

Research Article

Content Analysis of Stochastic Volatility Model in Discrete and Continuous Time Setting

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Abstract: This study investigated the popularity of stochastic volatility in recent literature. Stochastic volatility models are common in the financial markets and decision making process. Efficient managing scenarios to these problems will reduce risks in future valuations in many financial assets. A volatility model that is stochastic can better capture the time-varying elements mostly absent in its counterpart, a standard volatility model. In this study, a content analysis is conducted to extract information on mostly used enhancement-stochastic models available in literature. The finding indicates that stochastic volatility with long memory pioneers in SciVerse search engine, whereas stochastic volatility with jump is the highest numbers in publication, in particular the Google Scholar.

Keywords: Content analysis, stochastic volatility

INTRODUCTION

Stochastic volatility models are commonly used in the field of mathematical finance and gaining popularity in financial econometrics and management field in particular asset and risk management. These models are those in which the variance of a stochastic process is itself randomly distributed. Models with stochastic volatility in its element can specifically capture the time-varying volatility, which has been presence in many financial markets and decision making (Shephard and Andersen, 2009). The understanding of stochastic volatility is important in various field of study, in particular option pricing, asset pricing, efficient portfolio allocation and accurate risk assessment and management. They are able to provide more accurate insight of what is actually happening in the financial data. A good management of such financial problems will reduce unnecessary risks that normally pose threats to most of traders and investors-both the bulls and the bears.

Models assimilated by good volatility component are able to mimic closely to the actual phenomena and produce better generalization of forecasting and estimating of variables, in particular particulates in many financial models. As standard volatility model has been in the literature from early introduction of option pricing by Black and Scholes (1973), the early version of stochastic volatility was first being study by Taylor (1982). Early work on stochastic volatility models mostly focus on the inconsistency in implied volatility values (Taylor, 1982, 1986). However, these

works are limited in the literature due to the difficulties of deriving its mathematical formulation.

Works by Stein and Stein (1991) and Heston (1993) has improved the Black Scholes model's assumptions, in which innovations to volatility need not be perfectly correlated with innovations to the price of the underlying asset; and use the stochastic volatility model in their works. Such models can give details for some of the empirical features of the joint time-series behavior of option prices and stock, which cannot be captured by more limited models.

There are two main streams when discuss the stochastic volatility. One is based on the discrete time setting and the other is in a continuous-time setting. The discrete-time setting is dominated by variant of Autoregressive Conditionally Heteroskedastic (ARCH) model, while the representation of stochastic differential equations represents the continuous-time. Though the theoretical part of the ARCH type models has drawbacks and open to criticism, they have been favored over continuous time setting for their ease of estimation. We present some preliminaries of important definitions in stochastic volatility.

Definition 1: A stochastic process $W(t)$ is a Brownian Motion (BM) if it satisfies the following properties:

- $W(t)$ is a continuous function of time with $W(0) = 0$.
- $W(t)$ has independent increments, i.e., for all $t > s, v > u$ and $u > t, W(t) - W(s)$ and $W(v) - W(u)$ are independent.

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- $W(t)$ has normal increments, i.e., for all $t > s, W(t) - W(s) \sim N(0, t - s)$.

Definition 2: A stochastic process S_t is said to follow a Geometric Brownian Motion (GBM) if it satisfies the following Stochastic Differential Equation (SDE):

$$dS_t = \mu S_t dt + \sigma S_t dW_t \quad (1)$$

where,

W_t : A BM

μ : The percentage drift

σ : The percentage volatility are constants

In a stochastic volatility model, the constant volatility σ is replaced by a function v_t that models the variance of S_t as follows:

$$dS_t = \mu S_t dt + \sqrt{v_t} S_t dW_{1t} \quad (2)$$

$$dv_t = \alpha_{s,t} dt + \beta_{s,t} dW_{2t} \quad (3)$$

where, $\alpha_{s,t}$ and $\beta_{s,t}$ are some functions of v_t and W_{2t} is another standard Gaussian that is correlated with W_{1t} with constant correlation factor ρ .

Stochastic Volatility (SV) model is typically analyzed by using advanced models, which became helpful and accurate as computer technology emerges; in which shows significant increase in number of literatures that work on this problem in recent years. We can use the stochastic volatility model to evaluate derivatives securities such as option. We can say that the stochastic volatility model is introduced to account for inconsistency in implied values.

Hence, SV models are one way to give treatment to the drawback of the Black Scholes model. This original model assumes that the volatility is constant over time but this assumption was rejected as showed in previous empirical studies (Stein, 1989; Aït-Sahalia and Lo, 1998). This handicap exposed by the Black Scholes model motivated large number of authors to propose many alternative models, for example, generalized Lévy processes, fractional Brownian motion, the diffusions with jumps and stochastic volatility models.

Johnson (1979) was the first researcher who used continuous time SV models in option pricing by using time varying volatility models. For more details, reader are encourage to see Johnson and Shanno (1987), Wiggins (1985) and Hull and White (1987). The common target of all these authors was to find a new enhanced formula to generalize the Black and Scholes (1973) method to option pricing models with the volatility clustering.

DISCRETE AND CONTINUOUS STOCHASTIC VOLATILITY

In this section, we present some widely used basic models in discrete and continuous stochastic volatility

models as follows. In discrete setting, generalized autoregressive conditionally heteroskedasticity (GARCH (1, 1)) model introduced by Bollerslev (1986) as follows.

Definition 3: A process ϵ_t is called a GARCH (p, q) process if its first two conditional moments exist and satisfy:

$$E(\epsilon_t | \epsilon_u, u < t) = 0 \quad t \in Z \quad (4)$$

There exist constant $\omega, \alpha_i, i = 1, 2, \dots, q$ and $\beta_j, j = 1, 2, \dots, p$ such that:

$$\sigma^2 = \text{var}(\epsilon_t | \epsilon_u, u < t) = \omega + \sum_{i=1}^q \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2, \text{ where } t \in Z$$

Box and Jenkins (1970) has also introduced the Autoregressive Integrated Moving Average (ARIMA) model.

Definition 4: Given a time series of data X_t where t is an integer index and the X_t are real numbers, then an ARIMA (p, d, q) model is given by:

$$(1 - \sum_{i=1}^p \phi_i L^i)(1 - p)^d X_t = (1 + \sum_{i=1}^q \theta_i L^i) \epsilon_t \quad (5)$$

where,

L : The lag operator

ϕ : The parameters of the autoregressive part of the model

θ_i : The parameters of the moving average part

ϵ_t : Error terms

The continuous setting of stochastic volatility follows the Ornstein-Uhlenbeck (OU) model defined as below.

Definition 5: The Ornstein-Uhlenbeck (OU) model is the solution to the linear SDE:

$$dX_t = \theta(\mu - X_t)dt + \sigma dW(t) \quad (6)$$

where, $W(t)$ is a Brownian Motion (BM), $\theta \in \mathbb{R}$ and μ and $\sigma > 0$ are parameters.

A simple discrete SV was considered by Hamilton (1989), whereas Hull and White (1987) presented the continuous time diffusion model. These models assume that the underlying volatility is constant over the life of the derivatives and unmoved by the changes in the price level of the underlying security. Hence they cannot explain long-observed elements of the implied volatility surface such as volatility smile and skew, which showed that the implied volatility have a tendency to differ with respect to strike price and expiry. By

assuming that the volatility of underlying price is stochastic process rather than constant, it will be possible to model derivatives more accurately. However, in the empirical literature, the appearance of the basic SV is mostly in discrete approximation of the continuous time SV models, such as work by Hull and White (1987).

Many authors (Harvey, 1998; Breidt *et al.*, 1998) considered the discrete time models as a fractionally integrated process provided that the log of the volatility had been modeled. With respect to continuous time, Comte and Renault (1998) worked on modeling the log of volatility as fractionally integrated BM. The important paper by Comte *et al.* (2012) introduced the second root model driven by fractionally integrated BM, while Barndorff-Nielsen and Shephard (2001) presented the infinite superposition of non-negative Ornstein Uhlenbeck processes.

SOME EXTENSION WORKS ON STOCHASTIC VOLATILITY

Realizing the importance of Stochastic Volatility (SV) in financial models, we made extensive investigation on SV in various search engines available in digital prints. We are interested to see in which enhancement does works related to SV has been undertaken. This investigation will help us to narrow the potential works to be conducted in relation to SV.

To obtain the current state and evolution on works concerning SV model in both settings i.e., the discrete and continuous, we conducted a systematic literature investigation in some selected academic databases for the past ten years. Google scholar, EBSCO host and SciVerse were used as search engines, by using some keywords as follow:

- "Stochastic volatility" and "jumps" and "continuous"
- "Stochastic volatility" and "jumps" and "discrete"
- "Stochastic volatility" and "moment-based inference" and "continuous"
- "Stochastic volatility" and "moment-based inference" and "discrete"
- "Stochastic volatility" and "simulation-based inference" and "continuous"
- "Stochastic volatility" and "simulation-based inference" and "discrete"

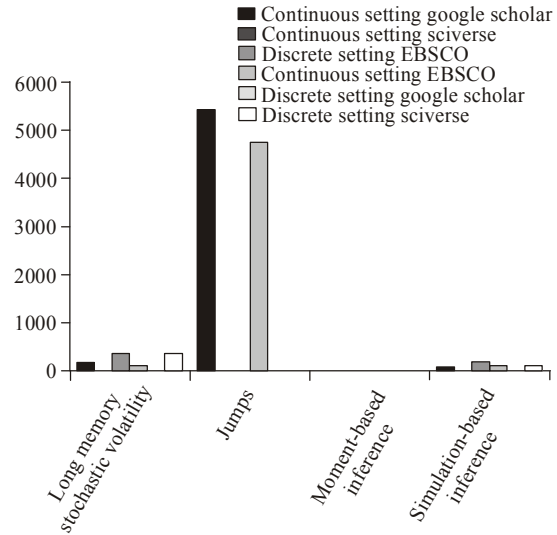


Fig. 1: Number of stochastic volatility literature from 2002 to 2012

- "Long memory stochastic volatility" and "continuous"
- "Long memory stochastic volatility" and "discrete"

We illustrate the findings in Table 1 and Fig. 1. From the table and figure, there is significant difference between stochastic volatility that has been published with jumps with the rest of other keywords. This is prominent in the Google Scholar database. The long memory comes second in frequency. However, for Sciverse database, works on long memory are more pronounce.

Works on simulation-based inference also interest researchers, making it being in the third place in frequency, while moment-based inference are very rare in literature.

In the following subsections, we introduce some selected extension works on stochastic volatility model under study.

JUMPS: A number of authors, in empirical studies, have improved standard SV models by adding jumps to the volatility dynamics or price process. Bates (1996) proved necessity to adding jumps to the SV, at least when volatility is Markovian.

Barndorff-Nielsen and Shephard (2001, 2002) designed the volatility model based from the pure jump

Table 1: Number of stochastic volatility literature from 2002 to 2012

	Continuous setting			Discrete setting			Total
	Google scholar	EBSCO	Sciverse	Google scholar	EBSCO	Sciverse	
Long memory	225	1	443	193	3	405	1270
Jumps	5510	31	1	4820	10	1	10373
Moment-based inference	8	0	12	7	0	11	38
Simulation-based inference	108	0	221	105	0	221	655
Total	5851	32	677	5125	13	638	12336

processes. With jumps in their model, σ represents the square root of the solution of the stochastic differential equation:

$$d\sigma_t^2 = -\lambda\sigma_t^2 dt + dz_{\lambda t}, \lambda > 0 \quad (7)$$

where, z represent a subordinator with independent non-negative and stationary increments.

Long Memory Stochastic Volatility (LMSV): Long memory also called long-range dependence, meaning that the various time observations are strongly correlated. For example, slowly decaying autocorrelation function. Via the long memory stochastic volatility models, there were attempts to describe a slowly decaying autocorrelations and recently there are increase numbers of attempts targeting this model, in discrete or continuous time setting.

Harvey (1998) and Breidt *et al.* (1998) introduced the first Long Memory Stochastic Volatility (LMSV). They proposed a discrete time model:

$$X_t = \sigma(Y_t) \varepsilon_t \quad (8)$$

where, (X_t) are the returns of stock, (ε_t) are i.i.d presenting shocks and the logarithm of (Y_t) is described by Autoregressive Fractionally Integrated Moving Average (ARFIMA). This model described the long-range behavior of the log-squared returns of market indexes successfully. With respect to continuous time, Comte and Renault (1998) presented a model of the price process such that the dynamics of the volatility are designed by the FOU process. In a work by Comte *et al.* (2003), the square root model driven by fractionally integrated Brownian motion is introduced. Comte *et al.* (2012) contributed via offering an extension of Heston option pricing model to continuous time SV such that the volatility process is defined by a square root long memory process.

Chronopoulou and Viens (2012) studied the accuracy of three different types of LMSV. One was a continuous time stochastic volatility when the stock price is geometric Brownian motion such that the volatility introduced as a FOU process. The others were discrete time models: a discretization of the previous continuous model and a discrete model when the returns are a zero mean independent identically distribution sequence and the volatility is a fractional ARIMA process. By working with simulated data and call option data of S and P500 index, they found that the continuous time model is more accurate than the other discrete models. However the main disadvantage of continuous time model is computationally expensive when applied on real data.

Wang and Zhang (2014) studied ordinary least square estimators of variogram parameters in long

memory stochastic volatility. They got two estimators of diffusion and drift parameters through finding the minimum of the distance function between the data periodogram and variogram.

Multivariate models: Volatility clustering into standard factor models which presented by Diebold and Nerlove (1989) are used in more than one areas of asset pricing. In continuous time the same author introduced the following models:

$$M_t = \sum_{j=1}^J \int B_{(j)s} dF_{(j)s} + G_t \quad (9)$$

where, G is a correlated multivariate BM and the factors $F(1), F(2), \dots, F(J)$ are independent univariate SV models. In the literature some related papers to this issue are form King *et al.* (1994) and Fiorentini *et al.* (2004). They claim that the factor loading vectors are constant over time. Harvey *et al.* (1994) study on multivariate discrete time:

$$M_t = C \int_0^T \sigma_s dW_s \quad (10)$$

where, C is a fixed matrix of constants such that the main diagonal all units, W_s is BM and σ is a diagonal matrix process. This implies that the risky part of prices is just a rotation of a p -dimensional vector of univariate SV independent processes.

Moment based inference: There were two separate approaches to estimate the efficiency of the stochastic volatility models. First, a computationally intensive method which can approximate the efficiency of likelihood-based inference well, but at the expense of using particular and time wasting techniques. Second, number of researchers built relatively simple papers that were inefficient estimators based on moments of the model that were easily to compute.

The task is to implement inference on $\theta = (\theta_1, \theta_2, \dots, \theta_k)'$ that is the SV parameter based on returns $y = (y_1, y_2, \dots, y_T)$ by using the moments method, Taylor (1982) calibrated the discrete time model. Melino and Turnbull (1990) developed the inference by basing on a larger set of moment conditions and clustering them more efficiently as they exploit the Generalized Method of Moments (GMM) procedure. Andersen and Sørensen (1996), Genon-Catalot *et al.* (2000) and Hoffman (2002) using (GMM) approach to give systematic study of which moments to heavily weight in stochastic volatility models. The class of the GMM inference is sensitive to both the choices of the number of moments to include and the particular choice of moments among the regular candidates.

Approach promoted by Harvey *et al.* (1994) in the discrete time log-normal SV models is as the follows. First, remove the predictable part of the return and then work with:

$$\log y_i^2 = h_i + \log \varepsilon_i^2 \quad (11)$$

In the case of short memory of the volatility this formula can be treated by Kalman filter, but the situation changed in the case of long memory models which is almost handle within the fluctuation domain. In both ways, this transports a Gaussian quasi-likelihood which can be helpful to estimate the parameters of the model.

One of the disadvantages of the continuous time SV models is that the moments y is not straightforward to compute when using moments based estimators. Nonetheless, Meddahi (2001) presents an approach for generating moment conditions for the full range of models within so-called Eigen function stochastic volatility class. Barndorff-Nielsen and Shephard (2001) studied the case of no leverage and obtained the properties of y in the second order and their squares.

Simulation based inference: Researchers began to use simulation based inference in the 1990s. Both Markov Chain Monte Carlo (MCMC) and Efficient Method of Moments (EMM) is two popular simulations to be used in order to treatment with SV models. To discuss the methods above it will be convenient to dealing with the simplest discrete log normal stochastic volatility given as follows:

$$m_i = \sigma_i \varepsilon_i \quad (12)$$

$$h_{i+1} = \mu + \phi(h_i - \mu) + \eta_i \quad (13)$$

where,

m_i : The risky part of returns

σ_i : Non-negative process

ε_i : Follows an autoregression with zero mean and unit variance

h_i : A non-zero mean Gaussian linear process

η_i : A zero mean Gaussian white noise process

To simulate from high dimensional later densities, for instance the smoothing variables $\Theta, h|y$, where $h = (h_1, h_2, \dots, h_T)'$ are the unobserved discrete time log-volatilities, we can use Markov chain Monte Carlo MCMC. Jacquier *et al.* (2004) applied a MCMC algorithm in attempt to solve this problem, while Kim *et al.* (1998) presented an extensively discussion in different MCMC algorithms.

Although most papers based on MCMC are formulated in discrete time there are some work, e.g., Eraker (2001), Elerian *et al.* (2001) and Roberts and Stramer (2001), that use the adaptable of general approach to deal with continuous time models. Based on particle filter, Kim *et al.* (1998) presented the first filter. In addition of the significant role in decision making, filtering method permits to computation of

one-step ahead predictions for model testing and marginal likelihood for model comparison.

From the literature, we can see that there are active discussions on SV-enhancement models. Most of the works involve with the theoretical development of the models. However, the empirical application of SV models were limited. This mainly because of the difficulties relating with their estimation. The biggest problem is in the finding their likelihood function and so the Maximum Likelihood (ML) estimation of the parameters is not direct.

These encouraged authors to find other estimation method for the SV models. There are two methods to estimate the parameter of autoregressive stochastic volatility ARSV (1). First, methods based on returns observation over the time. These include the Method of Moments (MM), Maximum Likelihood Estimation (MLE). Second, methods based on log of the square of the returns observation such as Quasi Maximum Likelihood (QML) estimation. For more details readers can refer to Broto and Ruiz (2004).

DISCUSSION AND CONCLUSION

This study uses content analysis to investigate the current state and evolution of stochastic volatility from 2002 to 2012. The result is illustrated in Table 1 and Fig. 1. There are significant difference between stochastic volatility has been published with jumps and long memory stochastic volatility in comparison with moment based and simulation based SV. Ten thousand three hundred and thirty works on jumps have been conducted when Google Scholar was investigated, making it the most dominant in the literature, while 848 works on long memory has been found in SciVerse database. Works on moment-based inference are very rare. However there are quite number of researches that concerned with simulation-based inference. We can also identified that researchers are more interested to look at the continuous setting involved with 6,560 articles compared to the discrete time setting with 5,776 articles.

Based from extensive amount of works that has undertaken in SV model, more potential works on the development of the SV model, in particular the estimation of important variables involve in the model are believe to be interesting to be investigated. A throughout understanding of the model will be beneficial to minimize the risk in the financial world in particular.

REFERENCES

- Ait-Sahalia, Y. and A.W. Lo, 1998. Nonparametric estimation of state-price densities implicit in financial asset prices. *J. Financ.*, 53(2): 499-547.

- Andersen, T.G. and B.E. Sørensen, 1996. GMM estimation of a stochastic volatility model: A Monte Carlo study. *J. Bus. Econ. Stat.*, 14(3): 328-352.
- Barndorff-Nielsen, O.E. and N. Shephard, 2001. Non-Gaussian Ornstein-Uhlenbeck-based models and some of their uses in financial economics. *J. Roy. Stat. Soc. B*, 63(2): 167-241.
- Barndorff-Nielsen, O.E. and N. Shephard, 2002. Estimating quadratic variation using realized variance. *J. Appl. Econom.*, 17(5): 457-477.
- Bates, D.S., 1996. Jumps and stochastic volatility: Exchange rate processes implicit in deutsche mark options. *Rev. Financ. Stud.*, 9: 69-107.
- Black, F. and M. Scholes, 1973. The pricing of options and corporate liabilities. *J. Polit. Econ.*, 18(3): 637-654.
- Bollerslev, T., 1986. Generalized autoregressive conditional heteroskedasticity. *J. Econometrics*, 31(3): 307-327.
- Box, G.E.P. and G.M. Jenkins, 1970. *Time Series Analysis: Forecasting and Control*. Holden-Day, San Francisco.
- Breidt, F.J., N. Crato and P. De Lima, 1998. The detection and estimation of long memory in stochastic volatility. *J. Econometrics*, 83(1): 325-348.
- Broto, C. and E. Ruiz, 2004. Estimation methods for stochastic volatility models: A survey. *J. Econ. Surv.*, 18(5): 613-649.
- Chronopoulou, A. and F.G. Viens, 2012. Estimation and pricing under long-memory stochastic volatility. *Ann. Financ.*, 8(2): 379-403.
- Comte, F. and E. Renault, 1998. Long memory in continuous-time stochastic volatility models. *Math. Financ.*, 8(4): 291-323.
- Comte, F., F. Coutin and E. Renault, 2003. *Affine Fractional Stochastic Volatility Models*. Working Paper, Université de Montréal.
- Comte, F., L. Coutin and É. Renault, 2012. Affine fractional stochastic volatility models. *Ann. Financ.*, 8(2): 337-378.
- Diebold, F.X. and M. Nerlove, 1989. The dynamics of exchange rate volatility: A multivariate latent factor ARCH model. *J. Appl. Econom.*, 4: 1-21.
- Elerian, O., S. Chib and N. Shephard, 2001. Likelihood inference for discretely observed nonlinear diffusions. *Econometrica*, 69(4): 959-993.
- Eraker, B., 2001. MCMC analysis of diffusion models with application to finance. *J. Bus. Econ. Stat.*, 19(2): 177-191.
- Fiorntini, G., E. Sentana and N. Shephard, 2004. Likelihood-based estimation of latent generalized ARCH structures. *Econometrica*, 72(5): 1481-1517.
- Genon-Catalot, V., T. Jeantheau and C. Laredo, 2000. Stochastic volatility models as hidden. *Bernoulli*, 6(6): 1051-1079.
- Hamilton, J.D., 1989. A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica*, 57(2): 357-384.
- Harvey, A.C., 1998. *Long Memory in Stochastic Volatility Forecasting Volatility in Financial Markets*. Butterworth-Haineman, Oxford, pp: 307-320.
- Harvey, A.C., E. Ruiz and N. Shephard, 1994. Multivariate stochastic variance models. *Rev. Econ. Stud.*, 61(2): 247-264.
- Heston, S.L., 1993. A closed-form solution for options with stochastic volatility with applications to bond and currency options. *Rev. Financ. Stud.*, 6(2): 327-343.
- Hoffman, J.L., 2002. The impact of student cocurricular involvement on student success: Racial and religious differences. *J. Coll. Student Dev.*, 43(5): 712-739.
- Hull, J. and A. White, 1987. The pricing of options on assets with stochastic volatilities. *J. Financ.*, 42: 281-300.
- Jacquier, E., N.G. Polson and P.E. Rossi, 2004. Bayesian analysis of stochastic volatility models with fat-tails and correlated errors. *J. Econometrics*, 122(1): 185-212.
- Johnson, H.E., 1979. *Option Pricing When the Variance is Changing*. Working Paper 11-79, Graduate School of Management, University of California, Los Angeles.
- Johnson, H. and D. Shanno, 1987. Option pricing when the variance is changing. *J. Financ. Quant. Anal.*, 22(2): 143-151.
- Kim, S., N. Shephard and S. Chib, 1998. Stochastic volatility: Likelihood inference and comparison with ARCH models. *Rev. Econ. Stud.*, 65: 361-393.
- King, M., E. Sentana and S. Wadhvani, 1994. Volatility and links between national stock markets. *Econometrica*, 62: 901-933.
- Meddahi, N., 2001. *An Eigenfunction Approach for Volatility Modeling*. CIRANO Working Papers 01/2001, CIRANO, Université de Montréal.
- Melino, A. and S.M. Turnbull, 1990. Pricing foreign currency options with stochastic volatility. *J. Econometrics*, 45(1): 239-265.
- Roberts, G. and O. Stramer, 2001. On inference for partial observed nonlinear diffusion models using the metropolis-hastings algorithm. *Biometrika*, 88(3): 603-621.
- Shephard, N. and T.G. Andersen, 2009. *Stochastic Volatility: Origins and Overview*. In: Andersen, T.G., R.A. Davis, J.P. Kreiss and T. Mikosch (Eds.), *Handbook of Financial Time Series, Part II*. Springer, Berlin, Heidelberg, pp: 233-254.

- Stein, J.C., 1989. Overreactions in the options market. *J. Financ.*, 44: 1011-1023.
- Stein, E.M. and J.C. Stein, 1991. Stock price distributions with stochastic volatility: An analytic approach. *Rev. Financ. Stud.*, 4: 727-752.
- Taylor, S.J., 1982. Financial Returns Modelled by the Product of Two Stochastic Processes: A Study of Daily Sugar Prices 1961-79. In: Anderson, O.D. (Ed.), *Time Series Analysis: Theory and Practice*. North-Holland, Amsterdam, 1: 203-226.
- Taylor, S., 1986. *Modelling Financial Time Series*. Wiley, New York, Vol. 113.
- Wang, X. and W. Zhang, 2014. Parameter estimation for long-memory stochastic volatility at discrete observation. *Abstr. Appl. Anal.*, 2014: 10.
- Wiggins, J.B., 1985. *Stochastic Variance Option Pricing*. Working Paper, Sloan School of Management, Massachusetts Institute of Technology, Cambridge, MA.