) with a mean of 0 and have the variance-covariance matrixes ψ_i and ψ_{ij}, which are the $q_1 \times q_1$ and $q_2 \times q_2$ variance-covariance matrixes of random effects in two levels, respectively. A_{ijk}, $B_{i,jk}$ and B_{ijk} are design matrices of size $r \times p$, $r \times q_1$ and $r \times q_2$ for the fixed and random effects of two levels specific to each sampling unit, respectively.

Height-diameter equations: Several functions to model the relationship between *dbh* and *h* of the trees in a stand are available (Curtis, 1967; Fang and Bailey, 1998; Huang *et al*., 2000, 1992). In this study, 23 functions (Table 2) were compared to determine which showed the best predictive ability and fit to the height-diameter data (Huang *et al*., 2000). All functions tested have two or three parameters and are nonlinear.

Using the R *nls* function to fit 23 above-mentioned equations by ONLS regression. Different initial values for the parameters were used to ensure that a global minimum was achieved. The best function was selected by applying three statistical criteria, which are Bias, Root Mean Square Error (RMSE) and the adjusted coefficient of determination (R^2_{adj}) (Zhang *et al*., 2011). The calculation formulas of these statistics were listed as follow:

$$Bias = \frac{\left| \sum_{i=1}^{M} \sum_{j=1}^{M_i} \sum_{k=1}^{n_{ij}} \left(y_{ijk} - \hat{y}_{ijk} \right) \right|}{n_{ij}} \qquad (3)$$

$$RMSE = \sqrt{ \sum_{i=1}^{M} \sum_{j=1}^{M_i} \sum_{k=1}^{n_{ij}} \frac{\left(y_{ijk} - \hat{y}_{ijk} \right)^2}{n_{ij} - r} } \qquad (4)$$

$$R^2_{\text{adj}} = 1 - \left(n_{ij} - 1 \right) \left[\frac{ \sum_{i=1}^{M} \sum_{j=1}^{M_i} \sum_{k=1}^{n_{ij}} \dfrac{\left(y_{ijk} - \hat{y}_{ijk} \right)^2}{n_{ij} - r} }{ \sum_{i=1}^{M} \sum_{j=1}^{M_i} \sum_{k=1}^{n_{ij}} \left(y_{ijk} - \bar{y} \right)^2 } \right] \qquad (5)$$

where,

y_{ijk} and $\hat{y}_{ijk}$ = The height measurements and predictions, respectively

$\bar{y}$ = The average of measurements

Mixed parameter ascertainment: Which parameters in the model should be considered as random effects and which can be treated as purely fixed effects is the key question to fit the NLME models. Pinheiro and Bates (2000) suggested that it could start with a model with random effects for all parameters and then examine the fitted object to decide which, if any, of the random effects can be eliminated from the model by Akaike's information criterion (Akaike, 1974), Bayesian information criterion (Weiss, 2005) and -2 logarithm likelihood (-2LL). An appropriate variance function structure for NLME models were determined by Likelihood Ratio Test (LRT) (Fang and Bailey, 2001; Pinheiro and Bates, 2000). All NLME models presented in this research were calibrated using the NLME function in the R statistical software (Ihaka and Gentleman, 2004; Pinheiro *et al.*, 2005).

Determining the structure of variance-covariance: The eigenvalues of variance-covariance matrixes ψ_i and ψ_{ij} are strictly positive (Pinheiro and Bates, 2000). A hypothetical 3×3 variance-covariance matrix is shown as follow (Calama and Montero, 2004; Fu *et al.*, 2013):

$$\begin{bmatrix} \sigma_u^2 & \sigma_{uv} & \sigma_{uw} \\ \sigma_{vu} & \sigma_v^2 & \sigma_{vw} \\ \sigma_{wu} & \sigma_{wv} & \sigma_w^2 \end{bmatrix}$$

where, σ_u^2, σ_v^2 and σ_w^2 are the variance for the random effects u, v and w, respectively, $\sigma_{uv} = \sigma_{vu}$ is the covariance among random effects u and v, $\sigma_{uw} = \sigma_{wu}$ is the covariance among random effects u and w, $\sigma_{vw} = \sigma_{wv}$ is the covariance among random effects v and w.

Determining the structure of R_{ijk}: To account for within-plot heteroskedasticity in R_{ijk}, which includes weighting factors (Davidian and Giltinan, 1995; Meng and Huang, 2009), the approach is given by Calama and Montero (2004) and Crecente-Campo *et al.* (2010a):

$$R_{ijk} = \sigma^2 G_{ij}^{0.5} I_{ij} G_{ij}^{0.5} \tag{6}$$

where, σ^2 is a scaling factor for the error dispersion (Grégoire *et al.*, 1995), G_{ij} is a $n_{ij} \times n_{ij}$ diagonal matrix within-plot error heteroscedasticity variances and I_{ij} is an $n_{ij} \times n_{ij}$ matrix showing the within-plot autocorrelation structure of error. In our case, because no correlation patterns emerged between measurements from the same plot, I_{ij} reduced to a $n_{ij} \times n_{ij}$ identity matrix.

In this study, the variance heterogeneity was removed by two variance functions: the power function and the exponential function (Pinheiro and Bates, 2000):

$$\operatorname{var} \exp\left(\varepsilon_{ijk}\right) = \sigma^2 \exp\left(2\delta d_{ijk}\right) \tag{7}$$

$$\operatorname{var} \operatorname{power}\left(\varepsilon_{ijk}\right) = \sigma^2 d_{ijk}^{2\delta} \tag{8}$$

where, δ is estimated parameter.

Parameter estimation: The parameters in equation were estimated using the R NLME function (Lindstrom and Bates, 1990; Pinheiro and Bates, 2000).

In this study, the random effects parameters can be calculated with the validation data of height and diameter, by the Empirical Best Linear Unbiased Predictors (EBLUPs) (Vonesh and Chinchilli, 1997):

$$\hat{b}_{ijk} \approx D Z_{ijk}^{\mathrm{T}} \left(R_{ijk} + Z_{ijk} D Z_{ijk}^{\mathrm{T}} \right)^{-1} \hat{\varepsilon}_{ijk} \tag{9}$$

where,

D = The estimated variance-covariance matrix for the random-effects $\hat{b}_{ijk}$

R_{ijk} = The estimated variance-covariance matrix for the error term

Z_{ijk} = The estimated partial derivatives matrix with respect to random effects parameters

RESULTS

Function selection: The results of 23 height-diameter functions' performance criteria show that the Eq. (4) has slightly better predictive ability than the others for two parameters and the Eq. (10) is the best for three parameters, which is better than the Eq. (4) and significantly different from it (F = 113.32, p<0.001). Therefore, the Eq. (10) was selected as the basic model for constructing the height-diameter relationship:

$$h_{ijk} = 1.3 + \phi_1 \Big/ \left(1 + \phi_2 \exp\left(-\phi_3 d_{ijk}\right)\right) \tag{10}$$

Constructing NLME models: Different combinations of random effects for Eq. (10) were compared by AIC, BIC, -2LL and LRT statistics and the results showed in Table 3. The models of Eq. (11) and (13), incorporating sites effects on ϕ_1 and ϕ_2, plots effects and the nested effects of sites and plots on ϕ_1, ϕ_2 and ϕ_3, respectively, yielded the smallest AIC, BIC and -2LL:

$$h_{ijk}^{\mathrm{s}} = 1.3 + \left(\phi_1 + u_i\right) \Big/ \Big[1 + \left(\phi_2 + v_i\right) \\ \exp\left(-\phi_3 d_{ijk}\right)\Big] + \varepsilon_{ijk} \tag{11}$$

$$h_{ijk}^{\mathrm{p}} = 1.3 + \left(\phi_1 + u_j\right) \Big/ \Big[1 + \left(\phi_2 + v_j\right) \\ \exp\left(-\left(\phi_3 + w_j\right) d_{ijk}\right)\Big] + \varepsilon_{ijk} \tag{12}$$

Table 3: Performance criteria of NLME models for combinations of random-effects

Effects	Mixed parameters	AIC	BIC	-2LL	LRT	p value
Sites effects	ϕ_1	8374.1	8401.9	8364.1		
	ϕ_1, ϕ_2	8134.6	8173.4	8120.6	243.56	<0.0001
	ϕ_1, ϕ_2, ϕ_3	8138.7	8194.2	8118.7	1.82	0.6117
Plots effects	ϕ_1	7454.4	7482.1	7444.4		
	ϕ_1, ϕ_2	7320.3	7359.2	7306.3	138.05	<0.0001
	ϕ_1, ϕ_2, ϕ_3	7309.9	7365.4	7289.9	16.43	0.0009
Nested effects of sites and plots	ϕ_1	7449.4	7482.7	7437.4		
	ϕ_1, ϕ_2	7317.0	7372.5	7297.0	140.46	<0.0001
	ϕ_1, ϕ_2, ϕ_3	7308.2	7397.0	7276.2	20.78	0.0020

Table 4: Performance criteria of NLME models with heteroscedasticity structure

Effect	Heteroscedasticity structure	AIC	BIC	-2LL	LRT	p value
Sites effects	None	8134.6	8173.4	8120.6		
	Power	7915.8	7960.2	7899.8	218.95	<0.0001
	Exponent	7930.0	7974.4	7914.0	116.56	<0.0001
Plots effects	None	7309.9	7365.4	7289.9		
	Power	6844.7	6905.7	6822.7	467.22	<0.0001
	Exponent	6872.3	6933.3	6850.3	340.10	<0.0001
Nested effects of sites and plots	None	7308.2	7397.0	7276.2		
	Power	6840.8	6935.1	6806.8	469.43	<0.0001
	Exponent	6868.5	6962.8	6834.5	334.60	<0.0001

$$h_{ijk}^{\text{s and p}} = 1.3 + \left(\phi_1 + u_i + u_{ij}\right) \Big/ \left[1 + \left(\phi_2 + v_i + v_{ij}\right)\right. \tag{13}$$
$$\left.\exp\left(-\left(\phi_3 + w_i + w_{ij}\right)d_{ijk}\right)\right] + \varepsilon_{ijk}$$

NLME models with heteroscedasticity: Table 4 lists the performance criteria of NLME models with different heteroscedasticity structure. Thus, the final models of plots effects, trees effects and the nested two effects are:

$$\text{Eqn. (11)} + \text{Eqn. (8)} \tag{14}$$

$$\text{Eqn. (12)} + \text{Eqn. (8)} \tag{15}$$

$$\text{Eqn. (13)} + \text{Eqn. (8)} \tag{16}$$

Parameter estimates: We can conclude the NLME *h-d* models with sites effects, plots effects and nested two effects of sites and plots for Chinese fir in southeast China as follow:

Sites effects:

$$h_{ijk}^{s} = 1.3 + \left(30.0793 + u_i\right) \Big/ \left[1 + \left(11.2303 + v_i\right)\right.$$
$$\left.\exp\left(-0.1275 d_{ijk}\right)\right] + \varepsilon_{ijk} \tag{17}$$
where,

$$\mu_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} \quad N\left\{\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \psi_i = \begin{pmatrix} 11.9899 & 0.9843 \\ 0.9843 & 10.3849 \end{pmatrix}\right\}$$

$$\varepsilon_{ijk} \quad N\left(0, \mathbf{R}_{ij} = 0.4275 I_{n_{ij}}\right)$$

$$\text{var power}\left(\varepsilon_{ijk}\right) = 0.4275 d_{ijk}^{1.2348}$$

Plots effects:

$$h_{ijk}^{p} = 1.3 + \left(16.1720 + u_j\right) \Big/ \left[1 + \left(5.6187 + v_j\right)\right.$$
$$\left.\exp\left(-\left(0.1966 + w_j\right)d_{ijk}\right)\right] + \varepsilon_{ijk} \tag{18}$$

where,

$$\mu_j = \begin{bmatrix} u_j \\ v_j \\ w_j \end{bmatrix} \quad N\left\{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \psi_j = \begin{pmatrix} 5.2871 & 0.7852 & -0.2415 \\ 0.7852 & 2.2331 & 0.0484 \\ -0.2415 & 0.0484 & 0.0245 \end{pmatrix}\right\}$$

$$\varepsilon_{ijk} \quad N\left(0, \mathbf{R}_{ij} = 0.1634 I_{n_{ij}}\right)$$

$$\text{var power}\left(\varepsilon_{ijk}\right) = 0.1634 d_{ijk}^{1.7726}$$

Nested effects of sites and plots:

$$h_{ijk}^{\text{s and p}} = 1.3 + \left(16.3144 + u_i + u_{ij}\right) \Big/ \left[1 + \left(5.6815 + v_i + v_{ij}\right)\right. \tag{19}$$
$$\left.\exp\left(-\left(0.1938 + w_i + w_{ij}\right)d_{ijk}\right)\right] + \varepsilon_{ijk}$$

where,

$$\mu_i = \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} \quad N\left\{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \psi_i = \begin{pmatrix} 3.0205 & 0.7621 & -0.9982 \\ 0.7621 & 0.0003 & -0.7624 \\ -0.9982 & -0.7624 & 0.0037 \end{pmatrix}\right\}$$

$$\mu_{ij} = \begin{bmatrix} u_{ij} \\ v_{ij} \\ w_{ij} \end{bmatrix} \quad N\left\{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \psi_{ij} = \begin{pmatrix} 3.8126 & 0.9000 & -0.0077 \\ 0.9000 & 2.4081 & 0.3151 \\ -0.007 & 0.3151 & 0.0262 \end{pmatrix}\right\}$$

$$\varepsilon_{ijk} \quad N\left(0, \mathbf{R}_{ij} = 0.1624 I_{n_{ij}}\right)$$

Table 5: Evaluation indices of each model

Model	Fitting data			Validation data		
	Bias	RMSE	R^2_{adj}	Bias	RMSE	R^2_{adj}
Eq. (10)	0.0208	2.8109	0.7206	0.1900	2.1982	0.8765
Eq. (17)	0.0026	2.0305	0.8542	0.1342	1.9122	0.9072
Eq. (18)	0.0050	1.5765	0.9121	0.0156	1.0177	0.9735
Eq. (19)	0.0047	1.5760	0.9123	0.0016	0.9798	0.9749
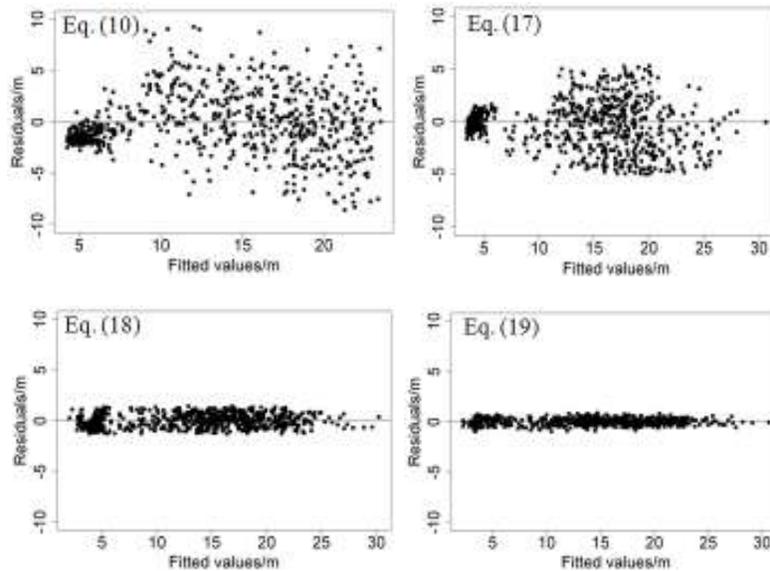
Fig. 2: Distribution of residuals of Eq. (10) and (17)-(19)
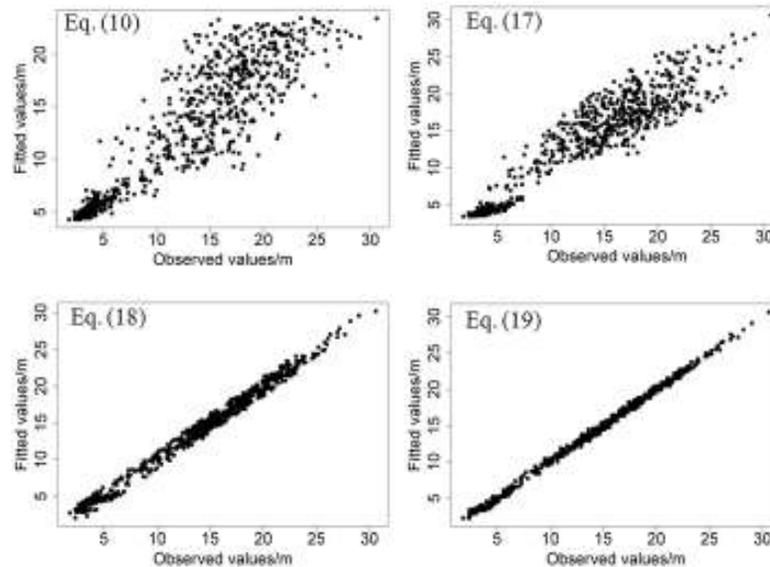
Fig. 3: Relationships between the fitted values and the observed values of Eq. (10) and (17)-(19)

$$\text{var power}\left(\varepsilon_{ijk}\right) = 0.1624 d_{ijk}^{1.7769}$$

Model prediction: The results of the three prediction statistics of Eq. (10), (17) and (19) were showed in Table 5. The prediction accuracy of the nested two levels NLME model Eq. (19) was higher than the others

for both fitting and validation data. Figure 2 shows the residual distributions of Eq. (10), (17) and (19) and Fig. 3 shows relationships between the fitted values and the observed values of these equations.

Based on the above analysis, we can conclude that Eq. (19), incorporating the nested effects of sites and plots, displays sufficiently high predictive power to

constitute a final model for predicting the height-diameter relationship of Chinese fir trees in a pure plantation in the study area.

CONCLUSION

Twenty three height-diameter equations were considered in this study for describing the height-diameter relationship for Chinese fir in pure plantation stands in Fujian province, southeast China. Then the best function Eq. (10) as the basic model was determined by Bias, RMSE and R_{adj}^2. The *ONLS* regression approach is commonly used to build the *h-d* models, but to a certain extent, there is a restriction of this method because of its assumption of independence and ignorance of heteroscedastiity. In this study, one level (site or plot) and nested two levels (plot nested within site) NLME models with variance function approach were used to estimate the height-diameter relationship based on the basic model because of the hierarchical structure of measurements data. We used the power function and the exponential function as the variance functions to eliminate the heteroscedasticity. The random effects parameters can be calculated with the measurements of the height and diameter (data not used in the fitting of the model), by EBLUPs Eq. (9). The results showed that the nested two levels NLME models Eq. (19) provided better model fitting and more precise estimations than the others Eq. (10), (17) and (18) (Table 5, Fig. 2 and 3). Therefore, NLME models could improve the fitting performance in modeling height-diameter relationship for Chinese fir, which is important for owners and managers of plantations to make appropriate sustainable management plans for different sites, even plots.

ACKNOWLEDGMENT

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