## Research Article

Group Eigenvalue Method for Food Supplier Selection Model with Ordinal Interval Preference Information

Wanzhen Liu<br>Changsha Vocational and Technology College, Changsha, 410010, P.R. China


#### Abstract

With the economic globalization, market competition is more and more fierce. The best food supplier selection is important for a food company maintaining a sustainable competitive advantage. The food supplier selection problem is a complex group decision making problem. To food supplier selection problem, which the evaluation information is the ordinal interval preference information, a new decision making method is proposed based on the concept of group eigenvalue method. A practical example is given to illustrate the effectiveness and feasibility of the proposed method.


$\underline{\text { Keywords: Food supplier selection, group decision making, group eigenvalue method, ordinal interval }}$

## INTRODUCTION

In recent years, food safety incidents often occur, which lead consumers' anxiety and panic about food safety issues, even these incidents cause the adverse impact to the society (Mazzocchi et al., 2013). The main reason of these accidents is not qualified food suppliers. With the economic globalization, market competition is more and more fierce, the food supplier selection is also very important for a food company maintaining a sustainable competitive advantage (Zhu, 2007; Hao and Jiang, 2010; Lao et al., 2012). Therefore, selecting of food suppliers a qualified and reliable suppliers are urgent problems need to solve in the management (Bi et al., 2007). Further, the fierce competitive environment characterized by high consumer expectations for quality products and short lead-times, companies are forced to take the advantage of any opportunity to optimize their business processes. Then a suit food supplier is very important for the food company.

The food supplier selection problem is a complex Group Decision Making (GDM) problem. The food supplier selection process is often in influenced by uncertainty in practice. Due to the fuzziness and uncertainty of the evaluation index or the limited knowledge of the expert or time pressure and lack of data, etc., in GDM with a number of experts and a set of alternatives, the experts usually provide their evaluations over the alternatives by means of uncertain preference information, in which each evaluation value cannot be specified, but a value range can be obtained ( $\mathrm{Xu}, 2013$ ). In GDM problem, a number of formats are already applied for ranking. These formats include ordinal (Cook and Kress, 1986; Wang et al., 2005; Fan et al., 2006; Cook, 2006) and ordinal interval numbers
(You and Fan, 2007; Fan and You, 2007; Tambouratzis, 2011). The choice of format depends on the way in which the constraints between alternative courses of action are expressed and on the means of aggregating and, subsequently, evaluating the alternative courses of action (Tambouratzis, 2011).

GDM problem with ordinal interval information is originally studied by Gonzalez-Pachon and Romero (2001). The ordinal interval can be defined as a collection of positive integer ranges given for providing the possible order positions of a set of alternatives. It is easy understand and can be easily used to express the experts' uncertain evaluation values. Taking the example provided by Fan et al. (2010). Suppose that a consumer wants to buy a car among four color cars and the preferences of him/her are as follows: The black one ranked top 2 , the white one is top 3 , the blue one is second or third and the yellow one is bottom 2 . Such preferences can be expressed with interval preference orderings but cannot or hard to be depicted by any other structures. GDM problems with ordinal interval preference information have been received much attention (Fan et al., 2010; Xu, 2013). Many method are also put forward. Such as, method with interval goal programming (Gonzalez-Pachon and Romero, 2001), linear programming method (Wang et al., 2005), TOPSIS method (Fan and You, 2007). More recently, Fan et al. (2010) proposed a possibility degree formula to compare two interval preference orderings and based on which and the collective expectation possibility degree matrix on pairwise comparisons of alternatives, an optimization model was built to solve group decision making problems with interval preference orderings. Xu (2013) proposed the uncertain additive weighted
averaging operator to fuse interval preference orderings. Then he established a nonlinear programming model by minimizing the divergences between the individual uncertain preferences and the group's opinions, to determine the experts' relative importance weights. Finally, he used the TOPSIS method to solve the GDM problem.

## PRELIMINARY KNOWLEDGE

Due to expert's limited knowledge and experience or limited time, for many decision problems, it is hard for the experts to provide the crisp preference rankings of alternatives. Ordinal interval preference information can well depict these situations. Thus, Gonzalez-Pachon and Romero (2001) firstly introduced the notion of interval preference orderings. And the mathematically definition of ordinal interval was complemented by Fan et al. (2010).

Definition 1: Fan et al. (2010) let $Z^{+}$be the set of positive integers. The interval preference ordering is expressed as:

$$
\tilde{r}=\left[r^{L}, r^{L+1}, \ldots, r^{U}\right]
$$

where, $r^{L}, r^{L+1}, \ldots, r^{U} \in Z^{+} . r^{L}$ and $r^{U}$ are the lower and upper bounds of the ordinal interval preference ordering $\tilde{r}$.

In particular, if $r^{L}=r^{U}$, then the ordinal interval preference ordering $\tilde{r}$ is reduced to an exact preference ordering.

For simplicity of representation, $\tilde{r}=\left[r^{L}, r^{L+1}, \ldots, r^{U}\right]$ is expressed as $\tilde{r}=\left[r^{L}, r^{U}\right]$.

Below we give a detailed description of GDM problems with interval preference orderings:

Consider a GDM problem, where there is a discrete set of n alternatives and a group of m experts $E=\left\{E_{1}, E_{2}, \ldots, E_{n}\right\}$. The expert $E_{j}$ provides his/her uncertain preferences on the set $\tilde{r}_{i}$, as a set of n interval preference orderings, $\quad \tilde{r}_{i}=\left(\tilde{r}_{i 1}, \tilde{r}_{i 2}, \ldots, \tilde{r}_{i n}\right)$, where $\tilde{r}_{i j}=\left[r_{i j}^{L}, r_{i j}^{U}\right]$ represents an interval-valued preference ordering given by the expert $E_{j}$ to the alternative $x_{i}$, each $\tilde{r}_{i}$ consists of a collection of positive integers ranked in increasing order, whose lower and upper limits are $r_{i j}^{L}$ and $r_{i j}^{U}$, respectively. For example, $\tilde{r}_{i j}=[1,3]$ which represents the possible ranking ordinals of an alternative $x_{i}$ given by the expert $E_{j}$ may be first, second and third. It is naturally assumed that the smaller the uncertain preference ordering value $\tilde{r}_{i j}$, the better the alternative $x_{i}$.

## GROUP EIGENVALUE METHOD FOR GDM WITH ORDINAL INTERVAL INFORMATION

Group Eigenvalue Method (GEM), proposed by Qiu (1997), is a good group decision method. This method fixes the indicators' weights by constituting experts' judgment matrix, which makes the decision process more simple and convenient (Luo et al., 2008; Jia and Fan, 2012; Ying, 2011).

It is usually impractical to request perfect evaluation from one expert because one person's knowledge and experience cannot be all-inclusive. To integrate experts' evaluations, suppose there is an idealized expert whose decision reliability is at the highest level and assume this expert's evaluation is consistent with other experts' evaluations. The ideal evaluation vector of this idealized expert can be expressed as $X^{*}=\left(x_{1}^{*}, x_{2}^{*}, \ldots, x_{n}^{*}\right)^{T}$, whose weight is assumed to be 1 .

The summation of angle values between the idealized expert's evaluation vector and every other expert's evaluation vector must be the minimum. Thus, X* can be worked out from:

$$
\max _{\|b\|=1} \sum_{i=1}^{m}\left(b^{T} X_{i}\right)^{2}=\sum_{i=1}^{m}\left(X^{* T} X\right)^{2}=\rho_{\max }
$$

where, $\rho_{\max }$ is the maximum single eigenvalue of matrix $\quad X^{T} X ; \quad X^{*}$ is the positive eigenvector corresponding to $\rho_{\max }$ and $\left\|X^{*}\right\|=1$; and $b$ is the weight vector satisfying $\forall b=\left(b_{1}, b_{2}, \ldots, b_{n}\right)^{T} \in E^{n}$ and $\|b\|=1$, with $E$ denoting the vector space. Each expert's standardized weight vector can be obtained after the eigenvector is normalized corresponding to the maximum eigenvalue.
The calculate steps of GEM is given as follows:
Step 1: Construct experts' scoring matrix: Let each one of the expert groups score all the indicators directly so as to form an $m \times n$ scoring matrix (integer $m$ is the number of experts; and integer $n$ is the number of indicators):

$$
X=\left(x_{i j}\right)_{m \times n}
$$

Here, $x_{i j}=\frac{r_{i j}^{L}+r_{i j}^{U}}{2}$ is the $j$ th expert's expected scoring value of $\tilde{r}_{i j}=\left[r_{i j}^{L}, r_{i j}^{U}\right]$ for the $i$ th alternative (Wan and Tian, 2010). Note, the larger $x_{i j}$ is, the better alternative of $x_{i}$.

Step 2: Calculate the ideal expert's scoring vector:
Calculate the matrix $F=X^{T} X$, then use the MATLAB software to obtain the largest eigenvalue $\rho_{\max }$ of matrix F and then the
corresponding eigenvector with respect to $\rho_{\max }$ is $X^{*}=\left(X_{1}^{*}, X_{2}^{*}, \ldots, X_{m}^{*}\right)$. At last, normalize the eigenvector to form the indicators' weights vector which is also called "the ideal expert's scoring vector". Thus the ideal expert's scoring vector is:

$$
S=\left(S_{1}, S_{2}, \ldots, S_{m}\right)
$$

where,

$$
S_{i}=X_{i}^{*} / \sum_{k=1}^{m} X_{k}, i=1,2, \ldots, m
$$

Step 3: Ranking alternative according to the ideal expert's scoring vector: Ranking order of the alternatives $x_{i}(j=1,2,3,4)$ according to $S_{i}(i=$ $1,2,3,4)$. The smaller value of $S_{i}(i=1,2,3,4)$ is, the better of alternative $x_{i}(j=1,2,3,4)$ is.

## A PARACTICAL EXAMPLE

A food supplier selection is given as an example to illustrate the validity and feasibility of the proposed method. Consider the following group decision making problem:

There are four alternative food suppliers $x_{1}, x_{2}, x_{3}, x_{4}$ for a company. To choose the best alternative, they hired four experts $E_{1}, E_{2}, E_{3}, E_{4}$ to evaluate these four alternatives. The evaluation information given by the experts are uncertain performance information and the information is shown in Table 1.

Suppose that the weights of every expert are equal, i.e., $w=(0.25,0.25,0.25,0.25)^{T}$. In order to solve the problem of group decision making, we use the proposed relative ratio value method to solve and the step is given as follows:

Step 1: Transform the uncertain preference information given in Table 1 into interval ordinal form and given in Table 2.
Step 2: Set $x_{i j}=\frac{r_{i j}^{L}+r_{i j}^{U}}{2}$, then we get the score decision matrix $X=\left(x_{i j}\right)_{m \times n}$ as follow:

$$
X=\left(x_{i j}\right)_{m \times n}=\left(\begin{array}{cccc}
1.5 & 2.5 & 3.5 & 2.5 \\
4 & 2.5 & 1.5 & 2 \\
3 & 2 & 3 & 2 \\
3 & 3 & 3 & 3
\end{array}\right)
$$

Step 3: Set $F=X^{T} X$,
Then we have:

Table 1: Uncertain preference information

|  | $E_{1}$ | $E_{2}$ | $E_{3}$ | $E_{4}$ |
| :--- | :--- | :--- | :--- | :--- |
| $x_{1}$ | $3 \leq r_{11} \leq 4$ | $r_{21}=1$ | $1 \leq r_{31} \leq 3$ | $1 \leq r_{41} \leq 3$ |
| $x_{2}$ | $1 \leq r_{12} \leq 4$ | $2 \leq r_{22} \leq 3$ | $2 \leq r_{32} \leq 4$ | $1 \leq \mathrm{r}_{42} \leq 3$ |
| $x_{3}$ | $1 \leq r_{13} \leq 2$ | $3 \leq r_{23} \leq 4$ | $1 \leq r_{33} \leq 3$ | $r_{43}=r_{41}$ |
| $x_{4}$ | $2 \leq r_{13} \leq 3$ | $2 \leq r_{24} \leq 4$ | $2 \leq r_{34} \leq 4$ | $1 \leq r_{44} \leq 3$ |

Table 2: Ordinal interval preference information

| Alternative | $E_{1}$ | $E_{2}$ | $E_{3}$ | $E_{4}$ |
| :--- | :--- | :--- | :--- | :--- |
| $x_{1}$ | $(1,2)$ | $(4,4)$ | $(2,4)$ | $(2,4)$ |
| $x_{2}$ | $(1,4)$ | $(2,3)$ | $(1,3)$ | $(2,4)$ |
| $x_{3}$ | $(3,4)$ | $(1,2)$ | $(2,4)$ | $(2,4)$ |
| $x_{4}$ | $(2,3)$ | $(1,3)$ | $(1,3)$ | $(2,4)$ |

$$
F=\left(\begin{array}{llll}
36.25 & 28.75 & 29.25 & 26.75 \\
28.75 & 25.50 & 27.50 & 24.25 \\
29.25 & 27.50 & 32.50 & 26.75 \\
26.75 & 24.25 & 26.75 & 23.25
\end{array}\right)
$$

The largest eigenvalue $\rho_{\max }$ of matrix $F$ is $\rho_{\max }=111.57$ and the corresponding engenvector is $X^{*}=(0.5449,0.4671,0.5205,0.4534)^{T}$.

The final evaluate score $S_{i}(i=1,2,3,4)$ of the alternatives $x_{i}(i=1,2,3,4)$ are, respectively:

$$
S_{1}=0.2744, S_{2}=0.2352, S_{3}=0.2621, S_{4}=0.2283
$$

Step 4: Rank the alternatives: Ranking order of the alternatives $x_{i}(i=1,2,3,4)$ according to $S_{i}(i=1,2,3,4)$. The smaller value of $S_{i}(i=1,2,3,4)$ is, the better of alternative $x_{i}(j=1,2,3,4)$ is. According to the order $S_{1}>S_{3}>S_{2}>S_{4}$, then the order is $x_{4} \succ x_{2} \succ x_{3} \succ x_{1}$ and this result coincides with Fan and You (2007).

## CONCLUSION

This article is focus on food supplier selection problem, which is a group decision making problem. To the evaluation information expressed with ordinal interval information, a new decision method is put forward based on group eigenvalue method. The proposed method is easy to calculation and has more advantage than the integer programming method and genetic algorithms. Finally, a food supplier selection problem is given as a case study to demonstrate and validate the application of the proposed method. The proposed method can also be extended to other multiattribute group decision making problems in which attribute values are expressed with interval numbers, triangular fuzzy numbers and intuitionistic fuzzy numbers.

## REFERENCES

Bi, X.Y., J. Zhang, T. Liu and P. Lv, 2007. Research on the establishment of selecting scheme of food suppliers. Food Sci. Technol., 6: 5-8.
Cook, W.D., 2006. Distance-based and ad hoc consensus models in ordinal preference ranking. Eur. J. Oper. Res., 172: 369-385.
Cook, W.D. and M. Kress, 1986. Ordinal ranking and preference strength. Math. Soc. Sci., 11: 295-306.
Fan, Z.P., J. Ma, Y.H. Sun and L. Ma, 2006. A goal programming approach to group decision making based on multiplicative preference relations and fuzzy preference relations. Eur. J. Oper. Res., 174: 311-321.
Fan, Z.P. and T.H. You, 2007. TOPSIS method to solve group decision problems with preference information in ordinal interval form. J. Northeastern Univ., Nat. Sci., 28(12): 1779-1781.
Fan, Z.P., Q. Yue, B. Feng and Y. Liu, 2010. An approach to group decision-making with uncertain preference ordinals. Comput. Ind. Eng., 58(1): 51-57.
Gonzalez-Pachon, J. and C. Romero, 2001. Aggregation of partial ordering rankings: An interval goal programming approach. Comput. Oper. Res., 28: 827-834.
Hao, S.C. and Y.N. Jiang, 2010. Evaluation model of food suppliers based on improved principal component analysis. Logist. Technol., 29(4): 62-64.
Jia, L.J. and D.C. Fan, 2012. A study of low carbon technology innovation project evaluation. Sci. Technol. Prog. Policy, 29(1): 128-132.
Lao, S.I., K.L. Choy, G.T.S. Ho, Y.C. Tsim, T.C. Poon and C.K. Cheng, 2012. A real-time food safety management system for receiving operations in distribution centers. Expert Syst. Appl., 39(3): 2532-2548.

Luo, J.J., J.J. Zheng, L. Sun and S.B. Zhang, 2008. An entropy-based multi-criterion group decision analysis method of selecting the optimum weaksubgrade treatment. J. Chongqing Univ., Engl. Edn., 7(3): 191-197.
Mazzocchi, M., M. Ragona and A. Zanoli, 2013. A fuzzy multi-criteria approach for the ex-ante impact assessment of food safety policies. Food Policy, 38: 177-189.
Qiu, W.H., 1997. Group eigenvalue method. Appl. Math. Mech., 18(11): 1027-1031.
Tambouratzis, T., 2011. Genetic algorithms for group decision problems using ordinal interval numbers. Proceedings of the 2011 International Conference on Genetic and Evolutionary Methods (GEM, 2011), pp: 65-70.

Wan, Z.G. and F.J. Tian, 2010. A note of the expected value and variance of fuzzy variables. Int. J. Nonlin. Sci., 9(4): 486-492.
Wang, Y.M., J.B. Yang and D.L. Xu, 2005. A preference aggregation method through the estimation of utility intervals. Comput. Oper. Res., 32: 2027-2049.
Xu, Z.S., 2013. Group decision making model and approach based on interval preference orderings. Comput. Ind. Eng., 64(3): 797-803.
Ying, M.A., 2011. Relative efficiency evaluation on water resource utilization. J. Northeast Agric. Univ., Engl. Edn., 18(3): 60-64.
You, T.H. and Z.P. Fan, 2007. A method for group decision making with preference information in ordinal interval form. J. Northeast. Univ., Nat. Sci., 28(2): 286-288.
Zhu, X.Z., 2007. Research on the selection of supplier in the green agricultural product supply chain. Issues Agric. Econ., 28(4): 67-70.

