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Research Article

Application of Improved Grey Relation Analysis to Water Quality Evaluation

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Abstract: The Evaluation of water quality is one of the important aspects of water resource management. Many indicators are needed to be considered in the evaluation process and thus the water quality evaluation problem is actually a Multiple Attribute Decision Making (MADM) problem. The aim of this study is to put forward a new water quality evaluation method based on grey analysis method. In order to avoid the subjective randomness on the weight of each evaluation indicator, the coefficient of variation method is adopted to determine the attribute weights of water quality evaluation. A practical example is given to illustrate the effectiveness and feasibility of the proposed method.

Keywords: Coefficient of variation, grey relation analysis, multiple attribute decision making, water quality evaluation

INTRODUCTION

It is all to know that water is very important for human in the life. With the rapid economic development, the supply and demand of water resources has become a serious problem because of the imbalance between the supply and demand of water (Huang and Xia, 2001). Environment pollution not only affects the biological integrity of aquatic systems, but also degrades the quality of water and affects human health directly and indirectly (Wong and Hu, 2014). Global water usage continues to increase at twice the rate of population growth. Clearly, water resource management is the key to a sustainable future for human. Evaluation of water quality is one of the main steps of water resource management. Therefore, effective evaluation methods and concrete evaluation criteria for assessing the quality of water resources must be developed in order to secure water safety for sustainable development and public health. Water quality evaluation is studied by many authors and many methods are also developed. At present, the water quality evaluation methods include fuzzy comprehensive evaluation method (Icaga, 2007), the matter element method (Kou, 2013; Liu and Zou, 2012; Wong and Hu, 2014), comprehensive index method (Chen *et al*., 2010), attribute recognition Method (Yu *et al*., 2013) and set pair analysis (Wang *et al*., 2012). In the practical application of these methods for water quality recognition problem, a key step is to determine indicators' weights. Different weights often lead to different evaluation results. Weighting methods, which try to define the importance of indicators, are categorized into subjective, objective and integrated methods. The subjective methods depend on the expert's preference information to determine the weights. For water quality evaluation problem, objective weighting method is more suitable than subjective methods. In this study we will use coefficient of variation method to determine the indicators' weight. Coefficient of variation method is an objective weighting method and has many applications in various fields.

In this study, we will use coefficient of variation method to determine indicators' weights. We will propose a new recognition method for the water quality problem. The new method is an improvement of Grey Relation Analysis (GRA) method, which combining the GRA method with TOPSIS method.

WATER QUALITY EVALUATION MODEL

Suppose that there are m objects (water samples) $A_1, A_2, \ldots, A_m$ waited to be evaluated about their water quality grades and each object belong to one grade of water quality standards which are denoted by $C_1, C_2, \ldots, C_K$. Each object has n indicators (index, evaluation attribute) $o_1, o_2, \ldots, o_n$. x_{ij} is the measurement value of object A_i with respect to indicator o_j. Thus the water sample A_i can be written as $A_i = (x_{i1}, x_{i2}, \ldots, x_{in})$, $i = 1, 2, \ldots, m$. Then the sample space matrix can be expressed with the following $n\times m$ matrix:

$$X = (x_{ij})_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} o_1 & o_2 & \cdots & o_n \end{array} \\ \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{pmatrix} \end{array}$$

Suppose F is some attribute space and C_1, C_2, …, C_K is an ordered series of grades in the attribute space F. The series satisfies the condition $C_1 > C_2 > \ldots > C_K$. Such a space can be established for the standard grades of every evaluation indicator. The standard grade matrix can then be expressed with:

$$A = (a_{ij})_{n\times K} = \begin{array}{c} \\ o_1 \\ o_2 \\ \vdots \\ o_n \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_K \end{array} \\ \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1K} \\ a_{21} & a_{22} & \cdots & a_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nK} \end{pmatrix} \end{array}$$

where, a_{ij} satisfies $a_{i1} < a_{i2} < \ldots < a_{iK}$ or $a_{i1} > a_{i2} > \ldots > a_{iK}$.

For further establishing the water quality evaluation decision model, the following discussion will calculate the attribute measure and attribute weights:

- **Calculate the attribute measure:** The attribute measure $\mu_{ijk} = \mu(x_{ij} \in C_k)$ of index value x_{ij}, which takes the attribute levels from the set C_k, is found in this way.

  Suppose $a_{i1} < a_{i2} < \ldots < a_{iK}$, then:

  - Take $\mu_{ij1} = 1, \mu_{ij2} = \cdots = \mu_{ijK} = 0$, if $x_{ij} \le a_{i1}$
  - Take $\mu_{ijK} = 1, \mu_{ij1} = \cdots = \mu_{ijK-1} = 0$, if $x_{ij} \ge a_{iK}$
  - Take $\mu_{ijl} = \dfrac{|x_{ij} - a_{i(l+1)}|}{|a_{il} - a_{i(l+1)}|}, \mu_{ij(l+1)} = \dfrac{|x_{ij} - a_{il}|}{|a_{il} - a_{i(l+1)}|}$ and $\mu_{ijk} = 0, k < l$ or $k > l+1$, if $a_{il} \le x_{ij} \le a_{i(l+1)}$

  Then we can get the attribute recognition decision matrix:

$$H_i = (\mu_{ijk})_{n\times K} = \begin{array}{c} \\ o_1 \\ o_2 \\ \vdots \\ o_n \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_K \end{array} \\ \begin{pmatrix} \mu_{i11} & \mu_{i12} & \cdots & \mu_{i1K} \\ \mu_{i21} & \mu_{i22} & \cdots & \mu_{i2K} \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{in1} & \mu_{in2} & \cdots & \mu_{inK} \end{pmatrix} \end{array}$$

- **Determine the indicators' weights:** After knowing indicators' attribute measure μ_{ijk} (i = 1, 2, …., m; j = 1, 2, …, n; k = 1, 2, …, K). The importance of every indicator may be the same or different. If lth indicator's values are equal, the indicator will doesn't work for determine the grade which the sample A_i belongs to, then we can make its weight is 0. Conversely, if the lth indicator's values have much difference among all attribute class, we should give it greater weight. The above demands can be complete by the coefficient of variation method. Coefficient of variation method is an objective method for determining index weights. The steps of coefficient of variation method can be given as follows:

- Normalize the sample space matrix $X = (x_{ij})_{m\times n}$.

An evaluation index can be classified as a benefit type (the sample is better with an increase in the index, such as Dissolved Oxygen (DO)) or as a cost type (the sample is better with a decrease in the index, such as NH3-N) depending upon its attributes. Thus normalization is necessary. We can transform the sample space matrix $X = (x_{ij})_{m\times n}$ into the normalized decision matrix $R_{il} = (r_{ijl})_{n\times K}$ with the following normalized method:

Among these indicators, to benefit type indexes, there are:

$$r_{ij} = \frac{x_{ij} - \min_i \{x_{ij}\}}{\max_i \{x_{ij}\} - \min_i \{x_{ij}\}}$$

While, to the cost types, there are:

$$r_{ij} = \frac{\max_i \{x_{ij}\} - x_{ij}}{\max_i \{x_{ij}\} - \min_i \{x_{ij}\}}$$

Obviously, r_{ij} is the data of the jth evaluating object on the indicator and $r_{ij} \in [0,1]$.

- Let $\bar{x}_j = \dfrac{1}{m}\sum_{i=1}^{m} x_{ij}$, $s_j = \sqrt{\dfrac{1}{m}\sum_{i=1}^{m}(x_{ij} - \bar{x}_j)^2}$ and $\tau_j = s_j / \bar{x}_j$, then the index weights can be calculated as $w_j = \tau_j / \sum_{j=1}^{n} \tau_j$, $j = 1, 2, \ldots, n$.

We can easily to show that the weights satisfy $w_j \ge 0$, $\sum_{j=1}^{n} w_j = 1, j = 1, 2, \ldots, n$.

IMPROVED GREY ANALYSIS METHOD FOR WATER QUALITY EVALUATION

The GRA method is firstly proposed by Deng (1989) is a quantitative analysis method, which can measure the degree of similarity and dissimilarity between two sequences. It is a well known decision making method and has many applications in MADM problems (Tseng, 2010; Lee and Lin, 2011; Cao *et al*., 2012).

In this section, we will develop a new water quality evaluation method, which is an improved GRA method. The specific calculation steps are given as follows:

Step 1: Establish the attribute recognition decision matrix $H_i = (\mu_{ijk})_{n\times K}$ $(i = 1, 2, \ldots, m)$

Step 2: Determine the attribute weights by coefficient of variation method

Step 3: Define the Positive Ideal Solution (PIS) and negative ideal solution (NIS) as follows.

The PIS is defined as:

$$\mu_i^+ = (\mu_{i1}^+, \mu_{i2}^+, \ldots, \mu_{in}^+)^T$$

where, $\mu_{ij}^+ = \max_{1\le k\le K}\{\mu_{ijk}\}$;

The NIS is defined as:

$$\mu_i^- = (\mu_{i1}^-, \mu_{i2}^-, \ldots, \mu_{in}^-)^T$$

where, $\mu_{ij}^- = \min_{1\le k\le K}\{\mu_{ijk}\}$.

Step 4: Calculate the grey relational coefficient of each sample from PIS and NIS using the following equation, respectively:

$$\xi(\mu_i^+, \mu_{ijk}) = \frac{\min_i \min_j |\mu_{ijk} - \mu_i^+| + \rho \max_i \max_j |\mu_{ijk} - \mu_i^+|}{|\mu_{ijk} - \mu_i^+| + \max_i \max_j |\mu_{ijk} - \mu_i^+|}$$

$$\xi(\mu_i^-, \mu_{ijk}) = \frac{\min_i \min_j |\mu_{ijk} - \mu_i^-| + \rho \max_i \max_j |\mu_{ijk} - \mu_i^-|}{|\mu_{ijk} - \mu_i^-| + \max_i \max_j |\mu_{ijk} - \mu_i^-|}$$

where, ρ is the identification coefficient, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$. Here we choose $\rho = 0.5$.

Step 5: Calculate the grey relational degree $\xi(\mu_i^+, \mu_{ik})$ of each object from PIS and grey relational degree $\xi(\mu_i^-, \mu_{ik})$ of each alternative from NIS by using the following equation, respectively:

$$\xi^+(\mu_{ik}) = \sum_{j=1}^{n} w_j \xi(\mu_i^+, \mu_{ijk})$$

and,

$$\xi^-(\mu_{ik}) = \sum_{j=1}^{n} w_j \xi(\mu_i^-, \mu_{ijk})$$

Step 6: Calculate the closeness of coefficient C_{ik} $(k = 1, 2, \ldots, K)$ of each object A_i with respect to k^{th} grade as follows:

$$C_{ik} = \frac{\xi^+(\mu_{ik})}{\xi^+(\mu_{ik}) + \xi^-(\mu_{ik})}, \quad k = 1, 2, \ldots, K$$

The larger of C_{ik} $(k = 1, 2, \ldots, K)$ is, the closer of sample A_i with PIS is.

Step 7: Water quality recognition rule:

If,

$$k_0 = \arg\max_{1\le k\le K}\{C_{ik}\}$$

Then the object A_i belonging to the grade k_0.

APPLICATION TO THE WATER QUALITY EVALUATION

To illustrate the practicability and feasibility of the proposed method, an example with the water quality evaluation discussed in Wang and Zou (2008) is given. Fuqiao River Reservoir located in Macheng City of Hubei province, is a large reservoir with the functions of irrigation water supply, flood control, tourism, power generation and fisheries and other functions. With the rapid growth of urban population, the problem of water quality has become the common concern of the whole sociality. Because of main pollutant of nutrient such as TN, TP and organic pollutants (pops), reservoir eutrophication tendency obvious, heavy metals and other toxic and harmful substances pollution is relatively small, so the choice dissolved oxygen, chemical oxygen demand, permanganate index, biochemical oxygen demand (cod), total phosphorus, ammonia nitrogen and total nitrogen 7 indicators as to participate in the evaluation of reservoir parameters according to Chinese Surface Water Environment Quality Standards (GB3838-2002). Theses selected evaluation indicators are briefly denoted by DO (o_1), COD (o_2), COD_{Mn} (o_3), BOD_5 (o_4), TP (o_5), NH_3N (o_6) and TN (o_7). The five water quality grades have been derived as follows: I (Good), II (Fine), III (Ordinary), IV (Poor) and V (Poor). The standard of water quality is reported in Table 1. Here, o_1 is the benefit indicator and others are cost indicators. The monitoring points (samples) are A_1, A_2, A_3, A_4 and A_5. The indicator measure values of samples are reported in Table 2.

Table 1: National quality standards of suface waters (GB3838-2002) of China (units of mg/L)

Grade	O_1	O_2	O_3	O_4	O_5	O_6	O_7
I	7.5	15	2	3	0.02	0.15	0.2
II	6	15	4	3	0.1	0.5	0.5
III	5	20	6	4	0.2	1.0	1.0
IV	3	30	10	6	0.3	1.5	1.5
V	2	40	15	10	0.4	2.0	2.0

Table 2: Water monitoring data of fuqiao river reservoir

	O_1	O_2	O_3	O_4	O_5	O_6	O_7
A_1	9.74	35	7.2	5.0	0.073	0.46	0.681
A_2	9.71	9	3.2	1.0	0.061	0.41	0.652
A_3	9.86	18	4.8	2.2	0.050	0.33	0.609
A_4	9.83	11	3.0	1.5	0.052	0.38	1.127
A_5	9.73	14	3.5	3.0	0.061	0.4	0.524

The steps of the proposed method are given as follows:

Step 1: According to Table 1 and 2, the sample space matrix and standard grade matrix are obtained as follows:

$$X=(x_{ij})_{5\times 5}=\begin{array}{c} \\ A_1\\ A_2\\ A_3\\ A_4\\ A_5\end{array}\begin{array}{c} \begin{array}{ccccc} o_1 & o_2 & o_3 & o_4 & o_5\end{array}\\ \begin{pmatrix} 9.74 & 35 & 7.2 & 5 & 0.073\\ 9.71 & 9 & 3.2 & 1 & 0.061\\ 9.86 & 18 & 4.8 & 2.2 & 0.05\\ 9.83 & 11 & 3 & 1.5 & 0.052\\ 9.73 & 14 & 3.5 & 3 & 0.061\end{pmatrix}\end{array}$$

$$A=(a_{ik})_{7\times 5}=\begin{array}{c} \\ o_1\\ o_2\\ o_3\\ o_4\\ o_5\\ o_6\\ o_7\end{array}\begin{array}{c} \begin{array}{ccccc} I & II & III & IV & V\end{array}\\ \begin{pmatrix} 7.5 & 6 & 5 & 3 & 2\\ 15 & 15 & 20 & 30 & 40\\ 2 & 4 & 6 & 10 & 15\\ 3 & 3 & 4 & 6 & 10\\ 0.02 & 0.1 & 0.2 & 0.3 & 0.4\\ 0.15 & 0.5 & 1 & 1.5 & 2\\ 0.2 & 0.5 & 1 & 1.5 & 2\end{pmatrix}\end{array}$$

Take monitoring point (sample) A_1 as the example, the steps of the proposed method are given as follows:

Step 2: The attribute recognition decision matrix $H_1 = (\mu_{1jk})_{7\times 5}$ are obtained as follows:

$$H_1=\begin{array}{c} \\ o_1\\ o_2\\ o_3\\ o_4\\ o_5\\ o_6\\ o_7\end{array}\begin{array}{c} \begin{array}{ccccc} I & II & III & IV & V\end{array}\\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0.5 & 0.5\\ 0 & 0 & 0.7 & 0.3 & 0\\ 0 & 0 & 0.5 & 0.5 & 0\\ 0.3375 & 0.6625 & 0 & 0 & 0\\ 0.1143 & 0.8857 & 0 & 0 & 0\\ 0 & 0.6380 & 0.3620 & 0 & 0\end{pmatrix}\end{array}$$

Step 3: By coefficient of variation method, the attribute weights obtained as follows:

$$w_1=0.1070, w_2=0.1713, w_3=0.1801, w_4=0.1408,$$
$$w_5=0.1343, w_6=0.0989, w_7=0.1676$$

Step 4: The PIS and NIS are:

$$C_1^+=(1, 0.5, 0.7, 0.5, 0.6625, 0.8857, 0.6380)$$

and,

$$C_1^-=(0,0,0,0,0,0,0)$$

Respectively,

Step 5: The grey relational degree $\xi(\mu_1^+,\mu_{1k})$ of each object from PIS and grey relational degree $\xi(\mu_1^-,\mu_{1k})$ of each alternative from NIS by using the following equation, respectively:

$$\xi^+(\mu_{11})=0.5320, \xi^+(\mu_{12})=0.6675, \xi^+(\mu_{13})=0.6437,$$
$$\xi^+(\mu_{14})=0.6149, \xi^+(\mu_{15})=0.5159$$

and,

$$\xi^-(\mu_{11})=0.8562, \xi^-(\mu_{12})=0.7663, \xi^-(\mu_{13})=0.7542,$$
$$\xi^-(\mu_{14})=0.7764, \xi^-(\mu_{15})=0.9143$$

Step 6: The closeness of coefficient of A_1 with respect to kth grade:

$$C_1=0.3832, C_2=0.4656, C_3=0.4605,$$
$$C_4=0.4420, C_5=0.3623$$

Step 7: Due to the maximum $C_2 = 0.4656$, so according to the water quality recognition rule, monitoring point (sample) A_1 belongs to the grade II standard and briefly denote $A_1\rightarrow$ II.

Similarly, other monitoring points' water quality results can be obtained as $A_2\rightarrow$ I, $A_3\rightarrow$ II, $A_4\rightarrow$ I, $A_5\rightarrow$ I. Also we can also get the ranking order of the five monitoring points' water quality as $A_4>A_2>A_5>A_3>A_1$.

CONCLUSION

This study is focus on water quality evaluation problem, which contains many evaluation indicators and thus this problem can be solved by using MADM method. Thus this study put forward an improved GRA method to deal with the water quality evaluation problem. The indicators' weights are determined by coefficient of variation method, which is an objective weighting method and thus can avoid the subjective randomness. The proposed method is easy to calculation and can be easily solve by software such as matlab 12.0. Finally, water quality evaluation of Fuqiao River Reservoir is given as a case study to demonstrate and validate the application of the proposed method.

The proposed method can also be used to other area, such as hydropower project investment decision-making, water resources carrying capacity and environmental air quality assessment.

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