

Research Article

Adaptive Neural Network Output Feedback Tracking Control for a Class of Complicated Agricultural Mechanical Systems

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Abstract: The study presents an adaptive neural network output feedback tracking control scheme for a class of complicated agricultural mechanical systems. The scheme includes a dynamic gain observer to estimate the unmeasurable states of the system. The main advantages of the authors scheme are that by introducing non-separation principle design neural network controller and the observer gain are simultaneously tuned according to output tracking error, the semi-globally ultimately bounded of output tracking error and all the states in the closed-loop system can be achieved by Lyapunov approach. With the universal approximation property of NN and the simultaneous parametrisation, no Lipschitz assumption and SPR condition are employed which makes the system construct simple. Finally the simulation results are presented to demonstrate the efficiency of the control scheme.

Keywords: Agricultural mechanical systems, higher relative degree, neural network, non-separation principle, output feedback

INTRODUCTION

After it was proven that Neural Network (NN) and Fuzzy Logic Systems (FLSs) are universal function approximators, the adaptive control algorithms of unknown or ill-defined nonlinear systems that employ NN and FLSs have been developed (Chen *et al.*, 2009; Yu *et al.*, 2011; Liu *et al.*, 2010; Hu *et al.*, 2010a; Liu and Wan, 2002; Chen and Jiao, 2010), especially when modern mechanical or electrical systems that are to be controlled become more and more complicated and, thus, their mathematical model is often hard to be established. To remove the assumption that the states of the system are available for measurement in the aforementioned control approaches, in the references (Leu *et al.*, 2005; Hu *et al.*, 2010b; Tong and Qu, 2005; Wang *et al.*, 2010; Ge and Zhang, 2003), the problem of adaptive fuzzy or neural network output feedback control for uncertain SISO, MIMO nonlinear system via state observers has been investigated and the stability of the resulting closed-loop adaptive control system has been analyzed. For state observers, likewise high gain observers, it is often very hard to choose a proper observer gain. In some schemes (Ge and Zhang, 2003), a low-pass filter is designed to make the estimation

error dynamics satisfy the Strictly Positive-Real (SPR) condition so that they can use Meyer-Kalmon-Yakubovitz (MKY) lemma, which makes the stability analysis of the closed-loop system and real implementation very complicated. And the parameters of filter are hard to be chosen. Above researches are all based on the concept of separation principle which can realize the original state feedback with the corresponding observer states and thus the corresponding output feedback controller can be constructed. But it is hard to choose the observer and controller design parameters. Such problems have been solved by using some non-separation principle designs by Qian and Liu (2002), Bullinger and Allgower (2005) and Du and Ge (2010), but they need to satisfy Lipschitz assumptions.

To simplify the system construct and relax the constraints, in this paper an adaptive neural network output feedback tracking control scheme for a class of affine nonlinear higher relative degree systems is presented. Combined with non-separation principle, the gains of the observer and the neural controller are simultaneously tuned according to output tracking error. The proposed scheme has few adapting parameters to be tuned and Lipschitz assumption, SPR condition are not required.

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PROBLEM FORMULATION

The following notations and definitions will be used extensively throughout this study. Let R be the real number and R^i represent the real i -vectors. $|k|$ denotes the usual Euclidean norm of a vector k . In case where k is a scalar, $|k|$ denotes its absolute value.

Consider the following SISO affine nonlinear uncertain system:

$$\begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{cases} \quad (1)$$

where, $x = [x_1, \dots, x_n] \in R^n$ are the states of the system and $y \in R, u \in R$ are system output and input, respectively. Only y is available for control design. $f(x), g(x)$ are unknown smooth nonlinear function. $h(x) \in R$ is smooth scalar function. The system has higher relative degree $r < n$. According to differential geometry theory of nonlinear system, there is a nonlinear coordinate transformation $T(x) = (\xi^T, \eta^T)^T$ which can change the original system (1) into the equivalent input-output description, namely:

$$\begin{cases} \frac{d\xi_i}{dt} = \xi_{i+1} \quad i = 1, \dots, r-1 \\ \frac{d\xi_r}{dt} = \alpha(\xi, \eta) + \beta(\xi, \eta)u \\ \frac{d\eta}{dt} = q(\xi, \eta) \\ y = \xi_1 \end{cases} \quad (2)$$

where, $\xi_i = L_f^{i-1} h(x), \alpha(\xi, \eta) = L_f^r h(x)$ and $\beta(\xi, \eta) = L_g L_f^{r-1} h(x) \neq 0$. For all $(x, u) \in \Omega_x \times R$ the function $\beta(\xi, \eta)$ is nonzero and bounded. This implies that $\beta(\xi, \eta)$ is strictly either positive or negative. Without loss of generality, we assume $\beta(\xi, \eta) > 0$ and there exist constants $\beta_{\max} \geq \beta_{\min} > 0$ such that $\beta_{\min} \leq |\beta(\bullet)| \leq \beta_{\max}$ on the compact set $\{\eta, \xi\} \in \Omega_c$; the smooth nonlinear functions $q(\xi, \eta), \alpha(\xi, \eta)$ and $\beta(\xi, \eta)$ are unknown and satisfy $q(0, 0) = 0$. The subsystem $\dot{\eta} = q(\xi, \eta)$ is unmodelled zero dynamics and the states (ξ_2, \dots, ξ_r) and η are not measurable.

The control objectives of this study is to utilize an adaptive neural network to determine a tracking controller for a class of affine nonlinear systems (1) with strong relative degree such that the system output y follows a desired trajectory y_d while all signals that are involved in the resulting closed-loop system are bounded.

Assumption 1: Zero dynamics $\frac{d\eta}{dt} = q(0, \eta)$ is exponentially stable and the function $q(\xi, \eta)$ is Lipschitz in ξ . By Lyapunov converse theorem, there is a Lyapunov function $V_0(\eta)$ which satisfies:

$$a_1 \|\eta\|^2 \leq V_0(\eta) \leq a_2 \|\eta\|^2 \quad (3)$$

$$\frac{\partial V_0(\eta)}{\partial \eta} q(0, \eta) \leq -a_3 \|\eta\|^2 \quad (4)$$

$$\left\| \frac{\partial V_0(\eta)}{\partial \eta} \right\| \leq a_4 \|\eta\| \quad (5)$$

where, $a_i, i = 1, 2, 3, 4$ are positive constant:

$$\|q(\xi, \eta) - q(0, \eta)\| \leq L_\xi \|\xi\| \quad (6)$$

where, L_ξ is Lipschitz constant.

STATE OBSERVER AND ADAPTIVE OUTPUT FEEDBACK CONTROLLER DESIGN

Define the vectory $\underline{y}_d = [y_d \dot{y}_d \dots y_d^{(r-1)}]^T \in R^r$, state vector $\underline{\xi} = [\xi_1 \xi_2 \dots \xi_r]^T \in R^r$. The reference signal y_d and its time derivative are assumed to be smooth and bounded. We also define the tracking error as $e = y - y_d$ and corresponding error vector as $\underline{e} = \underline{\xi} - \underline{y}_d = [\xi_1 - y_d, \dots, \xi_r - y_d^{(r-1)}]^T \in R^r$.

The error equation is as follows:

$$\begin{cases} \dot{\eta} = q(\xi, \eta) \\ \dot{\underline{e}} = A_0 \underline{e} + B[\alpha(\xi, \eta) + \beta(\xi, \eta)u - y_d^{(r)}] \\ e_1 = C^T \underline{e} \end{cases} \quad (7)$$

where,

$$A_0 = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{r \times r}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}_{r \times 1}, \quad C = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{r \times 1}$$

Consider the following observer that estimates the state vector \underline{e} in (7):

$$\begin{cases} \dot{\hat{e}}_i = \hat{e}_{i+1} + \frac{\lambda_i}{\rho^i} (e_i - \hat{e}_i) \quad (i = 1, \dots, r-1) \\ \dot{\hat{e}}_r = -K^T \hat{\underline{e}} + \frac{\lambda_r}{\rho^r} (e_1 - \hat{e}_1) \\ \hat{e}_1 = C^T \hat{\underline{e}} \end{cases} \quad (8)$$

Define:

$$A_k = \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ -k_1 & -k_2 & \dots & -k_r \end{bmatrix}_{r \times r}, \quad A_\lambda = \begin{bmatrix} -\lambda_1 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\lambda_{r-1} & 0 & \dots & 1 \\ -\lambda_r & 0 & \dots & 0 \end{bmatrix}_{r \times r}$$

Equation (8) can be rewritten as:

$$\begin{aligned} \dot{\hat{e}} &= A_k \hat{e} + \lambda_\rho \bar{e}_1 \\ \hat{e}_1 &= C^T \hat{e} \end{aligned} \quad (9)$$

where, $\hat{e} = [\hat{e}_1, \dots, \hat{e}_r]^T$, $\bar{e}_1 = e_1 - \hat{e}_1$, $\lambda_\rho = [\lambda_1/\rho, \dots, \lambda_r/\rho^r]^T$.

Choose the vectors $K = [k_1, \dots, k_r]^T$, $\lambda = [\lambda_1, \dots, \lambda_r]^T$ to make matrices A_k, A_λ be Hurwitz. Thus, there exist square matrices $P_K = P_K^T > 0$, $P_\lambda = P_\lambda^T > 0$, $Q_K = Q_K^T > 0, Q_\lambda = Q_\lambda^T > 0$ satisfying:

$$A_k^T P_K + P_K A_k = -Q_K, A_\lambda^T P_\lambda + P_\lambda A_\lambda = -Q_\lambda \quad (10)$$

The gain of the observer is time-variable ($0 < \rho \leq 1$) and is updated by:

$$\rho = e^{-\alpha}, \quad \dot{\alpha} = \begin{cases} \kappa(|e_1| - \Xi_0) & |e_1| > \Xi_0 \\ 0 & |e_1| \leq \Xi_0 \end{cases} \quad \alpha(0) \geq 0 \quad (11)$$

where, design parameters κ, Ξ_0 are positive.

Considering the Eq. (7), the ideal control law u^* is chosen as:

$$u^* = \frac{1}{\beta(\xi, \eta)} (-\alpha(\xi, \eta) + y_d^{(r)} - K^T \underline{e}) \quad (12)$$

Then, the following equation holds:

$$\begin{aligned} \alpha(\xi, \eta) + \beta(\xi, \eta)u - y_d^{(r)} &= \alpha + \beta u - y_d^{(r)} + \beta u^* - \beta u^* \\ &= \alpha + \beta u - \beta u^* - y_d^{(r)} + \beta^* \frac{1}{\beta} (-\alpha + y_d^{(r)} - K^T \underline{e}) \\ &= -K^T \underline{e} + \beta(u - u^*) \end{aligned} \quad (13)$$

Then system (7) can be rewritten into:

$$\begin{aligned} \dot{\eta} &= q(\xi, \eta) \\ \dot{e}_i &= e_{i+1} \quad (i=1, \dots, r-1) \\ \dot{e}_r &= -K^T \underline{e} + \beta(\bullet)(u - u^*) \\ \dot{e}_1 &= C^T \underline{e} \end{aligned} \quad (14)$$

Define the observing error as $\tilde{e} = \underline{e} - \hat{e}$. Subtracting (14) from (8), we have:

$$\begin{aligned} \dot{\tilde{\eta}} &= q(\xi, \eta) \\ \dot{\tilde{e}}_i &= \tilde{e}_{i+1} - \frac{\lambda_i}{\rho^i} \tilde{e}_i \quad (i=1, \dots, r-1) \\ \dot{\tilde{e}}_r &= -K^T \tilde{e} - \frac{\lambda_r}{\rho^r} \tilde{e}_1 + \beta(\bullet)(u - u^*) \\ \dot{\tilde{e}}_1 &= C^T \tilde{e} \end{aligned} \quad (15)$$

Considering the following change of coordinates

$$\tilde{z}_1 = \frac{\tilde{e}_1}{\rho^{r-1}}, \tilde{z}_2 = \frac{\tilde{e}_2}{\rho^{r-2}}, \dots, \tilde{z}_r = \frac{\tilde{e}_r}{\rho^0} = \tilde{e}_r, \quad (15)$$

$$\tilde{z}_1 = \frac{1}{\rho} \tilde{z}_{i+1} - \frac{\lambda_i}{\rho} \tilde{z}_i, \quad \tilde{z}_r = -\frac{\lambda_r}{\rho} \tilde{z}_1 - K^T \underline{\rho} \tilde{z} + \beta(\bullet)(u - u^*) \quad (16)$$

where,

$$\tilde{z} = [\tilde{z}_1, \dots, \tilde{z}_r]^T, \quad \underline{\rho} = \begin{bmatrix} \rho^{r-1} & & 0 \\ & \ddots & \\ 0 & & \rho^0 \end{bmatrix}$$

and $\tilde{e} = \underline{\rho} \tilde{z}$.

Considering (9), (10), (14) and (16), we have:

$$\begin{aligned} \dot{\tilde{z}} &= \tilde{A} \tilde{z} + \tilde{B} \beta(\bullet)(u - u^*) \\ \tilde{z}_1 &= \frac{1}{\rho} \tilde{z}_2 - \frac{\lambda_1}{\rho} \tilde{z}_1 \\ \tilde{z}_2 &= \frac{1}{\rho} \tilde{z}_3 - \frac{\lambda_2}{\rho} \tilde{z}_2 \\ &\vdots \\ \tilde{z}_r &= -\frac{\lambda_r}{\rho} \tilde{z}_1 - K^T \underline{\rho} \tilde{z} + \beta(\bullet)(u - u^*) \end{aligned} \quad (17)$$

The control design presented in this study employs a Radial Basis Function (RBF) neural network to approximate the unknown function over a compact region of the state space because of their good capabilities in function approximation. The following RBF NN is used to approximate the any continuous function $h(z): R^n \rightarrow R$ over a compact region $\Omega \subset R^n$ with arbitrary accuracy by choosing enough nodes:

$$h(z) = W^{*T} \phi(z) + \varepsilon(z), \quad \forall z \in \Omega \quad (18)$$

where, W^* is a vector of adjustable weights, $z \in \Omega_{ZNN} \subset R^n$ is the input vector and the kernel vector is $\phi(z) = [\phi_1(z), \dots, \phi_l(z)]^T$ with active function $\phi_i(z) = \exp[-\frac{\|z - \sigma_i\|^2}{v_i^2}]$, $i = 1, 2, \dots, l$. l is the hidden layer nodes number and $\varepsilon(z)$ is the approximation error. We define the ideal weight vector $W^* \triangleq \arg \min_{W \in R^l} \left\{ \sup_{z \in \Omega_{ZNN}} |h(z) - \hat{h}(z)| \right\}$ which is an artificial quantity required for analytical purposes. Since the functions are approximated over a compact set, we have the following relationship:

$$\|W^*\| \leq \omega_{\max}, \|\varepsilon\| \leq \varepsilon_{\max}, \quad \forall z \in \Omega_{ZNN} \quad (19)$$

where, $\omega_{\max}, \varepsilon_{\max} > 0$. According to Equation (15), we have:

$$W^{*T} \phi(z) + \varepsilon(z) \leq \|W^{*T} \phi(z)\| + \|\varepsilon(z)\| \leq \|\phi(z)\| \omega_{\max} + \varepsilon_{\max} \leq \zeta \psi(z) \quad (20)$$

where, $\psi(z) = \sqrt{\left(\sum_{m=1}^l \phi^2(z)\right)} + 1$, $\varsigma = \max\{\omega_{\max}, \varepsilon_{\max}\}$.

Lemma 1 (Du and Ge, 2010): If Gaussian radial basis function is used, then for $X = [x_1, \dots, x_n]^T, Y = [y_1, \dots, y_n]^T$, there exists a positive constant $L_p = 2\sqrt{n \sum_{i=1}^l (1/v_i^2)}$ on R^n such that:

$$\|\phi(X) - \phi(Y)\| \leq L_p \|X - Y\| \quad (21)$$

So the unknown function u^* in (17) can be approximated by RBF NNs:

$$u^*(\underline{e}, \underline{z}, y_d^{(r)}) = W^{*T} \phi(\underline{e}, \underline{z}, y_d^{(r)}) + \varepsilon \quad (22)$$

where, W^* is the ideal NN weight vector and ε is the approximating error. With Lemma 1, $\|\underline{\rho}\| \leq 1$ and $\|\underline{\tilde{e}}\| = \|\underline{\rho}\underline{\tilde{z}}\| \leq \|\underline{\tilde{z}}\|$, we can obtain:

$$\begin{aligned} u^*(\eta, \underline{e}, \underline{z}, \bar{y}_d, y_d^{(r)}) &= W^{*T} \phi(\eta, \underline{e}, \underline{z}, y_d^{(r)}) + \varepsilon \\ &\leq \|W^{*T} \phi(0, \hat{\underline{e}}, \hat{\underline{z}}, \bar{y}_d, y_d^{(r)}) + W^{*T} (\phi(\eta, \underline{e}, \underline{z}, y_d^{(r)}) - \phi(0, \hat{\underline{e}}, \hat{\underline{z}}, \bar{y}_d, y_d^{(r)}))\| + \varepsilon \\ &\leq \varsigma^* \psi(\hat{\underline{e}}, \hat{\underline{z}}, \bar{y}_d, y_d^{(r)}) + \omega_{\max} L_p (\|\hat{\underline{z}}\| + \|\eta\|) \end{aligned} \quad (23)$$

where, $\psi(\hat{\underline{e}}, \hat{\underline{z}}, \bar{y}_d, y_d^{(r)}) = \sqrt{\left(\sum_{m=1}^l \phi^2(\hat{\underline{e}}, \hat{\underline{z}}, \bar{y}_d, y_d^{(r)})\right)} + 1$, $\varsigma^* = \max\{\omega_{\max}, \varepsilon_{\max}\}$. ς^* is an unknown parameter and we define $\hat{\varsigma}$ as the estimation of unknown scalar $\frac{\beta_{\max}}{\beta_{\min}} \varsigma^*$, Define:

$$\tilde{\varsigma} = \hat{\varsigma} - \frac{\beta_{\max}}{\beta_{\min}} \varsigma^* \quad (24)$$

The adaptive and control laws are chosen as follows:

$$u = -\frac{\gamma}{\sqrt{\rho}} \hat{\underline{e}}^T P_K B - \hat{\varsigma} \frac{\hat{\underline{e}}^T P_K B \psi^2}{\|\hat{\underline{e}}^T P_K B\| \psi + \sigma}, \quad \dot{\hat{\varsigma}} = -\tau \sqrt{\rho} \hat{\varsigma} + \frac{\|\hat{\underline{e}}^T P_K B\|^2 \psi^2}{\|\hat{\underline{e}}^T P_K B\| \psi + \sigma} \quad (25)$$

where, design parameters γ, τ and σ are positive.

STABILITY ANALYSIS

In this section, stability analysis for the proposed output feedback tracking control scheme will be

presented. To this end, we firstly give the relationship according to (3), (4) and (5):

$$\begin{aligned} \dot{V}_0 &= \frac{\partial V_0(\eta)}{\partial \eta} q(0, \eta) + \frac{\partial V_0(\eta)}{\partial \eta} [q(\xi, \eta) - q(0, \eta)] \\ &\leq -a_3 \|\eta\|^2 + a_0 a_4 \|\eta\| \|\xi\| \\ &\leq -\frac{a_3}{2} \|\eta\|^2 + \frac{a_0 a_4}{2 a_3} \|\xi\|^2 \\ &\leq -\frac{a_3}{2} \|\eta\|^2 + \frac{a_0 a_4}{2 a_3} \|\underline{e} + \underline{y}_d\|^2 \\ &\leq -\frac{a_3}{2} \|\eta\|^2 + \frac{a_0 a_4}{a_3} (\|\underline{e}\|^2 + \|\underline{y}_d\|^2) \end{aligned} \quad (26)$$

We are now ready to establish the main theorem of this study.

Theorem 1: Consider the closed-loop system consisting of the plant (1) under the assumption 1, the adaptive controller and adaptation law (25), with an appropriate initial value $\alpha(0) \geq \alpha^*$ for α^* satisfying the condition (31), all closed-loop signals are semi-globally uniformly bounded over the following compact sets, namely:

- $\forall t \geq 0, \quad \Omega_{\eta(t)} = \left\{ \eta(t) \mid \|\eta(t)\| \leq \sqrt{\frac{2[V(0)q' + \sigma']}{q' a_1 \lambda_w}} \right\},$
 - $\Omega_{\underline{e}(t)} = \left\{ \underline{e}(t) \mid \|\underline{e}(t)\| \leq \sqrt{\frac{2[V(0)q' + \sigma']}{q' \lambda_{\min}(P_K)}} \right\},$
 - $\Omega_{\underline{\tilde{e}}(t)} = \left\{ \underline{\tilde{e}}(t) \mid \|\underline{\tilde{e}}(t)\| \leq \sqrt{\frac{2[V(0)q' + \sigma']}{q' \lambda_{\min}(P_L)}} \right\},$
 - $\Omega_{\tilde{\varsigma}(t)} = \left\{ \tilde{\varsigma}(t) \mid \|\tilde{\varsigma}(t)\| \leq \sqrt{\frac{2[V(0)q' + \sigma']}{q' \beta_{\min}}} \right\}$
 - $t \rightarrow \infty, \quad \Omega_{\underline{e}} = \left\{ \underline{e}(t) \mid \|\underline{e}(t)\| \leq \sqrt{\frac{2\sigma'}{q' \lambda_{\min}(P_K)}} \right\},$
 - $\Omega_{\underline{\tilde{e}}} = \left\{ \underline{\tilde{e}}(t) \mid \|\underline{\tilde{e}}(t)\| \leq \sqrt{\frac{2\sigma'}{q' \lambda_{\min}(P_L)}} \right\},$
 - $\Omega_{\tilde{\varsigma}_\infty} = \left\{ \tilde{\varsigma}(t) \mid \|\tilde{\varsigma}(t)\| \leq \sqrt{\frac{2\sigma'}{q' \beta_{\min}}} \right\}$ where
- $$q' = \min \left\{ \frac{q_1}{\lambda_{\max}(P_K)}, \frac{\eta^*}{\lambda_{\max}(P_L)}, \frac{\tau \sqrt{\rho}}{2} \right\}, \quad \sigma' = \beta_{\max} \varsigma^* \sigma$$

Proof: Define $q_1 = \lambda_{\min}(Q_K), q_2 = \lambda_{\min}(Q_\lambda), q_3 = \|P_\lambda B K^T\|, q_4 = \|P_\lambda B\| + \|P_K B\|, q_5 = \frac{\beta_{\max}^2 \omega_{\max}^2 L_p^2}{2\gamma \beta_{\min}}$:

$$q_6 = \frac{2\beta_{\max}^2 \omega_{\max}^2 L_p^2}{a_3}, c_1 = (\beta_{\max} + \beta_{\min})q_4\psi, c_2 = \beta_{\max} q_4 \omega_{\max} L_p, c_3 = \frac{c_1^2}{\beta_{\min} \tau \sqrt{\rho}} + c_2 + \frac{q_4^2 \beta_{\max}^2 \gamma}{2\sqrt{\rho} \beta_{\min}}, c_4 = \frac{2q_4^2 \beta_{\max}^2 \omega_{\max}^2 L_p^2}{a_3}$$

Let the Lyapunov candidate function be defined as:

$$V = V_0 + \frac{1}{2} e^T P_K e + \frac{1}{2} \tilde{z}^T P_1 \tilde{z} + \frac{1}{2} \beta_{\max} \tilde{s}^2 \tag{27}$$

With (25), $e = \hat{e} + \tilde{e}$ and $\|\rho\| \leq 1$, then $\|\tilde{e}\| \leq \|\rho \tilde{z}\| \leq \|\tilde{z}\|$, the derivative of (27) with respect to time is given by:

$$\begin{aligned} \dot{V} &= \dot{V}_0 - \frac{1}{2} \|\dot{e}\|_{Q_e}^2 + \dot{e}^T P_K B [\beta(\bullet)(u - u^*)] - \frac{1}{2\rho} \|\tilde{z}\|_{Q_z}^2 + \tilde{z}^T P_1 B [\beta(\bullet)(u - u^*) - K^T \rho \tilde{z}] + \beta_{\max} \tilde{s} \dot{\tilde{s}} \\ &\leq \dot{V}_0 - \frac{q_1}{2} \|e\|^2 - \frac{1}{2} \left(\frac{q_2}{\rho} + 2q_3 \right) \|\tilde{z}\|^2 + \dot{e}^T P_K B [\beta(\bullet)(u - u^*)] + \beta_{\min} \tilde{s} \dot{\tilde{s}} + \tilde{e}^T P_K B [\beta(\bullet)(u - u^*)] + \tilde{z}^T P_K B [\beta(\bullet)(u - u^*)] \\ &\leq \dot{V}_0 - \frac{q_1}{2} \|e\|^2 - \frac{1}{2} \left(\frac{q_2}{\rho} + 2q_3 \right) \|\tilde{z}\|^2 + \dot{e}^T P_K B [\beta(\bullet)(u - u^*)] + \beta_{\max} q_4 \|\tilde{z}\| (|u| + |u^*|) + \beta_{\min} \tilde{s} \dot{\tilde{s}} \end{aligned} \tag{28}$$

where, the item $\dot{e}^T P_K B [\beta(\bullet)(u - u^*)]$ satisfies:

$$\begin{aligned} \dot{e}^T P_K B [\beta(\bullet)(u - u^*)] &= -\frac{\gamma \beta(\bullet)}{\sqrt{\rho}} \|\dot{e}^T P_K B\|^2 - \beta(\bullet) \tilde{s} \frac{\|\dot{e}^T P_K B \psi\|^2}{\|\dot{e}^T P_K B\| \psi + \sigma} + \beta_{\max} \dot{e}^T P_K B u^* \\ &\leq -\frac{\gamma \beta_{\min}}{\sqrt{\rho}} \|\dot{e}^T P_K B\|^2 - \beta_{\min} \tilde{s} \frac{\|\dot{e}^T P_K B \psi\|^2}{\|\dot{e}^T P_K B\| \psi + \sigma} + \beta_{\max} \|\dot{e}^T P_K B u^*\| \\ &\leq -\frac{\gamma \beta_{\min}}{2} \left(\frac{2}{\sqrt{\rho}} - 1 \right) \|\dot{e}^T P_K B\|^2 - \beta_{\min} \tilde{s} \frac{\|\dot{e}^T P_K B \psi\|^2}{\|\dot{e}^T P_K B\| \psi + \sigma} + \beta_{\max} \tilde{s}^* \sigma + q_5 \|\tilde{z}\|^2 + q_6 \|\dot{e}^T P_K B\|^2 + \frac{q_3}{8} \|\eta\|^2 \end{aligned} \tag{29}$$

According to (25), the control law can be parameterized as $|u| \leq \frac{\gamma}{\sqrt{\rho}} \|\dot{e}^T P_K B\| + \psi |\hat{\zeta}|$. For the item $\beta_{\max} q_4 \|\tilde{z}\| (|u| + |u^*|)$ in (28), it shows that:

$$\begin{aligned} \beta_{\max} q_4 \|\tilde{z}\| (|u| + |u^*|) &\leq \beta_{\max} q_4 \|\tilde{z}\| \left(\frac{\gamma}{\sqrt{\rho}} \|\dot{e}^T P_K B\| + \psi |\hat{\zeta}| + \zeta^* \psi + \omega_{\max} L_p (\|\tilde{z}\| + \|\eta\|) \right) \\ &\leq \beta_{\max} q_4 \psi \|\tilde{z}\| (|\hat{\zeta}| + |\zeta^*|) + \frac{\gamma \beta_{\max} q_4}{\sqrt{\rho}} \|\tilde{z}\| \|\dot{e}^T P_K B\| + q_4 \beta_{\max} \omega_{\max} L_p \|\tilde{z}\|^2 + q_4 \beta_{\max} \omega_{\max} L_p \|\tilde{z}\| \|\eta\| \\ &\leq (c_3 + c_4) \|\tilde{z}\|^2 + \frac{\beta_{\min} \gamma}{2\sqrt{\rho}} \|\dot{e}^T P_K B\|^2 + \frac{\beta_{\min} \tau \sqrt{\rho}}{4} (|\hat{\zeta}|^2 + |\zeta|^2) + \frac{a_3}{8} \|\eta\|^2 \end{aligned} \tag{30}$$

Using (26), (29) and (30), (28) becomes:

$$\begin{aligned} \dot{V} &\leq -\frac{a_3}{4} \|\eta\|^2 - \frac{q_1}{2} \|e\|^2 - \frac{1}{2} \left(\frac{q_2}{\rho} + 2q_3 - 2q_6 - 2c_3 - 2c_4 \right) \|\tilde{z}\|^2 + \beta_{\max} \tilde{s}^* \sigma - \frac{\gamma \beta_{\min}}{2} \left(\frac{1}{\sqrt{\rho}} - 1 - q_6 \right) \|\dot{e}^T P_K B\|^2 + \dots \\ &\quad + \frac{\beta_{\min} \tau \sqrt{\rho}}{4} (|\hat{\zeta}|^2 + |\zeta|^2) - \beta_{\min} \tilde{s} \frac{\|\dot{e}^T P_K B \psi\|^2}{\|\dot{e}^T P_K B\| \psi + \sigma} + \beta_{\min} \tilde{s} \left(-\tau \sqrt{\rho} \dot{\tilde{s}} + \frac{\|\dot{e}^T P_K B\|^2 \psi^2}{\|\dot{e}^T P_K B\| \psi + \sigma} \right) \\ &\leq -\frac{a_3}{4} \|\eta\|^2 - \frac{q_1}{2} \|e\|^2 - \frac{1}{2} \left(\frac{q_2}{\rho} + 2q_3 - 2q_6 - 2c_3 - 2c_4 \right) \|\tilde{z}\|^2 - \frac{\gamma \beta_{\min}}{2} \left(\frac{1}{\sqrt{\rho}} - 1 \right) \|\dot{e}^T P_K B\|^2 \end{aligned} \tag{31}$$

In (31), there exists a constant α^* for the following relationships hold:

$$q_2 e^{\alpha^*} + 2q_3 - 2q_6 - 2c_3 - 2c_4 > 0, \quad e^{\frac{\alpha^*}{2}} - 1 \geq 0 \quad (32)$$

From (11), the variable α is no-decreasing. Considering two aspects:

- If $\alpha(0)$ is chosen large enough
- α Depends on the output tracking error e_1 , when e_1 increases, which makes (32) hold and thus e_1 is decreased until it equals to error tolerance Ξ_0 . So there exist $\alpha(0)$ and a finite time t^* such that:

$$q_2 e^{\alpha} + 2q_3 - 2q_6 - 2c_3 - 2c_4 > \eta^*, \quad e^{\frac{\alpha}{2}} - 1 \geq 0 \quad (33)$$

Then, (31) can be written as:

$$\dot{V} \leq -\frac{a_3}{4} \|\eta\|^2 - \frac{q_1}{2} \|\underline{e}\|^2 - \frac{1}{2} \eta^* \|\underline{z}\|^2 - \frac{\beta_{\min} \tau \sqrt{\rho}}{4} (|\hat{\zeta}| + |\tilde{\zeta}|)^2 + \beta_{\max} \zeta^* \sigma \leq -q' V + \sigma' \quad (34)$$

where,

$$q' = \min \left\{ \frac{a_3}{8a_2}, \frac{q_1}{\lambda_{\max}(P_K)}, \frac{\eta^*}{\lambda_{\max}(P_L)}, \frac{\tau \sqrt{\rho}}{2} \right\}, \quad \sigma' = \beta_{\max} \zeta^* \sigma$$

Multiplying both sides of (34) by $e^{-q't}$ and integrating over $[0, t]$, we obtain:

$$\|\underline{\tilde{e}}(t)\|_{P_L}^2 \leq \|\underline{\tilde{z}}(t)\|_{P_L}^2 \leq 2V(t) \leq 2V(0)e^{-q't} + \frac{2\sigma'}{q'}(1 - e^{-q't}) \quad (35)$$

So,

$$\|\underline{\tilde{e}}(t)\| \leq \sqrt{\frac{2[V(0)e^{-q't} + \sigma']}{q' \lambda_{\min}(P_L)}}, \quad \forall t \geq 0, \quad \|\underline{\tilde{e}}(t)\| \leq \sqrt{\frac{2\sigma'}{q' \lambda_{\min}(P_L)}}, \quad t \rightarrow \infty \quad (36)$$

Repeatedly, we can obtain the rest conclusions. With the boundedness of vectors \underline{e} and $\underline{\tilde{e}}$, it is easy to prove the boundedness of the vector $\underline{\hat{e}}$. This completes the proof.

SIMULATION STUDY

To verify the performance of the proposed adaptive neural network output feedback tracking controller, simulations will be taken for a class of affine nonlinear higher relative degree system described as follows:

$$\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= -2((\xi_1 - \eta_1)^2 - 1)(\xi_2 - \eta_2) - \eta_1 + (2 + \sin(\xi_1 \eta_1))u \\ \dot{\eta}_1 &= \eta_2 \\ \dot{\eta}_2 &= -2\eta_1 - 0.2\eta_2 + \xi_1 \\ y &= \xi_1 \end{aligned} \quad (37)$$

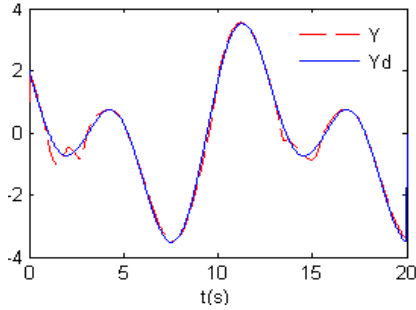


Fig. 1: Plots of output tracking of system

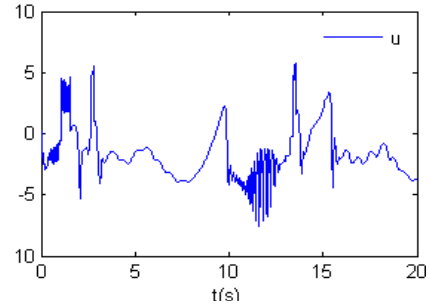


Fig. 3: Plots of control input

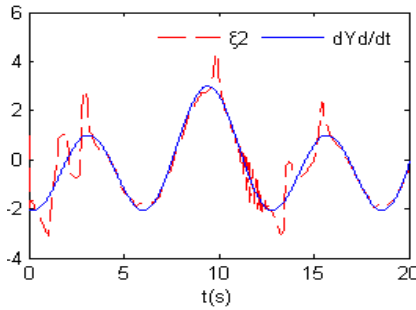


Fig. 2: Plots of state tracking of system

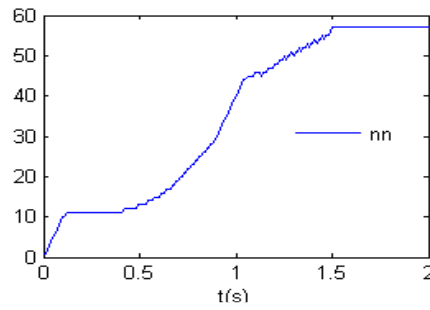


Fig. 4: Node number of hidden layer

where, $\beta(\xi, \eta) = 2 + \sin(\xi_1 \eta_1) > 0$. The system has an unmodelled zero dynamic. Only output is available for feedback design. The control objective is to force the system output y to follow the desired trajectory that is employed as $y_d = -2 \sin t + 2 \cos(0.5t)$. The tracking errors $e_1 = \xi_1 - y_d, e_2 = \xi_2 - \dot{y}_d$.

The system initial conditions are $x_1(0) = 2, x_2(0) = -2, \hat{e}_1(0) = 1, \hat{e}_2(0) = 2$. The simulation parameters are selected as follows:

$$K = \begin{bmatrix} 4 \\ 10 \end{bmatrix}, Q_k = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}, A_k = \begin{bmatrix} 0 & 1 \\ -2 & -10 \end{bmatrix}, P_k = \begin{bmatrix} 10.6 & 1 \\ 1 & 0.3 \end{bmatrix},$$

$$\lambda = \begin{bmatrix} 140 \\ 4 \end{bmatrix}, Q_\lambda = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix},$$

$$A_\lambda = \begin{bmatrix} -140 & 1 \\ -4 & 0 \end{bmatrix}, P_\lambda = \begin{bmatrix} 0.357 & 1 \\ 1 & 140.143 \end{bmatrix}$$

Choose other design parameters as follows:

$$\Xi_0 = 0.04, \kappa = 1.6, \alpha(0) = 3.8, \sigma = 0.2, \gamma = 0.02, \tau = 0.1$$

The simulation results using MATLAB is shown in Fig. 1 to 4, where the neural network initial structure and parameters is adjusted on-line by using GGP-RBF algorithm.

Figure 1 and 2 shows the results of output and state tracking. It can be seen that the actual trajectory converges rapidly to the desired one. The control input

signal is shown in Fig. 3. The growing and pruning automatically of hidden layer nodes is shown in Fig. 4. These simulation results demonstrate the tracking capability of the proposed controlled and its effectiveness for control systems tracking of affine nonlinear higher relative degree systems.

CONCLUSION

A new adaptive neural network output feedback tracking control scheme is presented for a class of affine nonlinear higher relative degree systems. The scheme does not require Lipschitz assumption and SPR condition which makes the system construct simple. By using the non-separation principle and the universal approximation property of NN few adapting parameters are required and the observer gains and controller parameters can be simultaneously tuned according to the tracking error. Output tracking error and all states in the closed-loop system are guaranteed to be semi-globally ultimately bounded by Lyapunov approach. Simulation results are provided to demonstrate the effectiveness of the proposed control scheme.

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REFERENCES

- Bullinger, E. and F. Allgower, 2005. Adaptive λ -tracking for nonlinear higher relative degree systems. *Automatica*, 41(7): 1191-2000.
- Chen, W.S. and L.C. Jiao, 2010. Adaptive tracking for periodically time-delaying and nonlinearly parameterized system using multilayer neural networks. *IEEE T. Neural Networ.*, 21(2): 345-351.
- Chen, B., X.P. Liu, K.F. Liu and C. Lin, 2009. Direct adaptive fuzzy control of nonlinear strict-feedback systems. *Automatica*, 45(6): 1530-1535.
- Du, H. and S.S. Ge, 2010. Output feedback adaptive neural control for a class of non-affine nonlinear systems with a dynamic gain observer. *IET Control Theory A.*, 5(3): 486-497.
- Ge, S.S. and J. Zhang, 2003. Neural-network control of nonaffine nonlinear system with zero dynamics by state and output feedback. *IEEE T. Neural Networ.*, 14: 900-918.
- Hu, H., G.R. Liu, H.Z. Tang and P. Guo, 2010a. Robust output tracking control for mismatched uncertainties nonlinear systems. *Nat. Sci. J. Xiangtan Univ.*, 32(4): 108-111.
- Hu, H., G.R. Liu, D.B. Liu and P. Guo, 2010b. Output feedback tracking control for a class of uncertain nonlinear MIMO systems using neural network. *Control Theor. Appl.*, 27(8): 382-386.
- Leu, Y.G., W.Y. Wang and T.T. Lee, 2005. Observer-based direct adaptive fuzzy-neural control for nonaffine nonlinear systems. *IEEE T. Neural Networ.*, 16(4): 853-861.
- Liu, G.R. and B.W. Wan, 2002. Direct adaptive fuzzy robust control for a class of nonlinear MIMO systems. *Control Theor. Appl.*, 19(5): 693-698.
- Liu, Y.J., W. Wang and S.C. Tong, 2010. Robust adaptive tracking control for nonlinear systems based on bounds of fuzzy approximation parameters. *IEEE T. Syst. Man Cy. A*, 40(1): 170-184.
- Qian, C. and W. Liu, 2002. Output feedback control of a class of nonlinear systems: A non-separation principle paradigm. *IEEE T. Automat. Contr.*, 47(10): 1710-1715.
- Tong, S.C. and L.J. Qu, 2005. Fuzzy adaptive output feedback control for a class of MIMO nonlinear systems. *Control Decision*, 27: 781-793.
- Wang, Y.F., T.Y. Chai and Y.M. Zhang, 2010. State observer-based adaptive fuzzy output feedback control for a class of uncertain nonlinear systems. *Inform. Sciences*, 180: 5029-5040.
- Yu, J.P., Y.M. Ma, B. Chen and H.S. Yu, 2011. Adaptive fuzzy backstepping position tracking control for a permanent magnet synchronous motor. *Int. J. Innov. Comput. I.*, 7(4): 1589-1602.