

Research Article

Investigating the Markov Property on Stock Returns: A Case Study of Ghana Stock Exchange

Sampson Wiredu, Abubakari A. Ghaniyyu and W. Abazing Mavis

Department of Statistics, Faculty of Mathematical Sciences, University for Development Studies,
P.O. Box 24, Navrongo, Ghana, West Africa

Abstract: This study was conducted to investigate the Markov property using daily, weekly and monthly stock returns of Accra Brewery Limited (ABL) of the Ghana Stock Exchange (GSE) spanning from the period of November, 1990 to August, 2007. Using Shapiro-Wilk normality test, the study revealed that the returns are not normally distributed as they were leptokurtic in nature indicating high volatility. Using several tests namely, the correlogram, ADF, PP and KPSS, Runs and Wright's non-parametric Variance ratio tests, the research concluded that the daily, weekly and monthly returns of GSE were stationary at level and do not follow random walk, hence do not have the Markov property.

Keywords: Market efficiency, Markov property, returns series, stationarity, unit root test

INTRODUCTION

The issue of whether stock returns are predictable is a very important issue in finance and yet controversial. The understanding and possible prediction of the way stock markets behave could result in investors getting a lot of returns for their investments. However, there are some contradictions in the findings of various researches on the behavior of stock markets due to differences in markets, sample periods and frequency of observations as well as the methodologies employed, regardless of whether the study was conducted in a developed or developing market.

Stationarity, as a characteristic of stock markets, is fundamental to the analysis of financial time series as this determines how the data could be handled. A non-stationary data can mislead one to generating spurious regressions and concluding that there is a significant relationship between the variables in the regression model even though no such relationship really exists. It can also limit the empirical and behavioral study of the underlying series to only one period and prevents generalization to other periods. Also, stationarity is a characteristic of an inefficient stock market.

Olweny and Omondi (2011), while studying the effect of macro-economic factors on stock return volatility in the Nairobi Stock Exchange, Kenya, found out that, upon the application of ADF test, the stock returns of Nairobi Stock Exchange (NSE) was stationary at 5% level of significance. The study used monthly time series data for NSE all share index that

covered a period of 10 years from 2001 to 2010. This result is supported by similar research by Malik *et al.* (2009), who examined the relationship between aggregate stock market, trading volume and daily stock returns from the Karachi Stock Exchange (KSE). At 1, 5 and 10%, respectively the return series was stationary using Phillips Perron test. Similarly, Ozkaya and Ozkaya (2012) and Gupta and Basu (2007), all found out that return series of different markets are stationary using one or more tests on various frequencies of return series.

Contrary to these findings are the following: Adjasi *et al.* (2008), on the effect of exchange rate volatility on the Ghana Stock Exchange, found out that the return series were not stationary by using ADF test. Also, Francis and Tewari (2011), in trying to investigate the relationship between stock returns and inflation rate in Ghana found out that both variables were not stationary at levels using the ADF and KPSS tests.

Other researchers include Aktham (2004) and Darrat *et al.* (2000) who all concluded that different return series of various markets were not stationary.

The GSE, as an emerging stock market and has a number of setbacks which affect its efficiency. As there is no consensus regarding the basic characteristics of stock markets, this study seeks to investigate the Markov property on the stock returns of GSE.

MATERIALS AND METHODS

The sample used for the study was the daily stock prices spanning from November, 1990 to September,

Corresponding Author: Sampson Wiredu, Department of Statistics, Faculty of Mathematical Sciences, University for Development Studies, P.O. Box 24, Navrongo, Ghana, West Africa

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2007, ignoring non-trading days and holidays obtained from the Ghana Stock Exchange (GSE). Because of the complete data within the time horizon, the closing stock price series of Accra Brewery Limited (ABL) was selected for the study. The logarithmic differences of the daily, weekly and monthly average returns were used for the study. This is given by the formula below:

$$R_t = \ln \left(\frac{P_t}{P_{t-1}} \right)$$

where,

R_t = The stock return at time t

\ln = The natural logarithm

P_t = The stock price at time t

P_{t-1} = The stock price at time t-1

The correlogram was used to check for stationarity of the various series. This involves a plot of the sample autocorrelation functions against the lag length. The sample autocorrelation at lag k is given by:

$$r_k = \frac{\sum_{t=1}^{N-k} (x_t - \bar{x})(x_{t+k} - \bar{x})}{\sum_{t=1}^N (x_t - \bar{x})^2}$$

where,

$$\bar{x} = \frac{\sum_{t=1}^N x_t}{N}$$

Augmented Dickey-Fuller (ADF) test: This method was used to test for the stationarity and is based on the assumption that the series follows a random walk and test a null hypothesis of non-stationarity (existence of a unit root) against the alternative of stationarity:

$$X_t = \rho X_{t-1} + \varepsilon_t$$

Subtracting X_{t-1} from both sides, the equation becomes:

$$X_t - X_{t-1} = \rho X_{t-1} - X_{t-1} + \varepsilon_t$$

$$\Delta X_t = \beta X_{t-1} + \varepsilon_t$$

where,

$$\Delta X_t = X_t - X_{t-1}$$

and

$$\beta = \rho - 1$$

The null hypothesis is $H_0: \beta = 0$, which implies $\rho = 1$ and alternative $H_1: \beta < 0$, thus $\rho < 1$.

This test requires the estimation of the regression:

$$\Delta X_t = \beta_1 + \beta X_{t-1} + \sum_{t=1}^m \alpha_i \Delta X_{t-1} + \varepsilon_t$$

where,

X_t = The time series

ΔX_t = The first difference of the X_t

β = A coefficient

m = The optimum lag length in the ADF regression which ensures that the residuals do not violate the assumption of serial correlation and initiates a white noise process, ε_t

$\sum_{t=1}^m \alpha_i \Delta X_{t-1}$ = The sum of the lagged values of the dependent variable, ΔX_t

Philip-Perron (PP) test: This test corrects the statistics for serial correlation and possible heteroskedastic error terms.

The null hypothesis is given by $H_0: \beta = 0$, existence of unit root and the alternative $H_1: \beta < 0$. This is based on the regression equation:

$$\Delta X_t = \alpha + \beta X_{t-1} + \delta t + \mu_t$$

where, X_t is the time series, ΔX_t is the first difference of the X_t , α is the intercept, β is the coefficient of interest, t is the time or trend variable and μ_t is the disturbance term. The ordinary least square standard errors are adjusted to allow for serial correlation in the disturbance term, μ_t .

Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test: The KPSS test is based on the null that a series is trend stationary or stationary around a level. The test starts with a model given by:

$$X_t = \alpha t + Y_t + \varepsilon_t$$

where, t is the deterministic trend, Y_t is a random walk and ε_t is a stationary error term. The random walk is given by:

$$Y_t = Y_{t-1} + \mu_t$$

With variance σ_μ^2 and $\mu \sim WN(0, \sigma_\mu^2)$. Where WN stands for white noise. A white noise is a completely random time series.

The hypothesis is given by $H_0: \sigma_\mu^2 = 0$ (The series is stationary) and alternative $H_1: \sigma_\mu^2 > 0$. With $H_0: \sigma_\mu^2 = 0$ implies Y_t is not a random walk process, but rather a constant and in turn, X_t becomes a stationary process.

Runs test: This is a non-parametric test and was used in checking for serial independence in the series. It examines whether the return movements are independent of each other by comparing the number of

runs observed in the series with the expected number of runs of the series. If the actual (observe) number of runs are equal or at least close to the expected number of runs, then the observations are independent and the series is random. The null hypothesis of randomness is tested by observing the sequence of successive price change with the same sign, positive, zero or negative. Each change is classified according to its position with respect to the mean. Positive change when greater than the mean, negative when less than the mean and zero when equals the mean. By comparing the actual (observed) number runs to the expected number of runs (μ) using the equation below, the test is carried out:

$$\mu = \frac{N(N + 1) - \sum_{i=1}^3 n_i^2}{N}$$

where, N is number of observations, i is signs of plus, minus and no change and n_i is total number of changes of each of each category of signs. For large sample sizes, the expected number of runs is approximately normally distributed with standard deviation:

$$\sigma_\mu = \sqrt{\frac{\sum_{i=1}^3 n_i [\sum_{i=1}^3 n_i^2 + N(N+1)] - 2N \sum_{i=1}^3 n_i^2 - N^3}{N^2(N-1)}}$$

The standard normal Z-statistic is given by:

$$Z = \frac{X - \mu \pm 0.5}{\sigma_\mu}, Z \sim N(0,1)$$

where, X is the actual number of runs, μ is the expected number of runs and ± 0.5 is the continuity correction which takes positive sign if $X < \mu$ and negative otherwise. A negative Z value indicates a positive serial correlation, whereas a positive value indicates a negative serial correlation.

Wright's non-parametric variance ratio test: Wright (2000) proposed an alternative test for standard variance ratio tests using ranks and signs in place of Lo-MacKinlay's test. Wright's modified variance ratio test is as follows:

Let $r(Y_t)$ be the rank of Y_t among Y_1, Y_2, \dots, Y_T :

$$r_{1t} = \frac{(r(Y)_t - \frac{T+1}{2})}{\sqrt{\frac{(T-1)(T+1)}{12}}}$$

$$r_{2t} = \phi^{-1} \left(\frac{r(Y)_t}{T+1} \right)$$

where, ϕ is the standard normal cumulative distribution function. The series r_{1t} is a simple linear transformation of the ranks, standardized to have sample mean 0 and sample variance 1. The series r_{2t} , known as the inverse normal or Van der Weerden scores, has sample mean 0 and sample variance approximately equal to 1.

The tests proposed by Wright (2000) are:

$$R_1 = \left[\frac{\frac{1}{Tk} \sum_{t=k+1}^T (r_{1t} + r_{1t-1} + \dots + r_{1t-k})^2}{\frac{1}{T} \sum_{t=1}^T r_{1t}^2} - 1 \right] * \left[\frac{2(2k-1)(k-1)}{3kT} \right]^{-1/2}$$

$$R_2 = \left[\frac{\frac{1}{Tk} \sum_{t=k+1}^T (r_{2t} + r_{2t-1} + \dots + r_{2t-k})^2}{\frac{1}{T} \sum_{t=1}^T r_{2t}^2} - 1 \right] * \left[\frac{2(2k-1)(k-1)}{3kT} \right]^{-1/2}$$

Y_t is assumed to be independent and identically distributed, $r(Y_t)$ is a random permutation of the numbers 1, 2, ..., T, each with equal probability. Wright (2000) also modified variance ratio tests by using the signs of returns. For any series Y_t , let $\mu(Y_t, q) = 1 (Y_t > q) - 0.5$. Choosing $q = 0$, $\mu(Y_t, 0)$ is 0.5 if Y_t is positive, otherwise it is -0.5. Let $S_t = 2\mu(Y_t, 0)$. This implies that S_t is equal to 1 when Y_t is positive and -1 otherwise, with equal probability of 0.5. S_t is an independent and identically distributed series with mean 0 and variance 1. The variance ratio statistic S_1 is defined as:

$$S_1 = \left[\frac{\frac{1}{Tk} \sum_{t=k+1}^T (s_t + s_{t-1} + \dots + s_{t-k})^2}{\frac{1}{T} \sum_{t=1}^T s_t^2} - 1 \right] * \left[\frac{2(2k-1)(k-1)}{3kT} \right]^{-1/2}$$

RESULTS AND DISCUSSION

The coefficients of skewness and kurtosis in Table 1 shows that the daily, weekly and monthly returns were skewed to the right and leptokurtic in nature indicating non-normality of the returns. The leptokurtic nature means the data were peaked and have most of their values concentrated around the centre. This indicates high volatility. That is, high returns are followed by low returns and extreme returns can be obtained at any point in time, a phenomenon known as volatility clustering. Using Shapiro-Wilk normality test, also shown in Table 1, the null hypothesis of a normal distribution was rejected at 5% level of significance with p-values of 0.00.

Table 1: Descriptive statistics of the returns

Returns	No.	Mean	S.D.	Skewness	Kurtosis	Shapiro-wilk
Daily	2589	0.000827	0.039706	4.650	406.804	0.00
Weekly	952	0.002155	0.053197	3.421	128.508	0.00
Monthly	192	0.001058	0.012973	1.154	15.412	0.00

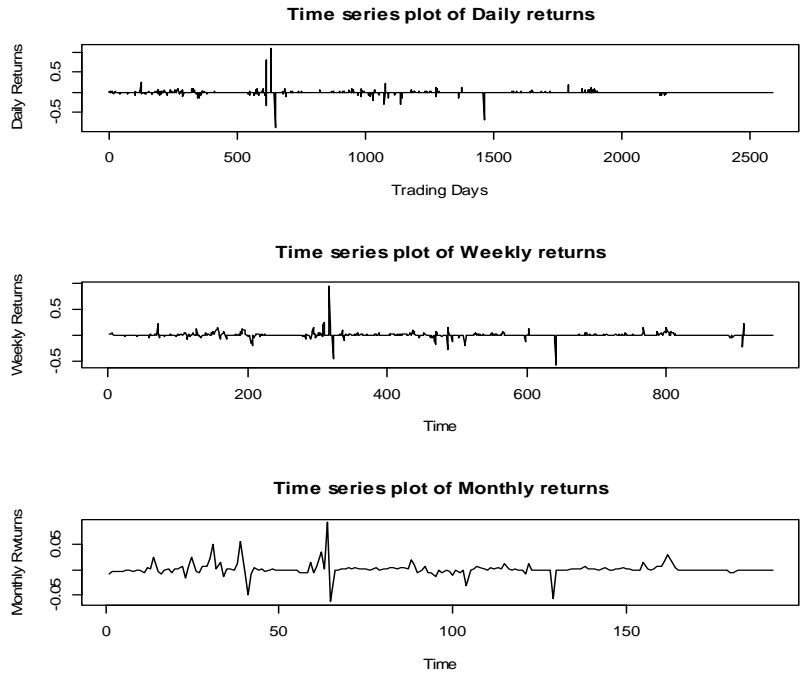


Fig. 1: Time series plot of daily, weekly and monthly returns

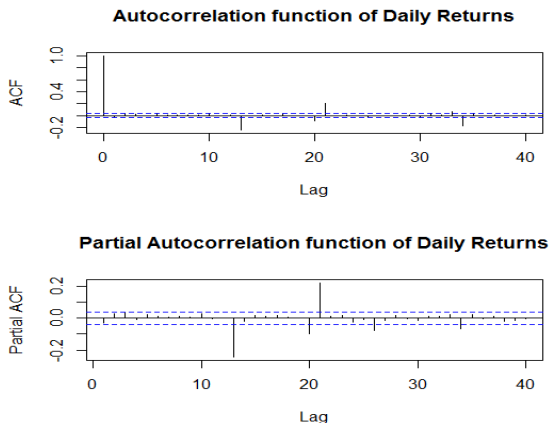


Fig. 2: ACF and PACF of daily return series

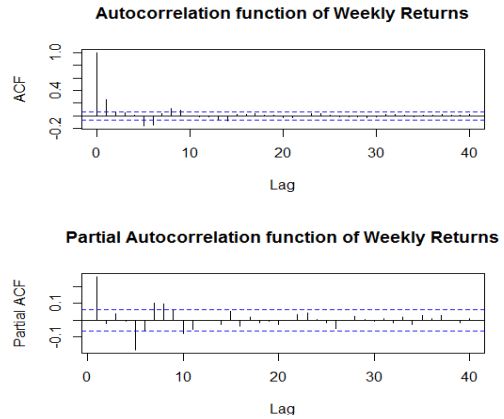


Fig. 3: ACF and PACF of weekly return series

Figure 1 shows the time series plots of the daily, weekly and monthly returns. It can be observed that the points, in all cases, hover around a fixed point and this show stationarity of the returns.

Also, observing the ACF and PACF of the daily return series in Fig. 2, there is a rapid decay of the spikes and most of them are within the acceptance region indicating stationarity. Similarly, the ACF and PACFs of the weekly and monthly series in Fig. 3 and 4 respectively behave the same way indicating stationarity.

The results of the stationarity test in Table 2 goes to confirm the stationarity of the daily, weekly and monthly series as the p-values generated for the ADF and PP are less than the level of significance

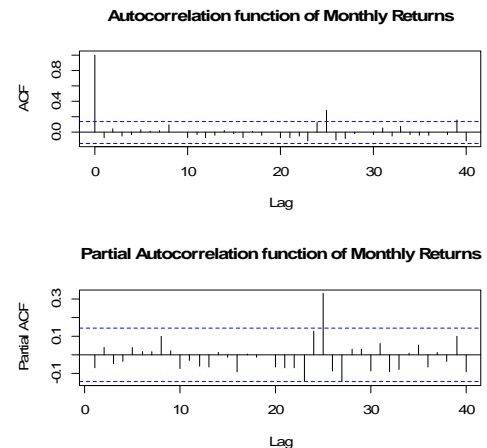


Fig. 4: ACF and PACF of monthly return series

Table 2: Stationarity test of daily, weekly and monthly returns in level form

Returns	ADF		PP		KPSS	
	Test statistic	p-value	Test statistic	p-value	Test statistic	p-value
Daily	-16.475	0.01	-2831.382	0.01	0.069	0.10
Weekly	-9.270	0.01	-659.443	0.01	0.055	0.10
Monthly	-5.520	0.01	-202.639	0.01	0.068	0.10

Table 3: Results of runs test using the mean as test value

Returns	Test value (mean)	Total cases	Cases<test value	Cases> = test value	Number of runs	Z-value	p-value
Daily	0.000827	2589	2328	261	355	-12.51	0.000
Weekly	0.002155	952	717	235	172	-15.96	0.000
Monthly	0.001058	192	131	61	47	-6.22	0.000

Table 4: Results of Wright’s non-parametric variance ratio test

	K = 2			K = 10			K = 15			K = 30		
	R ₁	R ₂	S ₁	R ₁	R ₂	S ₁	R ₁	R ₂	S ₁	R ₁	R ₂	S ₁
Daily	11.18	10.89	36.93	24.70	24.02	96.82	25.94	24.88	118.54	26.73	24.51	166.07
Weekly	17.59	16.96	19.84	25.38	22.66	38.38	26.27	22.91	43.88	27.22	22.80	52.82
Monthly	4.82	3.75	6.50	7.34	5.41	10.13	6.30	4.55	8.70	1.64	0.90	2.17

Table 5: Critical values of Wright’s non-parametric variance ratio test

Returns	K = 2			K = 10			K = 15			K = 30		
	R ₁	R ₂	S ₁	R ₁	R ₂	S ₁	R ₁	R ₂	S ₁	R ₁	R ₂	S ₁
Daily	-1.69	-1.70	-1.71	-1.76	-1.73	-1.57	-1.64	-1.69	-1.60	-1.71	-1.70	-1.64
Weekly	-1.63	-1.69	-1.69	-1.72	-1.73	-1.66	-1.73	-1.71	-1.68	-1.70	-1.72	-1.70
Monthly	-1.71	-1.84	-1.73	-1.71	-1.74	-1.52	-1.72	-1.73	-1.52	-1.60	-1.63	-1.51

of 5%. This is confirmed by the KPSS tests results in the same table.

Table 3 shows the results of Runs test. The return series were all found to be dependent using the mean as test value, as the p-values were all less than the 5% level of significance indicating that the return series do not follow the random walk theory. The negative z-values indicate that the actual number of runs fall short of the expected number of runs implying that there is positive serial correlation between the returns.

Finally, Table 4 shows the results of Wright’s Variance Ratio test. The test statistic for R₁, R₂ and S₁ for all the return series, are greater than their critical values, shown in Table 5, hence we rejected the null hypothesis of random walk for the holding periods of 2, 10, 15 and 30, respectively at 5% level of significance.

CONCLUSION

In this study, the Markov property in the stock returns of GSE was investigated. The stock returns are found to be non-normal in distribution and stationary at level and linearly dependent. Stationarity means that the series have a finite mean and variance and therefore have most of their values fluctuating around a constant long-run mean. Hence, the returns exhibit short term memory and have a mean reverting property. In other words, stock returns will return to their original equilibrium after a structural change in financial

markets. In short, the returns of the GSE do not follow random walk, and hence the Markov property. Since the random walk is consistent with an efficient stock market and the GSE violates it and not normally distributed, the GSE can be classified as an inefficient market.

An economic implication of this is that investors can predict the stock returns using historical data, however, it should be noted that stock returns are affected by economic conditions. This is represented by different economic variables.

It is recommended that the regulators of the GSE should ensure that all players of the market comply with the policies and regulations to ensure effectiveness and efficiency as a stock market is an important institution in the economy of a country.

REFERENCES

Adjasi, C., S.K. Harvey and D. Agyapong, 2008. Effect of exchange rate volatility on the Ghana stock exchange. *Afr. J. Account. Econ. Financ. Bank. Res.*, 3(3): 28-47.

Aktham, M., 2004. Oil price shocks and emerging stock markets: A generalized VAR approach. *Int. J. Appl. Econometrics Quant. Stud.*, 1(2): 27-40.

Darrat, A.F., K. Elkhail and R.H. Sam, 2000. On the integration of emerging stock markets in the Middle East. *J. Econ. Dev.*, 25(2): 119-129.

- Francis, T. and D.D. Tewari, 2011. Stock returns and inflation: An Auto Regressive Distributed Lag (ARDL) econometric investigation on Ghana. *Afr. J. Bus. Manage.*, 5(26): 10550-10556.
- Gupta, R. and P.K. Basu, 2007. Weak form efficiency in Indian stock markets. *Int. Bus. Econ. Res. J.*, 6(3): 57-64.
- Malik, S., S. Hussain and S. Ahmed, 2009. Impact of political event on trading volume and stock returns: The case of KSE. *Int. Rev. Bus. Res. Papers*, 5(4): 354-364.
- Olweny, T. and K. Omondi, 2011. The effects of macro-economic factors on stock return volatility in the Nairobi stock exchange, Kenya. *Econ. Financ. Rev.*, 1(10): 34-48.
- Ozkaya, A. and R. Ozkaya, 2012. Uncovering the chaotic behaviour of ISE 100 stock market index. *Afr. J. Bus. Manage.*, 6(7): 2727-2737.
- Wright, J.H., 2000. Alternative variance-ratio tests using ranks and signs. *J. Bus. Econ. Stat.*, 18: 1-9.