

## Research Article

### Application of Fuzzy Projection Method to Water Resource Planning

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**Abstract:** Water resource planning is very important for water resources management. A desirable water resource planning is typically made in order to satisfy multiple objectives as much as possible. Thus the water resource planning problem is actually a Multiple Attribute Decision Making (MADM) problem. The aim of this study is to put forward a new decision method to solve the problem of water resource planning in which attribute values expressed with triangular fuzzy numbers. The new method is an extension of projection method. To avoid the subjective randomness, the coefficient of variation method is used to determine the attribute weights. A practical example is given to illustrate the effectiveness and feasibility of the proposed method.

**Keywords:** Coefficient of variation, multiple attribute decision making, projection method, water resources planning

## INTRODUCTION

Water resource is an important natural resource. It is not only the main part of the ecological environment, but also played an important role in human life and production. Issues of water resources have become a global problem and many countries and regions have already been in serious shortage of fresh water supplies. Water resources planning are very important for water resources management and electric energy production. A decision in water resource planning and management is typically made in order to satisfy multiple objectives (attributes) as much as possible (Alipour *et al.*, 2010). In majority of cases, there are a number of alternatives that each one is able to provide different level of satisfaction for every objective. A water resources planning should consider institutional, economic, environmental, social and other effects as the evaluation attributes and thus ordinary selection and evaluation of the water resources planning is a Multiple-Attribute Decision-Making (MADM) problem (Goodman and Edwards, 1992; Chen *et al.*, 2011; Wang *et al.*, 2000; Zarghaami *et al.*, 2007; Alipour *et al.*, 2010). Many MADM methods are proposed and developed to support such a decision, such as TOPSIS method (Hwang and Yoon, 1981; Kim *et al.*, 2013), VIKOR method (Kim and Chung, 2013; Opricovic, 2011), generalized induced ordered weighted averaging operator (GIOWAO) method (Ding *et al.*, 2011) and interval-fuzzy multistage programming method (Li *et al.*, 2008).

In many situations, due to the limitation of time or the limitation of decision makers' knowledge and incomplete understanding of the world, some evaluation attributes' value are usually hard to express with crisp numbers. Fuzzy numbers such as interval numbers, triangular fuzzy numbers and linguistic variables are suitable to depict these attributes' characters. Thus in many water resources planning problems, the decision makers use the fuzzy MADM methods to solve them (Kim and Chung, 2013; Zarghaami *et al.*, 2007). Projection method proposed by Wang (1999), is a well MADM method and has many applications (Yue, 2012; Zheng *et al.*, 2010; Ju and Wang, 2013).

The aim of this study is to develop the projection method to solve the problem of water resources planning, in which the attribute values are expressed with triangular fuzzy numbers.

## PRELIMINARIES

In this section, we first recall some basic definitions of triangular fuzzy number and projection method and then we will construct a MADM model for water resource planning problem, in which the evaluation attribute values are expressed with triangular fuzzy numbers:

**Definition 1:** A triple  $\tilde{A} = (a, b, c)$  called triangular fuzzy number, if its membership function is defined as follows (Xu, 2002):

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$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \leq a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{c-x}{c-b}, & b \leq x \leq c \\ 0, & x \geq c \end{cases} \quad (1)$$

Here  $a, b$  and  $c$  are real numbers and satisfy  $a \leq b \leq c$ . Note that, when  $a = b = c$ ,  $\tilde{A} = (a, b, c)$  degenerates a crisp number and we briefly note  $\tilde{A} = a$ .

**Definition 2:** An important concept regarding the applications of fuzzy numbers is defuzzification task which transforms a fuzzy number into a crisp value (Ebrahimnejad *et al.*, 2012). The most commonly used defuzzification method is the centroid defuzzification method given as follows (Yager, 1981):

$$\bar{x}_0(\tilde{A}) = \frac{\int_a^c x \mu_{\tilde{A}}(x) dx}{\int_a^c \mu_{\tilde{A}}(x) dx} = \frac{a+b+c}{3}$$

Projection method was initially put forward by Wang (1999). In this method, the projection value is calculated based on the product of the norm and the angle cosine between the decision alternative and the ideal alternative, which not only reflects the direction but also reflects the distance (Ju and Wang, 2013). Recently, many researchers have paid a great deal of attention to the method (Yue, 2012; Zheng *et al.*, 2010; Ju and Wang, 2013). In what follows, some basic concepts of the projection method are introduced briefly.

**Definition 3:** Let  $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)$  and  $\beta = (\beta_1, \beta_2, \dots, \beta_n)$  be two any vector, Then:

$$\cos(\alpha, \beta) = \frac{\sum_{j=1}^n \alpha_j \beta_j}{\sqrt{\sum_{j=1}^n \alpha_j^2} \sqrt{\sum_{j=1}^n \beta_j^2}} \quad (2)$$

is called the included angle cosine of  $\alpha$  and  $\beta$ .

**Definition 4:** Let  $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)$  be a vector, then:

$$\|\alpha\| = \sqrt{\sum_{j=1}^n \alpha_j^2} \quad (3)$$

is called the mould of vector  $\alpha$ .

As a fact that a vector is composed of the direction and the mould and the cosine of the angle between the vector value can only measure

whether their direction consistent, while it does not reflect the size of the mould. The mould and angle cosine value are must consideration together in order to reflect the degree of similarity. Therefore we will define a vector projection.

**Definition 5:** A vector  $\tilde{z}^+$  is called the triangular fuzzy positive ideal point, if:

$$\tilde{z}^+ = (\tilde{z}_1^+, \tilde{z}_2^+, \dots, \tilde{z}_n^+) \quad (4)$$

where,

$$\tilde{z}_j^+ = (z_j^{+L}, z_j^{+M}, z_j^{+U}) = (\max_i z_{ij}^L, \max_i z_{ij}^M, \max_i z_{ij}^U)$$

$$j = 1, 2, \dots, n.$$

**Definition 6:** Let  $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)$  and  $\beta = (\beta_1, \beta_2, \dots, \beta_n)$  be two vectors, the projection of vector  $\alpha$  on  $\beta$  is defined as follows:

$$\text{Pr } j_\beta(\alpha) = \cos(\alpha, \beta) \|\alpha\|$$

$$= \frac{\sum_{j=1}^n \alpha_j \beta_j}{\sqrt{\sum_{j=1}^n \alpha_j^2} \sqrt{\sum_{j=1}^n \beta_j^2}} \sqrt{\sum_{j=1}^n \alpha_j^2} = \frac{\sum_{j=1}^n \alpha_j \beta_j}{\sqrt{\sum_{j=1}^n \beta_j^2}} \quad (5)$$

Generally speaking, the larger value of projection  $\text{Pr } j_\beta(\alpha)$ , the more closer of vector  $\alpha$  to vector  $\beta$ .

Let  $\tilde{z}_i = (\tilde{z}_{i1}, \tilde{z}_{i2}, \dots, \tilde{z}_{in})^T, i = 1, 2, \dots, m$  are  $m$  alternatives, where  $\tilde{z}_{ij} = (z_{ij}^L, z_{ij}^M, z_{ij}^U)$  ( $i = 1, 2, \dots, m, j = 1, 2, \dots, n$ ). Then the projection of the  $i$ th alternative  $\tilde{z}_i$  on the triangular fuzzy positive ideal point  $\tilde{z}^+$  can be derived as follows:

$$\text{Pr } j_{\tilde{z}^+}(\tilde{z}_i) = \frac{\sum_{j=1}^n (z_{ij}^L z_j^{+L} + z_{ij}^M z_j^{+M} + z_{ij}^U z_j^{+U})}{\sqrt{\sum_{j=1}^n [(z_j^{+L})^2 + (z_j^{+M})^2 + (z_j^{+U})^2]}} \quad (6)$$

where,  $\tilde{z}_i = (\tilde{z}_{i1}, \tilde{z}_{i2}, \dots, \tilde{z}_{in})^T, i = 1, 2, \dots, m$

Obviously, the larger value of  $\text{Pr } j_{\tilde{z}^+}(\tilde{z}_i)$ , the more closer of alternative  $x_i$  to the triangular fuzzy positive ideal point  $\tilde{z}^+$ . That is to say, the alternative  $x_i$  is better.

### FUZZY PROJECTION METHOD FOR WATER RESOURCE PLANNING

Consider a water resource planning problem with a set of  $m$  candidate schemes (alternatives)  $A = \{A_1, A_2, \dots, A_m\}$  and a set of  $n$  evaluation attributes  $O = \{o_1, o_2, \dots, o_n\}$ . Suppose the evaluation attribute value of  $i$ th alternative  $A_i = (i, 1, 2, \dots, m)$  with respect to  $j^{\text{th}}$  attribute  $o_j$  ( $j = 1, 2, \dots, n$ ) given by decision maker is a

triangular fuzzy number with the form  $\tilde{a}_{ij} = (a_{ij}^l, b_{ij}^m, c_{ij}^u)$ . Hence, the water resource planning model is actually a MADM model with the decision matrix  $\tilde{D} = (\tilde{a}_{ij})_{m \times n}$  with the following form:

$$D = (\tilde{a}_{ij})_{m \times n} = \begin{matrix} & o_1 & o_2 & \cdots & o_n \\ A_1 & \left( \begin{matrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \cdots & \tilde{a}_{mn} \end{matrix} \right) \\ A_2 & & & & \\ \vdots & & & & \\ A_m & & & & \end{matrix}$$

Suppose  $w = (w_1, w_2, \dots, w_n)$  be the attribute weight vector which satisfies  $w_j \geq 0, j = 1, 2, \dots, n$  and  $\sum_{j=1}^n w_j = 1$ .

In general, attribute set can be classified into two types: benefit attribute set and cost attribute set. In other words, the attribute set can be divided into two subsets:  $I_1$  and  $I_2$ , which mean the subset of benefit attribute set and cost attribute set, respectively.

To eliminate the impact of different dimension, we need firstly to normalize the decision matrix  $\tilde{D} = (\tilde{a}_{ij})_{m \times n}$  by some normalization method. The normalization method is to preserve the property that the range of a normalized triangular fuzzy number  $\tilde{r}_{ij}$  belongs to the closed interval  $[0, 1]$ . Here, the fuzzy decision matrix  $\tilde{D} = (\tilde{a}_{ij})_{m \times n}$  is transformed into the normalized fuzzy decision matrix  $\tilde{R} = (\tilde{r}_{ij})_{m \times n}$ , where  $\tilde{r}_{ij} = (r_{ij}^l, r_{ij}^m, r_{ij}^u)$  by the following normalization method (Xu, 2002):

$$\begin{cases} r_{ij}^L = a_{ij}^L / \sqrt{\sum_{i=1}^m (a_{ij}^U)^2} \\ r_{ij}^M = a_{ij}^M / \sqrt{\sum_{i=1}^m (a_{ij}^M)^2} \\ r_{ij}^U = a_{ij}^U / \sqrt{\sum_{i=1}^m (a_{ij}^L)^2} \\ i \in M, j \in I_1 \end{cases} \quad (7)$$

and

$$\begin{cases} r_{ij}^L = (1/a_{ij}^U) / \sqrt{\sum_{i=1}^m (1/a_{ij}^L)^2} \\ r_{ij}^M = (1/a_{ij}^M) / \sqrt{\sum_{i=1}^m (1/a_{ij}^M)^2} \\ r_{ij}^U = (1/a_{ij}^L) / \sqrt{\sum_{i=1}^m (1/a_{ij}^U)^2} \\ i \in M, j \in I_2 \end{cases} \quad (8)$$

where  $M = \{1, 2, \dots, m\}$ .

When  $\tilde{a}_{ij} = (a_{ij}^l, b_{ij}^m, c_{ij}^u)$  is a crisp number, that is  $a_{ij} = b_{ij} = c_{ij}$ , then the normalization method adopts the following formulas:

$$r_{ij} = \frac{a_{ij} - \min_i \{a_{ij}\}}{\max_i \{a_{ij}\} - \min_i \{a_{ij}\}}, i \in M, j \in I_1 \quad (9)$$

and

$$r_{ij} = \frac{\max_i \{a_{ij}\} - a_{ij}}{\max_i \{a_{ij}\} - \min_i \{a_{ij}\}}, i \in M, j \in I_2 \quad (10)$$

In the MADM process, a key step is to determine attribute weights. Different weights often lead to different decision results. Weighting methods, which try to define the importance of attribute, are categorized into subjective, objective and integrated methods. The subjective methods depend on the expert's preference information to determine the weights. In this study we will use coefficient of variation method to determine the attribute weight. The coefficient of variation is defined as the ratio of the standard deviation  $\sigma$  to the mean  $\mu$ :  $CV = \sigma/\mu$ . It shows the extent of variability in relation to the mean of the population (Wang *et al.*, 2013). The steps of coefficient of variation method are given as follows:

- According to the centroid defuzzification method, the triangular fuzzy number  $\tilde{r}_{ij} = (r_{ij}^l, r_{ij}^m, r_{ij}^u)$  can be transformed into crisp number the crisp number decision matrix  $X = (x_{ij})_{m \times n}$ , where:

$$x_{ij} = \frac{r_{ij}^l + r_{ij}^m + r_{ij}^u}{3} \quad (11)$$

- The mean and standard deviation of a column vector in matrix  $X = (x_{ij})_{m \times n}$  is  $\bar{x}_j$  and  $s_j$ . Then we have  $\bar{x}_j = \frac{1}{m} \sum_{i=1}^m x_{ij}$  and  $s_j = \sqrt{\frac{1}{m} \sum_{i=1}^m (x_{ij} - \bar{x}_j)^2}$ ,  $j = 1, 2, \dots, n$ . Then the attribute weights are obtained by coefficient of variation method as follows (Yue, 2012):

$$w_j = \frac{s_j / \bar{x}_j}{\sum_{j=1}^n s_j / \bar{x}_j}, j = 1, 2, \dots, n \quad (12)$$

Obviously,  $w_j$  satisfies  $w_j \geq 0$  and  $\sum_{j=1}^n w_j = 1$ .

In the following, we will develop the triangular fuzzy projection method to the water resource planning with the following steps:

- Step 1:** Establish triangular fuzzy number decision matrix  $\tilde{D} = (\tilde{a}_{ij})_{m \times n}$ .
- Step 2:** Normalize the decision matrix  $\tilde{D} = (\tilde{a}_{ij})_{m \times n}$  to the normalized decision making matrix  $\tilde{R} = (\tilde{r}_{ij})_{m \times n} = ((r_{ij}^l, r_{ij}^m, r_{ij}^u))_{m \times n}$  by Eq. (7) to Eq. (10).
- Step 3:** Calculate the attribute weights by coefficient of variation method.
- Step 4:** Calculate the weighted normalized decision matrix  $\tilde{Z} = (\tilde{z}_{ij})_{m \times n}$ , where  $\tilde{z}_{ij} = w_j \tilde{r}_{ij}$ .
- Step 5:** Determine the triangular fuzzy positive ideal point  $\tilde{z}^+$  according to Eq. (4).
- Step 6:** Calculated projection of each alternative  $A_i$  on The triangular fuzzy positive ideal point  $\text{Pr } j_{\tilde{z}^+}(\tilde{z}_i)$  according to Eq. (6).
- Step 7:** Rank the alternatives according to the projection  $\text{Pr } j_{\tilde{z}^+}(\tilde{z}_i)$  ( $i = 1, 2, \dots, m$ ). The larger of the value of  $\text{Pr } j_{\tilde{z}^+}(\tilde{z}_i)$ , the better of the alternative  $A_i$ .

**CASE STUDY**

To illustrate the applicability and feasibility of the proposed method, an example discussed in Opricovic (2011) dealing with the water resource planning is given. In Serbia, the Mlava water resources systems have selected six potential dam sites  $A_1, A_2, \dots, A_6$  for reservoirs to provide water. The designed reservoir systems are evaluated according to the following attribute:

- Investment costs ( $o_1$ ) (in 106 US\$)
- Water supply discharge-yield ( $o_2$ ) ( $m^3/s$ )
- Social impact ( $o_3$ ) (%) on urban and agricultural area expressing local regret as percentage of the regret in the alternative with maximum social impact
- Impact on the monastery Gornjak (grade) ( $o_4$ )

Here, attribute  $o_2$  is the benefit attribute and others are the cost attribute. The attribute values are reported in Table 1.

In the following, we will use the proposed method to solve this problem and the detail calculation steps are given as follows:

**Step 1:** According to Table 1, we get the triangular fuzzy number decision matrix:

Table 1: Attribute values for water resource planning

	Attribute			
	$o_1$	$o_2$	$o_3$	$o_4$
$A_1$	(38,40.01, 48)	(3.26,4.08, 4.08)	(43,47,48)	10
$A_2$	(20,21.06, 24)	(2.57,2.87, 2.87)	(6,6,6)	10
$A_3$	(24.58,25.87, 29.75)	(2.82,2.97, 2.97)	(38,42,50)	1
$A_4$	(44.54,46.89, 56.27)	(2.46,2.73, 2.73)	(60,62,68)	0
$A_5$	(33.33,33.33, 43.33)	(2.25,2.50, 2.62)	(6,6,6)	2
$A_6$	(33.86,33.86, 42.32)	(2.47,2.74, 2.85)	(6,6,6)	3

$$\tilde{D} = \begin{pmatrix} (38,40.01,48) & (3.26,4.08,4.08) \\ (20,21.06,24) & (2.57,2.87,2.87) \\ (24.58,25.87,29.75) & (2.82,2.97,2.97) \\ (44.54,46.89,56.27) & (2.46,2.73,2.73) \\ (33.33,33.33,43.33) & (2.25,2.50,2.62) \\ (33.86,33.86,42.32) & (2.47,2.74,2.85) \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} (43,47,48) & 10 \\ (6,6,6) & 10 \\ (38,42,50) & 1 \\ (60,62,68) & 0 \\ (6,6,6) & 2 \\ (6,6,6) & 3 \end{pmatrix}$$

**Step 2:** Accord to Eq. (7)-(10), we get the normalized decision making matrix:

$$R = \begin{pmatrix} (0.2468,0.3076,0.3836) & (0.4351,0.5506,0.6266) \\ (0.4937,0.5844,0.7288) & (0.3430,0.3873,0.4408) \\ (0.3982,0.4758,0.5930) & (0.3764,0.4008,0.4561) \\ (0.2106,0.2625,0.3273) & (0.3283,0.3684,0.4193) \\ (0.2734,0.3693,0.4373) & (0.3003,0.3374,0.4024) \\ (0.2800,0.3635,0.4305) & (0.3296,0.3697,0.4377) \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} (0.0715,0.07310,0.0801) & 0 \\ (0.5722,0.5730,0.5737) & 0 \\ (0.0687,0.0819,0.0906) & 0.9 \\ (0.0505,0.0554,0.0574) & 1 \\ (0.5722,0.5730,0.5737) & 0.8 \\ (0.5722,0.5730,0.5737) & 0.7 \end{pmatrix}$$

**Step 3:** By the coefficient of variation method, attribute weights are obtained as:

$$w_1 = 0.1450, w_2 = 0.0807, w_3 = 0.4020, w_4 = 0.3724$$

**Step 4:** The projection  $\text{Pr } j_{\tilde{z}^+}(\tilde{z}_i)$  of each alternative are obtained as:

$$\text{Pr } j_{\tilde{z}^+}(\tilde{z}_1) = 0.0490, \text{Pr } j_{\tilde{z}^+}(\tilde{z}_2) = 0.2397$$

$$\Pr j_{\tilde{z}_3}(\tilde{z}_3) = 0.5416, \Pr j_{\tilde{z}_4}(\tilde{z}_4) = 0.5748$$

$$\Pr j_{\tilde{z}_5}(\tilde{z}_5) = 0.6577 \text{ and } \Pr j_{\tilde{z}_6}(\tilde{z}_6) = 0.6051$$

Then the ranking order of these candidate alternatives is obtained as  $A_5 > A_6 > A_4 > A_3 > A_2 > A_1$  and the desirable candidate alternative is  $A_5$ .

From the meaning of these alternatives  $A_1, A_2, \dots, A_6$ , when we select  $A_5$  as the dam site to provide water, the loss of agricultural is relatively small, which is one advantage of alternative  $A_5$ . Note that, when we given the attribute weights satisfy  $w_1 = w_2 = w_3 = w_4 = 0.25$ , the projection  $\Pr j_{\tilde{z}_i}(\tilde{z}_i)$  of each alternative are:

$$\Pr j_{\tilde{z}_1}(\tilde{z}_1) = 0.1630, \Pr j_{\tilde{z}_2}(\tilde{z}_2) = 0.2800$$

$$\Pr j_{\tilde{z}_3}(\tilde{z}_3) = 0.4511, \Pr j_{\tilde{z}_4}(\tilde{z}_4) = 0.4289$$

$$\Pr j_{\tilde{z}_5}(\tilde{z}_5) = 0.4724, \Pr j_{\tilde{z}_6}(\tilde{z}_6) = 0.4466$$

Then the ranking order of these candidate alternatives is obtained as  $A_5 > A_3 > A_6 > A_4 > A_2 > A_1$  and the set of compromise solutions is  $\{A_3, A_5, A_6\}$  and this result is also in agreement with the one obtained in Opricovic (2011).

### CONCLUSION

Water resources planning need to consider many influence factors and thus it is actually a MADM problem. For water resources planning problem with attribute values expressed with triangular fuzzy numbers, this study develop a new decision method to solve it based on projection method. Attribute weights play an important role in water resource planning and different weights often lead different result. The coefficient of variation method used in this study is an objection weighting method, which can avoid the subjective randomness. Finally, a water resource planning problem is given as a case study to demonstrate and validate the application of the proposed method. The proposed method can also be applied to other MADM problems such as project selection, material selection.

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