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Research Article

Modelling Rates of Inflation in Ghana: An Application of Arch Models

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Abstract: This study sought to model rates of inflation in Ghana using the Autoregressive Conditional Heteroscedastic models. In particular, the ARCH, GARCH and EGARCH models were compared. Monthly rates of inflation from January 2000 to December 2013 were used in the study with the rates from January 2000 to December 2012 serving as the training set and January 2013-December 2013 serving as the validation set. The result revealed that the EGARCH (1, 2) model with a mean equation of ARIMA $(3, 1, 2) \times (0, 0, 0)_{12}$ was appropriate for modelling Ghana's monthly rates of inflation. A one year out-of-sample forecast for the year 2014 shows that Ghana would experience double digit inflation with an end of year inflation rate of 15.0% and a margin of error of 0.9%. This study would inform and guide policy-makers as well as investors and businessmen on management of expected future rates of inflation.

Keywords: ARCH models, ARIMA models, Ghana, inflation, performance

INTRODUCTION

Price stability is one of the main objectives of every government as it is an important economic indicator that governments, politicians, economists and other stakeholders use as basis of argument when debating on the state of the economy (Suleman and Sarpong, 2012). In recent years, rising inflation has become one of the major economic challenges facing most countries in the world especially developing countries such as Ghana. David (2001) described inflation as a major focus of economic policy worldwide. This is rightly so as inflation is the frequently used economic indicator of the performance of a country's economy due to the fact that it has a direct effect on the state of the economy. In Ghana, the debate on achieving a single digit inflation value has been the major concern for both the government and the opposition parties. While the government boasts of a stable economy with consistent single digit inflation, the opposition parties doubt these figures and believe that the figures had been cooked up and do not reflect the true situation in the economy. Despite the different opinions on the inflation figures, it is important to point out that, both the government and the opposition parties are concerned about the inflation (general level of prices) in the country as it affects all sectors of the economy. Webster (2000) defined inflation as the persistent increase in the level of consumer prices or a persistent decline in the purchasing power of money. Hall (1982) also expresses inflation as a situation where the demand for goods and services exceeds their supply in the economy.

Traditional time series models assume a constant conditional variance. However, to a large extent most economic and financial series often exhibit non constant conditional variance (Heteroscedastic) and hence traditional time series do not perform well when used to forecast such series. The heteroscedasticity affects the accuracy of forecast confidence limits and thus has to be handled by constructing appropriate non-constant variance models (Amos, 2010). Several models such as the Autoregressive Conditionally Heteroscedastic (ARCH) model and its variants like the Generalized Autoregressive Conditionally Heteroscedastic (GARCH) and Exponential Generalized Autoregressive Conditionally Heteroscedastic (EGARCH) models have therefore been developed to model the non constant volatility of such series. The ARCH model was introduced by Engle (1982) and later it was modified by Bollerslev (1986) to a more generalized form known as the GARCH. The GARCH model has been used most widely for the specification of the ARCH. The GARCH model imposed restrictions on the parameters to assure positive variances. Nelson (1991) therefore presented an alternative to the GARCH model by modifying the GARCH to Exponential GARCH (EGARCH) model. Unlike the GARCH, the EGARCH does not need the inequality restrictions on the parameters to assume a positive variance.

Empirical researches have been carried out in the area of inflation modelling and forecasting in Ghana. Examples include Alnaa and Ahiakpor (2011) and Suleman and Sarpong (2012), etc. All these researchers

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attempted to model inflation in Ghana using models that did not capture the conditional heteroscedasticity of the time series inflation data. However, it has been argued by Campbell *et al*. (1997) that it is both statistically inefficient and logically inconsistent to use and model volatility measures that are based on the assumption of constant variance over some period when the resulting series progress over time.

Abledu and Agbodah (2012) found ARIMA (1, 1, 0) to be the best model in an attempt to analyze and forecast the macroeconomic impact of oil price fluctuations in Ghana using annual data from 2000-2011. Suleman and Sarpong (2012) concluded that the *ARIMA* (3, 1, 3) × $(2, 1, 1)_{12}$ best represent the behaviour of inflation rates. Alnaa and Ahiakpor (2011) found *ARIMA* (6, 1, 6) to be the best fitted model for forecasting inflation in Ghana.

The volatility in the consumer prices of some selected commodities in the Nigerian market were also examined by Awogbemi and Oluwaseyi (2011) and the results showed that ARCH and GARCH models are better models because they give lower values of AIC and BIC as compared to the conventional Box and Jenkins ARIMA models.

Existing literature have modelled the rates of inflation for Ghana using models that assume constant variance over time such as the ARIMA of the Box-Jenkins. Not much work had been done on modelling the rates of inflation in Ghana using models that assume non-constant variance over time. This indicates a gap in literature and as such the novelty of this paper. The paper provides empirical evidence on modelling rates of inflation in Ghana using the Autoregressive Conditional Heteroscedastic models.

MATERIALS AND METHODS

The study used sample data spanning from January 2000 to December 2013 with the rates from January 2000 to December 2012 serving as the training set and January-December of 2013 serving as the validation set. The data were obtained from the official website of the Ghana Statistical Service (GSS) and modelling was done with the aid of the EVIEWS 5.0 and PASW/SPSS 20.0 statistical software. The models used in this study are briefly described as follows.

Autoregressive Conditional Heteroscedastic (ARCH) model: An ARCH process is a mechanism that includes past variance in the explanation of future variances (Engle, 2004). The ARCH model was developed by Engle (1982) which provides a systematic framework for volatility modelling. ARCH models specifically take the dependence of the conditional second moments in consideration when modelling. Let $\{x_t\}$ be the mean-corrected return, ε_t be the Gaussian white noise with zero mean and unit variance and I_t be the information set at time t given by $I_t = \{x_1, x_2, \ldots, x_{t-1}\}$. Then the ARCH ($m$) model is specified as:

$$x_t = \sigma_t \varepsilon_t$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 x_{t-1}^2 + \cdots + \alpha_m x_{t-m}^2$$

where, $\alpha_0>0$ and $\alpha_i \geq 0$, $i = 1, \ldots,$ m and:

$$\mathrm{E}\left(x_t \mid I_t\right) = \mathrm{E}\left[\mathrm{E}\left(x_t \mid I_t\right)\right] = \mathrm{E}\left[\sigma_t \mathrm{E}\left(\varepsilon_t\right)\right] = 0$$
$$\mathrm{V}\left(x_t \mid I_t\right) = \mathrm{E}\left(x_t^2\right) = \sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i x_{t-i}^2$$

and the error term ε_t is such that E (ε_t |I_t) = 0 and V (ε_t |I_t) = 1.

Generalized Autoregressive Conditional Heteroscedastic (GARCH) model: The ARCH formulation can lead to complexity if the order of the model is higher. This necessitated the introduction of the GARCH model as an extension of the ARCH models by Bollerslev (1986). Let $x_t = r_t - u_t$ be the mean corrected return, where r_t is the return of an asset, u_t is the conditional mean of x_t. Then x_t follows a GARCH (m, s) model if:

$$x_t = \sigma_t \varepsilon_t$$
$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i x_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2$$

where, $\{\varepsilon_t\}$ is a sequence of independent, identically distributed random variable with mean zero and unit variance and the parameters of the model are α_i, $i = 0, \ldots, m$ and β_j, $j = 1, \ldots, s$ such that $\alpha_i \geq 0$ and $\beta_j \geq 0$; $\sum_{i=1}^{v}(\alpha_i + \beta_i) < 1$, where $v = \max(m, s)$ and $\alpha_i = 0$ for $i>m$ and $\beta_j = 0$ for $j>s$. The constraints on $\alpha_i + \beta_i$ implies that the unconditional variance of x_t is finite, whereas its conditional variance σ^2_t evolves over time.

Exponential Generalized Autoregressive Conditional Heteroscedastic (EGARCH) model: Despite the added advantage that the GARCH model brought to the ARCH-type models, the GARCH model also had the weakness of an inability to capture the asymmetry effect that is inherent in most real life financial data (Frimpong and Oteng-Abayie, 2006). To circumvent this problem of asymmetric effects on the conditional variance, Nelson (1991) extended the ARCH framework by proposing the Exponential GARCH (EGARCH) model. The EGARCH (m, s) model can be stated alternatively as:

$$x_t = \sigma_t \varepsilon_t$$

$$ln\left(\sigma_t^2\right) = \alpha_0 + \frac{\sum_{i=1}^{s} \alpha_i \left|x_{t-i}\right| + \gamma_i x_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{m} \beta_j ln\left(\sigma_{t-j}^2\right)$$

A positive x_{t-i} contributes α_i $(1+\gamma_i)$ $|\varepsilon_{t-i}|$ to the log volatility, whereas a negative x_{t-i} contributes α_i $(1 - \gamma_i)$ $|\varepsilon_{t-i}|$, where $\varepsilon_{t-i} = \frac{x_{t-i}}{\sigma_{t-i}}$. The parameter γ signifies the leverage effect and is expected to be negative. The use of the ln (σ^2_t) enables the model to respond asymmetrically to positive and negative lagged values of x_t.

RESULTS AND DISCUSSION

Figure 1 shows the time series plot of the monthly inflation data showing a general overview of the series. From Fig. 1, it is can be seen that the data was not stationary as shown by a slow decay in the ACF of the series and a very significant spike at lag 1 of the PACF with marginal spikes at other lags as shown by Fig. 2. Furthermore, Fig. 2 shows significant spikes at lag 12 of the PACF which indicates that there is the presence of seasonal variation in the data set.

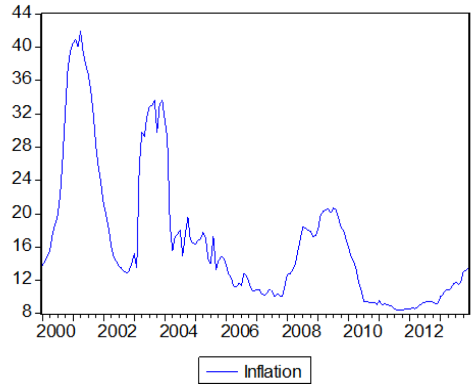


Fig. 1: Time series plot of inflation from 2000 to 2013

The Augmented Dickey-Fuller (ADF) and Philips-Perron (PP) tests were performed and the results shown by Table 1 indicated that the series were not stationary over the period. Hence the ordinary differencing transformation was carried out and the ADF and PP

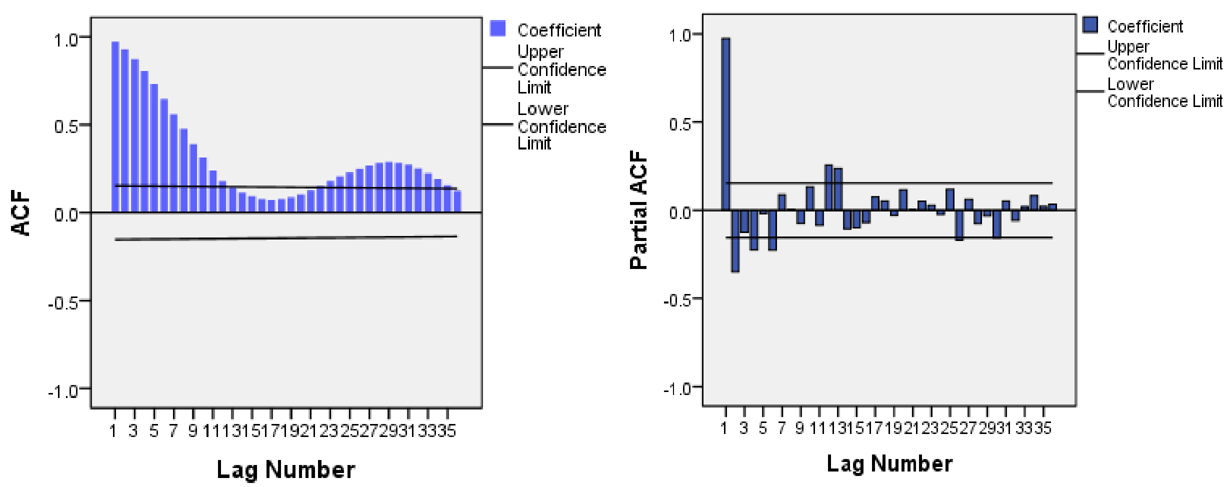


Fig. 2: ACF and PACF plots for the monthly rates of inflation

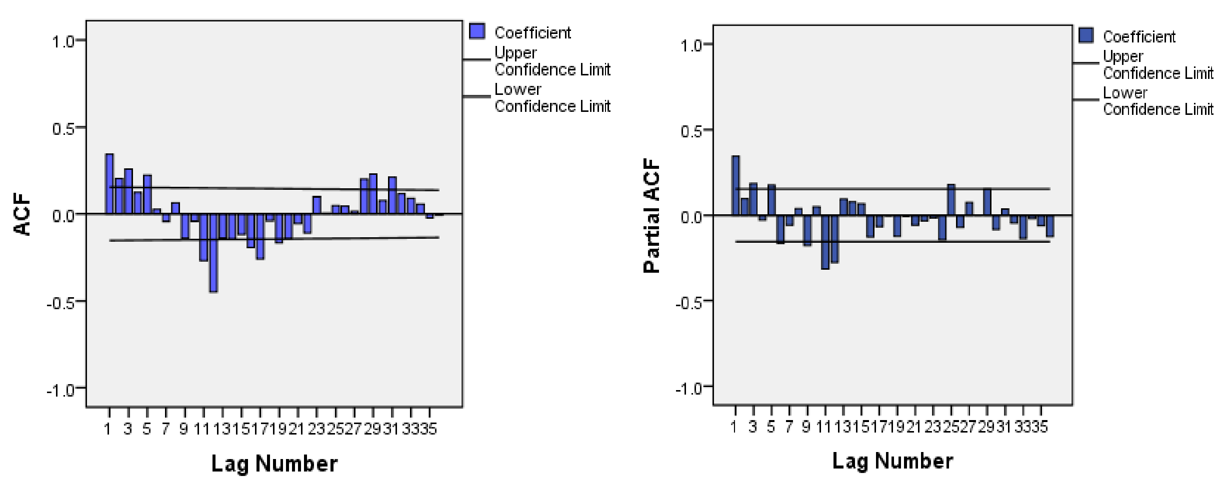


Fig. 3: ACF and PACF plots for the first differenced monthly rates of inflation

Table 1: Unit root test of the inflation data in its level form

Test	Statistic	p-value
Augmented dickey-fuller	-2.1096	0.2413
Phillips-perron	-2.3958	0.1445

Table 2: Unit root test for the differenced transformed inflation data

Test	Statistic	p-value
Augmented dickey-fuller	-7.2881	0.0000
Phillips-perron	-9.3143	0.0000

Table 3: Different ARIMA (p, 1, q) × $(P, 0, Q)_{12}$ models fitted

Model	AIC	BIC	Log-likelihood
ARIMA (1, 1, 1) × $(0, 0, 0)_{12}$	3.47	3.58*	-241.56
ARIMA (1, 1, 2) × $(0, 0, 0)_{12}$	3.48	3.61	-241.29
ARIMA (1, 1, 3) × $(0, 0, 0)_{12}$	3.49	3.64	-240.95
ARIMA (2, 1, 1) × $(0, 0, 0)_{12}$	3.49	3.61	-239.92
ARIMA (2, 1, 2) × $(0, 0, 0)_{12}$	3.50	3.64	-239.61
ARIMA (2, 1, 3) × $(0, 0, 0)_{12}$	3.49	3.66	-238.11
ARIMA (3, 1, 1) × $(0, 0, 0)_{12}$	3.50	3.65	-238.12
ARIMA (3, 1, 2) × $(0, 0, 0)_{12}$	3.41*	3.58*	-230.93*
ARIMA (3, 1, 3) × $(0, 0, 0)_{12}$	3.45	3.63	-232.16

*: Best based on the selection criterion

Table 4: Estimates of ARIMA (3, 1, 2) × $(0, 0, 0)_{12}$ model

Variable	Coefficient	S.E.	t-statistics	p-value
C	-0.041	0.044	-0.934	0.352
AR (1)	-1.317	0.078	-16.883	0.000
AR (2)	-0.608	0.125	-4.859	0.000
AR (3)	0.227	0.079	2.885	0.005
SAR (12)	0.187	0.080	2.323	0.022
MA (1)	1.587	0.030	52.937	0.000
MA (2)	0.995	0.018	54.969	0.000
SAR (12)	-0.961	0.015	-62.438	0.000

Table 5: Diagnostics test statistics

Test	Statistics	p-value
Breusch-Godfrey LM	0.752	0.832
ARCH LM	47.688	0.000

tests performed on the transformed data indicated stationarity as shown in Table 2. The rapid decay in the ACF and the PACF of the transformed data as shown in Fig. 3 confirms the stationarity of the data after the ordinary differencing. Figure 3 shows significant spikes at lags 12 of the ACF and also at lags 12 of the PACF which suggest that as seasonal moving average and seasonal autoregressive components need to be added respectively to the model.

Due to the differencing of the data and the presence of seasonality, several ARIMA $(p, d, q) \times (P, D, Q)_{12}$ models were fitted to the transformed data and the best model selected based on the maximum value of log-likelihood and minimum values of Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC). From Table 3, the ARIMA (3, 1, 2) × $(0, 0, 0)_{12}$ was the best model based on the selection criteria used. This model was then estimated with all the parameters being significant as shown in Table 4. The model diagnosis as given by Fig. 4 shows that the standardized residuals exhibit random variation about their mean. From Table 5, the Breusch-Godfrey LM test of the residuals

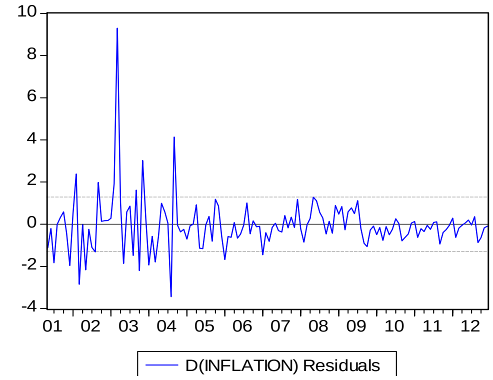


Fig. 4: Time plot of the standardized residuals from the ARIMA (3, 1, 2) × $(0, 0, 0)_{12}$

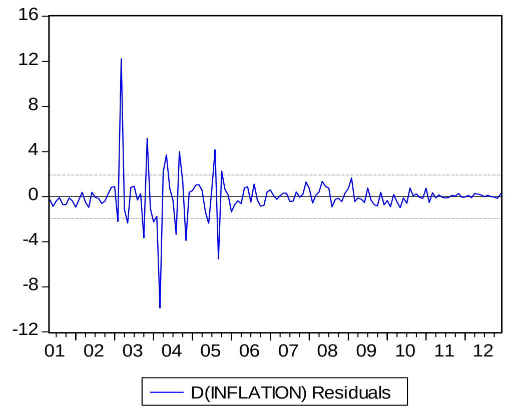


Fig. 5: Time plot of the standardized residuals from the EGARCH (1, 2)

indicates that there was no autocorrelation whilst the ARCH LM test indicates the presence of ARCH effects. This suggests that an ARCH model would provide more reliable results.

Hence, several ARCH models with a mean equation of ARIMA (3, 1, 2) × $(0, 0, 0)_{12}$ were estimated and the best model selected based on the maximum value of log-likelihood and minimum values of Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC). From Table 6, the EGARCH (1, 2) model was the best model based on the selection criteria used. The model was then estimated with all the parameters except AR (2) significant as shown by Table 7. The model diagnosis as given by Fig. 5 shows that the standardized residuals exhibit random variation about their mean. From Table 8, the ARCH LM test indicates the absence of ARCH effects. These results imply that the EGARCH (1, 2) model is the most appropriate for the monthly rates of inflation in Ghana. The forecast performance of the EGARCH (1, 2) is shown in Fig. 6.

Finally, a one year out-of-sample monthly forecast for the year 2013 were obtained as shown in Table 9. Comparing the observed values and the forecast values,

Table 6: Different ARCH family models fitted

Model	AIC	BIC	Log-likelihood
ARCH (1)	3.11	3.32	-208.01
ARCH (2)	2.92	3.15	-193.22
ARCH (3)	2.90	3.16	-191.33
GARCH (1, 1)	2.95	3.18	-195.33
GARCH (1, 2)	2.84	3.10	-187.02
GARCH (1, 3)	2.92	3.19	-191.37
GARCH (2, 1)	2.69	2.95	-176.52
GARCH (2, 2)	3.21	3.48	-211.65
GARCH (2, 3)	3.80	4.09	-252.02
GARCH (3, 1)	3.61	3.88	-239.69
GARCH (3, 2)	2.80	3.10	-182.31
GARCH (3, 3)	3.86	4.18	-255.26
EGARCH (1, 1)	3.12	3.38	-206.06
EGARCH (1, 2)	2.49*	2.76*	-161.20*
EGARCH (1, 3)	2.51	2.81	-163.44
EGARCH (2, 1)	2.78	3.06	-181.74
EGARCH (2, 2)	2.99	3.28	-195.29
EGARCH (2, 3)	2.59	2.90	-166.05
EGARCH (3, 1)	3.04	3.33	-198.57
EGARCH (3, 2)	2.57	2.88	-164.73

*: Best based on the selection criterion

Table 7: Estimates of EGARCH (1, 2) model

Variable	Coefficient	S.E.	Z-statistic	p-value
Mean equation				
C	-0.112	0.046	-2.463	0.014
AR (1)	-0.185	0.063	-2.931	0.003
AR (2)	0.068	0.010	0.685	0.494
AR (3)	0.255	0.038	6.783	0.000
SAR (12)	-0.357	0.014	-24.997	0.000
MA (1)	0.625	0.078	8.015	0.000
MA (2)	0.265	0.075	3.535	0.000
SMA (12)	0.449	0.011	41.259	0.000
Variance equation				
α_0	-1.448	0.117	-12.379	0.000
α_1	1.663	0.157	10.587	0.000
γ	-0.207	0.041	-4.997	0.005
β_1	0.223	0.029	7.825	0.000
β_2	0.680	0.032	21.071	0.000

Table 8: Diagnostics test statistics for EGARCH (1, 2)

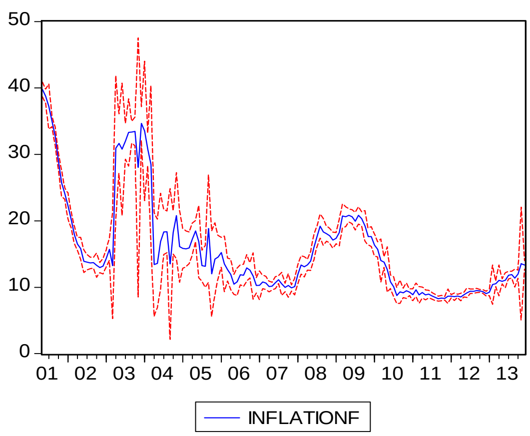
Test	Statistic	p-value
ARCH LM	0.715	0.862
Normality (Jarque-Bera)	1.398	0.497

Table 9: One year out-sample forecast for the year 2013

			95% confidence interval	
Month	Observed	Forecast	Lower	Upper
January	10.1	9.2	9.0	9.4
February	10.4	10.4	8.9	11.9
March	10.8	10.5	10.3	10.7
April	10.9	11.0	9.8	12.2
May	11.0	10.9	10.7	11.1
June	11.6	11.1	10.5	11.7
July	11.8	11.8	11.5	12.1
August	11.5	11.9	11.6	12.2
September	11.9	11.4	10.7	12.1
October	13.1	12.0	11.7	12.3
November	13.2	13.5	9.2	17.8
December	13.5	13.4	13.1	13.7

Table 10: Estimates of EGARCH (1, 2) model

Variable	Coefficient	S.E.	z-statistic	p-value
Mean equation				
C	-0.181	0.104	-1.729	0.084
AR (1)	-0.347	0.153	-2.264	0.024
AR (2)	0.123	0.059	2.086	0.037
AR (3)	0.354	0.075	4.702	0.000
SAR (12)	-0.414	0.049	-8.494	0.000
MA (1)	0.912	0.170	5.364	0.000
MA (2)	0.400	0.119	3.350	0.001
SMA (12)	0.487	0.047	10.312	0.000
Variance equation				
α_0	-1.134	0.156	-7.253	0.000
α_1	1.367	0.190	7.202	0.000
γ	-0.194	0.069	-2.816	0.005
β_1	0.154	0.041	3.745	0.000
β_2	0.701	0.056	12.607	0.000



Forecast: INFLATIONF	
Actual: INFLATION	
Forecast sample: 2000M01 2013M12	
Adjusted sample: 2001M05 2013M12	
Included observations: 152	
Root Mean Squared Error	1.764792
Mean Absolute Error	0.888968
Mean Abs. Percent Error	5.321618
Theil Inequality Coefficient	0.050841
Bias Proportion	0.000006
Variance Proportion	0.016036
Covariance Proportion	0.983958

Fig. 6: Forecasting performance of the EGARCH (1, 2)

it is seen that the EGARCH (1, 2) model was able to produce values that mimic the behavior of the observed values. The model parameters were estimated again using all the data set (January 2000 to December 2013) and a one year out-of-sample forecast for the year 2014 was obtained as shown by Table 10 and 11 respectively.

From Table 11, it is seen that although the rates of inflation would be in double digits, there would be a decrease up to the end of the first quarter of 2014. However, it would increase from the beginning of the second quarter till the end of the third quarter and then decrease marginally toward the end of the year.

Table 11: One year out-of -sample forecast for the year 2014

		95% confidence interval	
Month	Forecast	Lower	Upper
January	13.7	11.3	16.1
February	13.6	13.0	14.2
March	11.7	6.9	16.5
April	14.6	13.8	15.4
May	14.8	11.5	18.1
June	14.9	14.1	15.7
July	15.0	12.4	17.6
August	15.0	14.4	15.6
September	15.1	13.7	16.5
October	15.1	14.5	15.7
November	15.0	14.2	15.8
December	15.0	14.6	15.4

CONCLUSION

The study sought to provide empirical evidence on modelling rates of inflation in Ghana using the Autoregressive Conditional Heteroscedastic models. In particular, the ARCH, GARCH and EGARCH models were compared. Several other forms of these models were fitted using the monthly rates of inflation in Ghana and based on the AIC and BIC values the best performing model was selected. There was significant evidence of ARCH effects in the monthly rates of inflation. As a result, the EGACRH (1, 2) model with a mean equation of ARIMA $(3, 1, 2) \times (0, 0, 0)_{12}$ was appropriate for modelling Ghana's monthly rates of inflation. A one year out-of-sample forecast of the monthly rates of inflation from the model produced very close values as compared to the observed values for the year 2013 implying that the model was able to mimic the underlying stochastic behavior of the monthly rates of inflation for Ghana. Subsequently a one year out-of-sample forecast for the year 2014 shows that Ghana would experience double digit inflation with an end of year inflation rate of 15.0% with a margin of error of 0.9%. These findings would benefit a host of people including economists, monetary policy markers and international businessmen. It also has important implications for investment processes on the stock markets. The study therefore recommends to all concerned to make use of the autoregressive Heteroscedastic models when forecasting monthly rates of inflation due to the presence of significant ARCH effect in the monthly series.

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