Current Research Journal of Economic Theory 6(1): 7-10, 2014
DOI:10.19026/crjet.6.5530
ISSN: 2042-4841, e-ISSN: 2042-485X

Submitted: November 04, 2013 Accepted: November 13, 2013 Published: March 20, 2014

**Research Article**

# Using Grey Theory to Predict Shanghai Unit GDP Energy Consumption

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**Abstract:** Energy consumption per unit of GDP, reflecting the environmental costs of economic development, pointed out the direction for urban construction, environmental protection is our common responsibility. In real life, the impact of energy consumption per unit of GDP is complex and uncertain factors, according to a series of known and unknown information, we can predict the energy consumption per unit of GDP as a grey system, so you can use the grey system theory. Grey model requires only a limited amount of data to estimate the unknown system behavior. In this study, first of all, by using the known data we established GM (1, 1) model, Verhulst model and the DGM (2, 1) model predictive analysis. The results show that GM (1, 1) model's prediction accuracy is higher than the prediction accuracy of Verhulst model and DGM (2, 1) model. Then, Shanghai next unit GDP energy consumption is predicted by GM (1, 1) model.

**Keywords:** GM (1, 1) model, grey model, unit GDP energy consumption

## INTRODUCTION

Community in the development process will certainly be different levels of energy consumption and Shanghai as the center of China's economic development, economic development has been in a leading position, while Shanghai is also practical action to efforts to achieve energy reduction targets. According to the Shanghai Bureau of Statistics data, we can easily develop Shanghai unit GDP energy consumption showed a clear downward trend. Although the unit GDP energy consumption of many factors, but there are many uncertain factors effect, but according to the relevant statistical indicators, or to the relevant statistical indicators derived on the basis of scientific prediction. Accurate predictions Shanghai unit GDP energy shouting, not only for the Government to formulate appropriate policies, but also for the healthy development of social economy are very important.

Grey system theory is composed by Deng (1982) and has already had a considerable development about three decades. Grey system was "part of the information clearly, some information is known, others are unknown. And the information is the small sample size, poor information, uncertainty problem and based on the information covered by the sequence operator role in the movement of things and explores the reality of the law. It has been widely used in several areas, such as agricultural, industrial and environmental system.

In recent years, the grey system theory has been widely used to forecast in various fields and demonstrated satisfactory results. For instance, Qiao *et al*. (2006) had used Grey system theory to investigate the interactive, intimidating relation between Urbanization and the environment in an arid area. Ma and Wang (2007) had used grey system theory to predict the pitting corrosion behavior for stainless SUS 630. He *et al*. (2009) had used grey system theory to take a new approach to performance analysis of ejector refrigeration system. Huang *et al*. (2011) had used grey system theory and multivariate linear regression to study the application and research of a new combinatorial analysis and forecasting method in real estate area. Guo *et al*. (2011) had used grey system theory to study a new approach to energy consumption prediction of domestic heat pump water heater. Li *et al*. (2012) had used grey system theory to study a new approach to performance analysis of disinfection. Wang *et al*. (2012) had used grey system theory to analyze an urban rail transit hazard evaluation methodology. Zheng *et al*. (2013) had used grey systems theory to predict the seawater quality in rigs-to-reefs area. Zhang and Zhang (2013) had used grey system theory to analyze the aerobic granular sludge formation.

Because of unit GDP energy consumption factors are complex and uncertain, therefore, to forecast the unit GDP energy consumption, there are a lot of known information, there are also a lot of unknown information, which can be regarded as a grey system, so it can be analyzed by using grey system theory.

In this study, we first introduce the GM (1, 1) model, Verhulst model and the DGM (2, 1) model of the basic concepts and then compare the prediction

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accuracy with three models. Finally, the GM (1, 1) model is used to predict Shanghai unit GDP energy consumption in the future.

## METHODOLOGY

**Grey models:** Generally, it consists of three parts in the GM (1, 1) modeling: preprocessing data by AGO (Accumulation Generating Operation), which is to reduce the randomness inherent in data; data estimation with first-order difference equation, is solved where two parameters, developing coefficient and grey input; post-processing by IAGO (Inverse Accumulated Generation Operation) to explore the final estimate.

In order to establish GM (1, 1) model, a raw data series is needed. Let:

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\} \quad (1)$$

The raw data can be selected from the experiments and/or the statistics. These data are fluctuating in a definite range. In order to find out the suitable pattern of the data series, The AGO of $X^{(0)}$ is defined to be:

$$X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n)\} \quad (2)$$

and

$$x^{(1)}(k) = \sum_{m=1}^{k} x^{(0)}(m) \quad (k = 1, 2, \cdots, n) \quad (3)$$

The GM (1, 1) model can be constructed by establishing a first order differential equation for $x^{(1)}(k)$ to be:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b \quad (4)$$

where,
$a$ = Developing coefficient
$b$ = Grey input

Therefore, the solution of the differential Eq. (4) can be obtained by using the least square method:

$$\hat{x}^{(1)}(k+1) = [x^{(0)}(1) - \frac{\hat{b}}{\hat{a}}]e^{-\hat{a}k} + \frac{\hat{b}}{\hat{a}} \quad (k = 2, 3, \cdots, n) \quad (5)$$

where,

$$\hat{P} = (\hat{a}, \hat{b})^T = (B^T B)^{-1} B^T Y$$

and

$$B = \begin{bmatrix} -\frac{1}{2}(x^{(1)}(1) + x^{(1)}(2)) & 1 \\ -\frac{1}{2}(x^{(1)}(2) + x^{(1)}(3)) & 1 \\ \vdots & \vdots \\ -\frac{1}{2}(x^{(1)}(n-1) + x^{(1)}(n)) & 1 \end{bmatrix} \quad (6)$$

$$Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \quad (7)$$

By applying the Inverse Accumulated Generation Operation (IAGO). That is:

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) = (1 - e^{\hat{a}})[x^{(0)}(1) - \frac{\hat{b}}{\hat{a}}]e^{-\hat{a}k} \quad (8)$$

and

$$\hat{x}^{(1)}(1) = \hat{x}^{(0)}(1) \quad (k = 2, 3, \Lambda, n).$$

Grey Verhulst model is also a time series forecasting model and we can construct the Verhulst model just as the above GM (1, 1) by establishing a first order differential equation for $X^{(1)}$ (k) as:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b(x^{(1)})^2 \quad (9)$$

where, $a$ is development coefficient, $b$ is grey action quantity. The solution of the parameter vector $\hat{p} = (\hat{a}, \hat{b})^T$ can be obtained by utilizing the least square method:

$$\hat{P} = (\hat{a}, \hat{b})^T = [(A\text{M}B)^T (A\text{M}B)]^{-1} (A\text{M}B)^T Y$$

and

$$A = \begin{bmatrix} -\frac{1}{2}(x^{(0)}(1) + x^{(0)}(2)) \\ -\frac{1}{2}(x^{(0)}(2) + x^{(0)}(3)) \\ \vdots \\ -\frac{1}{2}(x^{(0)}(n-1) + x^{(0)}(n)) \end{bmatrix} \quad (10)$$

$$B = \begin{bmatrix} [\frac{1}{2}(x^{(0)}(1) + x^{(0)}(2))]^2 \\ [\frac{1}{2}(x^{(0)}(2) + x^{(0)}(3))]^2 \\ \vdots \\ [\frac{1}{2}(x^{(0)}(n-1) + x^{(0)}(n))]^2 \end{bmatrix} \quad (11)$$

$$Y = \begin{bmatrix} (x^{(0)}(2) - x^{(0)}(1)) \\ (x^{(0)}(3) - x^{(0)}(2)) \\ \vdots \\ (x^{(0)}(n) - x^{(0)}(n-1)) \end{bmatrix} \quad (12)$$

The resolution of (9) is:

$$\hat{x}^{(0)}(k+1) = \frac{\hat{a}\, x^{(0)}(1)}{\hat{b}\, x^{(0)}(1) + [\hat{a} - \hat{b}\, x^{(0)}(1)] e^{\hat{a}k}} \quad (k = 0,1,\ldots,n) \tag{13}$$

The DGM (2, 1) model is a single sequence second-order linear dynamic model and is fitted by differential equations.

The new sequence $X^{(1)}$ is used to construct the albino equation of DGM (2, 1) model by setting up a second-order differential equation, directly. That is:

$$\frac{d^2 x^{(1)}}{dt^2} + a\frac{dx^{(1)}}{dt} = b \tag{14}$$

where,

$$\hat{P} = (\hat{a}, \hat{b})^T = (B^T B)^{-1} B^T Y$$

and

$$B = \begin{bmatrix} -x^{(0)}(2) & 1 \\ -x^{(0)}(3) & 1 \\ \vdots & \vdots \\ -x^{(0)}(n) & 1 \end{bmatrix} \tag{15}$$

$$Y = \begin{bmatrix} (x^{(0)}(2) - x^{(0)}(1) \\ (x^{(0)}(3) - x^{(0)}(2) \\ \vdots \\ (x^{(0)}(n) - x^{(0)}(n-1) \end{bmatrix} \tag{16}$$

according to (14), we have:

$$\hat{x}^{(1)}(k+1) = \left(\frac{\hat{b}}{\hat{a}^2} - \frac{x^{(0)}(1)}{\hat{a}}\right)e^{-\hat{a}k} + \frac{\hat{b}}{\hat{a}}(k+1) + \left(x^{(0)}(1) - \frac{\hat{b}}{\hat{a}}\right)\frac{1+\hat{a}}{\hat{a}} \tag{17}$$

The prediction values of original sequence can be obtained by applying inverse AGO to $X^{(1)}$, Namely:

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) = \hat{x}^{(1)}(k) \quad (k = 0,1,\ldots,n)$$

## CASE STUDY

In this section, the GM (1, 1), the Verhulst model and the DGM (2, 1) are used for comparison. The unit GDP energy consumption data in Shanghai from 2003 to 2011 is adopted to demonstrate the effectiveness and practicability of these models. The unit GDP energy consumption data in 2003-2008 is employed to set up the three grey prediction models and the unit GDP energy consumption data from 2009 to 2011 is used as data set to compare the three models accuracy. The evaluation criterion is the MRPE (the Mean Relative Percentage Error), which measures the percent of prediction accuracy:

$$MRPE = \frac{1}{n}\sum_{k=1}^{n}\left[\frac{|x^{(0)}(k) - \hat{x}^{(0)}(k)|}{x^{(0)}(k)}\right]$$

The real and predictive values are shown in Table 1 to compare the three model accuracy and relative error. MRPE are shown in Table 2.

Table 1 demonstrates that GM (1, 1) prediction model is more accurate than the others by comparing the relative error.

From Table 2, it can be seen that the MRPE of the GM (1, 1) model, the Verhulst model and the DGM (2, 1) from 2009 to 2011 are 6.23, 11.36 and 7.81%, respectively. The effectiveness and accuracy of GM (1, 1) model is higher than the Verhulst model and the DGM (2, 1) model.

Table 1: Model values and prediction error of unit GDP energy consumption in Shanghai

Shanghai unit GDP energy consumption (2003-2011)

| | | | GM (1, 1) | | Verhulst | | DGM (2, 1) | |
|---|---|---|---|---|---|---|---|---|
| | Year | Real value (tons of standard coal/ten thousand Yuan) | Model value | Error R (%) | Model value | Error R (%) | Model value | Error R (%) |
| Model Building stage | 2003 | 1.015 | 0.982 | 3.251 | 1.126 | -10.936 | 1.103 | -8.670 |
| | 2004 | 0.917 | 0.857 | 6.543 | 0.847 | 7.634 | 0.896 | 2.290 |
| | 2005 | 0.889 | 0.906 | -1.912 | 0.713 | 19.798 | 0.801 | 9.899 |
| | 2006 | 0.851 | 0.943 | -10.811 | 1.035 | -21.622 | 0.925 | -8.696 |
| | 2007 | 0.805 | 0.761 | 5.466 | 0.732 | 9.068 | 0.721 | 10.435 |
| | 2008 | 0.775 | 0.742 | 4.258 | 0.852 | -9.935 | 0.637 | 17.806 |
| Ex-post building stage | 2009 | 0.727 | 0.704 | 3.164 | 0.681 | 6.327 | 0.702 | 3.439 |
| | 2010 | 0.712 | 0.725 | -1.826 | 0.653 | 8.287 | 0.728 | -2.247 |
| | 2011 | 0.618 | 0.673 | -8.900 | 0.706 | -14.239 | 0.694 | -12.298 |

Table 2: Error analytical results

| Stage | GM (1, 1) MRPE (%) | Verhulst MRPE (%) | DGM (2, 1) MRPE (%) |
|---|---|---|---|
| 2003-2008 | 5.31 | 12.74 | 6.37 |
| 2009-2011 | 6.23 | 11.36 | 7.81 |

Table 3: Forecasting values

| Year | 2009 | 2010 | 2011 | 2012 | 2013 |
|---|---|---|---|---|---|
| Real values | 0.727 | 0.712 | 0.618 | | |
| Model values | 0.704 | 0.725 | 0.673 | 0.652 | 0.647 |

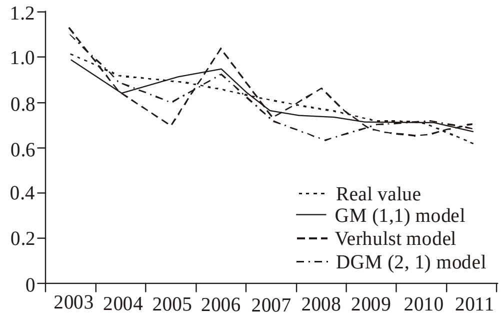


Fig. 1: Real values and models values

Figure 1 shows that the GM (1, 1) model and the DGM (2, 1) model have the better forecasting precision in 2009-2011, but the GM (1, 1) prediction model seems to obtain the lowest post-forecasting errors and it is more suitable to make a short-term prediction, so the GM (1, 1) model uses to predict Shanghai unit GDP energy consumption for 2012 and 2013 (Table 3).

## CONCLUSION

In this study, we compare the accuracy of the grey forecasting models to predict Shanghai unit GDP energy consumption. The grey model requires as few as four data for estimation, such as fitting and forecasting. And through AGO method, the irregular data of system can become regular sequences that it can identify the uncertainties of system and predict the parameters of it. With its simplicity and effectiveness, this study uses it.

The result shows that the accuracy of GM (1, 1) model in forecast value for 2009 to 2011 is higher than the Verhulst grey model and the DGM (2, 1) model. With its simplicity and effectiveness, we use the GM (1, 1) model to predict Shanghai unit GDP energy consumption for 2012 and 2013. The results show that Shanghai unit GDP energy consumption is expected to decrease and it will be tons of 0.647 tons of standard coal/ten thousand Yuan by 2013.

## ACKNOWLEDGMENT

This study is supported by the National Natural Science Foundation of China (11101265, 41171101, 61075115), National Social Science Fund (12BGL088), Shanghai Education Research and Innovation Key Project (14ZZ157).

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