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Research Article

Group Acceptance Sampling Plan for Re-Submitted Lots Under Generalized Pareto Distribution

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Abstract: In this study, a Group Acceptance Sampling Plan (GASP) for lot resubmitting is developed for situations in which the lifetime of a product follows the generalized Pareto distribution. The design parameters such as minimum group size and acceptance number are observed when the consumer's risk, number of testers and the test termination time are pre-specified. The proposed plan requires less sample size than the ordinary GASP. The condition of lot re-sampling was examined and measurement of a resubmitted method having a GASP for inspection.

Keywords: Consumer's risk, group sampling plan, generalized Pareto distribution, producer's risk, resubmitted method

INTRODUCTION

More recently, quality assurance expanded the scope of final inspection that consists of all features of manufacturing e.g. statistical process control, HACCP, Six sigma and ISO 9000. Acceptance sampling is an important field of statistical quality control in making decision whether to accept or reject the lot under inspection. Acceptance sampling plans were frequently used during the Second World War, for instance MIL-STD-105, were developed by Dodge and others and became commonly used as quality standards. The applications stem from real life scenarios: if every bullet is inspected prior to war, no bullet is at hand for time of action and if no bullet is inspected, then tragedy may occur in the war. Dodge described that a sample is chosen from a lot or batch and the outcome of the products totally depends on the characteristic collected from this sample. This procedure is called as acceptance sampling plan or lot acceptance sampling. It is used to accept or reject of a submitted product but not to estimate the quality of the lot. Acceptance sampling is a middle way between no inspection and hundred percent inspections. Among all acceptance sampling methods, the single attribute acceptance sampling is the most commonly used because it is easy for practical implementation. The inference about acceptance or rejection of a submitted lot by the single attribute acceptance sampling is based on the truncated life test. In order to examine the lifetime of a product, one must follow the destructive or truncated life test. It requires considerably long experiment time to thoroughly observe the absolute or complete life time of a high reliability product. The life test must complete within a pre-specified schedule and such life test is known as truncated life test.

In the past few decades, much effort has gone into the investigation of single acceptance plans under a truncated life test. Epstein (1954) proposed an acceptance sampling plans followed on truncated life test assuming that the lifetime of a product based on exponential distribution. He considered two approaches to design the acceptance sampling plan for this lifetime distribution. The first is the replacement case, in which, if a product fails before the end of the experiment time, the failed item is replaced by a new product. In non-replacement case, a product is not replaced by a new one. Goode and Kao (1961) described the sampling reliability testing for truncated life tests under the assumption that the lifetime of a product is needed on the Weibull distribution. This method is used when inspection of products is by attribute and the exponential distribution is included as a special case of the Weibull distribution. Gupta (1962) presented accepting sampling plans for truncated life tests from the normal and lognormal distribution and also discussed the table that gives the minimum values of sample size necessary to ensure a certain mean life. In order to facilitate selection of an appropriate acceptance sampling plan, producer's risk is also discussed.

In many situations a submitted lot is not accepted by the consumer based on a single acceptance sampling plan. The producer can dispute the first sample information, select the second sample of the same size

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for inspection and make an inference discarding the previous results. Hence, the rejected lot may be resubmitted, this repeated acceptance sampling process is called resubmitted acceptance sampling plan. Govindaraju and Ganesalingam (1997) proposed the performance measures of the re-sampling technique with the single sampling plan for examining resubmitted lot that are different from the ordinary acceptance sampling plans.

Kantam and Rosaiah (1998) designed a new probability density function known as the half logistic distribution in the area of acceptance sampling plan and also developed an acceptance sampling plan. Meanwhile, ordinary acceptance sampling plans have been proposed by Kantam *et al.* (2001), Baklizi (2003), Baklizi and EI Masri (2004), Rosaiah and Kantam (2005),Kantam *et al.* (2006), Tsai and Wu (2006) and Rosaiah *et al.* (2006). Recently, Rosaiah *et al.* (2007), Aslam and Shahbaz (2007), Balakrishnan *et al.* (2007), Rosaiah and Kantam (2008) and Aslam and Kantam (2008) also developed the above mentioned plan for varios lifetime distributions.

A number of products are put in a tester in order to save cost, time and other important resources. In such a life test, the tester is known as a group, while the number of products in each tester is the group size. The acceptance sampling accomplished through the group life test is called the Group Acceptance Sampling Plan (GASP). The GASP is also frequently implemented using the truncated life test. The first introduction of GASP is by Aslam and Jun (2009), which was based on life test when the lifetime of products follows either an inverse Rayleigh or a log-logistic distribution. The minimum number of groups, probability of lot acceptance and minimum ratio of true average life to specified average life are investigated for different producer's and consumer's risks. Subsequently, Aslam and Jun (2009) designed a GASP for the Weibull distribution with known shape parameter and determined the number of groups and the acceptance number by satisfying producer's and as well as consumer's risks for a given termination time. Further, Aslam *et al.* (2010a) studied GASP based on life test assuming that the lifetime follows the Pareto distribution of the second kind with known shape parameters. The design parameters such as number of groups and probability of lot acceptance are determined for specified values of termination time, mean ratio and number of testers. The obtained results are explained with lifetime examples. Aslam *et al.* (2010b) also proposed a time truncated GASP based on life test considering that the lifetime follows generalized exponential distribution and generalized Pareto distribution. The minimum number of groups and the acceptance number were obtained by satisfying the producer's and consumer's risks at the pre-assumed quality standards.

More recently, Ramaswamy and Anburajan (2012), Ramaswamy and Sutharani (2013), Rao *et al.* (2013) and Rao *et al*. (2014) proposed various acceptance sampling plans for a truncated life. As discussed earlier, acceptance sampling plans are helpful and lead to the final inference of the lot on the basis of random sampling, then two risks are involved with a sample. If a good lot is rejected on the basis of sample information, it will be harmful to producer and if a bad lot is accepted, it will be a disadvantage to consumer. The probability of rejecting a good lot is called the producer's risk denoted by α and the probability of accepting the bad lot is called the consumer's risk denoted by β, respectively. The main objective of the acceptance sampling plan is to develop the design parameters which satisfy both risks.

RE-SUBMITTED GROUP ACCEPTING SAMPLING PLAN

An attribute acceptance sampling plan has much utilization in several ways. For example, it is used to inspect the submitted products to satisfy the pre-requisite assumptions before they are manufactured. The final product must satisfy the consumer's conditions. The attribute acceptance sampling plan has three design parameters: lot or batch size N, sample size n and the acceptance number or the number of failures c. The plan is carried out as; select specific n products from a lot of size N with the acceptance number c. If the number of defective products is less than c, then accept the lot; otherwise reject the lot. The quality measure p indicates the fraction of defective products. If p = 0% or p = 100%, there is no argument to use the attribute acceptance sampling plan. On the other hand, if p lies between 0% and 100%, the attribute acceptance sampling plan is very helpful to accept or reject the product on the basis of random sample taken from the lot.

The single attribute acceptance sampling is the mixture of sample size, acceptance number and termination time. To inspect the quality standard of a product, the null and alternative hypotheses are formulated such that $\mu \geq \mu_0$ and $\mu < \mu_0$, where μ and μ_0 are the true and specified average life of a product respectively. In a single attribute acceptance sampling plan, a sample size n is randomly selected from a lot and put on test. An experimenter continues this testing for a pre-assumed experiment time *t*. The acceptance number c is constant for an experiment, which is truncated if more than c or $(c+1)$ failures are observed during the experiment time or the time of experiment is finished, whichever is earlier. A lot is accepted if no more than c failures are recorded during the testing time, otherwise rejected. The current study proposesa specially designed for certain situations. Following are the steps for its applications:

i) Find the group size g when number of testers r is pre-specified. Choose $(r \times g)$ products from a lot and allocate r products to each group g.
ii) The required sample size in the life test is $(r \times g)$.

iii) Determine the acceptance number c for every group and specify the termination time t_0.
iv) Truncate the life test and reject the lot if more than c or $(c+1)$ failures occur in any group.
v) The producer must respect the consumer's confidence and must not take undue advantage of the re-sampling. Perform the GASP i.e., steps (i) to (iv), on non acceptance of the original GASP, apply the proposed plan m times and reject the submitted lot if it is not accepted on $(m-1)$th resubmission.

It is important to note that the proposed plan for the resubmitted lot based on the number of failures is not an extension of the existing plan. It cannot be compared with the single sampling plan for the resubmitted lot developed by Govindaraju and Ganesalingam (1997). Although the two-stage and the multiple-stage group acceptance sampling plan based on a truncated life test could be formulated for the resubmitted lots. The group sampling under a truncated life test is preferred due to it easiness of implementation. The resubmitted acceptance sampling plan should not be confused with the two-stage group acceptance sampling plan because in two-stage group acceptance sampling, the conclusion about the submitted lot is decided by combining the results from the first and the second samples.

Pareto (1897) discussed the Pareto distribution as a model for income and much later, Baklizi (2003) developed an ordinary acceptance sampling plan for Pareto distribution of the 2nd kind. Choulakian and Stephens (2001), Zhang (2007) and Abd Elfattah *et al.* (2007) have derived the properties of Generalized Pareto distribution (GPD). Recently, Mughal and Aslam (2011b) and Mughal and Ismail (2013) developed a Group Acceptance Sampling Plan (GASP) for family of Pareto distributions. The Probability Density Function (PDF) and Cumulative Distribution Function (CDF) of survival $S(t)$ and hazard function $H(t)$ of the Generalized Pareto Distribution are given respectively:

$$f(t;\alpha,\beta,\lambda,\delta)=\frac{\delta\alpha}{\beta}\left(\frac{t-\lambda}{\beta}\right)^{\delta-1}\left[1+\left(\frac{t-\lambda}{\beta}\right)^{\delta}\right]^{-(\alpha+1)} \quad (1)$$

$$F(t;\alpha,\beta,\lambda,\delta)=1-\left[1+\left(\frac{t-\lambda}{\beta}\right)^{\delta}\right]^{-\alpha} \quad (2)$$

$$S(t)=\left[1+\left(\frac{t-\lambda}{\beta}\right)^{\delta}\right]^{-\alpha} \quad (3)$$

$$H(t)=\frac{\delta\alpha}{\beta}\left(\frac{t-\lambda}{\beta}\right)^{\delta-1}\left[1+\left(\frac{t-\lambda}{\beta}\right)^{\delta}\right]^{-1} \quad (4)$$

where, $\lambda < t < \infty$, $\beta > 0$, $\alpha > 0$, $\delta > 0$, λ is the location parameter, β is the scale parameter and (a, δ) are shape parameters respectively. The mean and variance of generalized Pareto distribution are given by:

$$\mu=\beta\frac{\Gamma\left(\alpha-\frac{1}{\delta}\right)\Gamma\left(1+\frac{1}{\delta}\right)}{\Gamma(\alpha)}+\lambda \quad (5)$$

$$\sigma^2=\beta^2\left[\frac{\Gamma\left(1+\frac{2}{\delta}\right)\Gamma\left(\alpha-\frac{2}{\delta}\right)}{\Gamma(\alpha)}-\left(\frac{\Gamma\left(1+\frac{1}{\delta}\right)\Gamma\left(\alpha-\frac{1}{\delta}\right)}{\Gamma(\alpha)}\right)^2\right] \quad (6)$$

For the validity of mean and variance, we consider the following two conditions $a > 1/\delta$ and $a > 2/\delta$, respectively. The above defined generalized Pareto distribution (1) can be converted to different distributions; for more explanations one may refer to Abd Elfattah *et al.* (2007). If the total number of failures occurred from every group is at most the pre-fixed acceptance number c, then the probability of accepting the lot for the ordinary group sampling plan based on the number of failures from all groups is given by (7):

$$L(p)=\sum_{i=0}^{c}\binom{rg}{i}p^i(1-p)^{rg-i} \quad (7)$$

where, p is the probability that a product fails in any group during the test termination time t_0. It would be easiest to find the test termination time t_0, that is equivalent to multiply the specified averagelife μ_0 and test termination ratio a, then it can be written as (8):

$$t_0=a\mu_0 \quad (8)$$

Then the lot acceptance probability for Generalized Pareto Distribution is (9):

$$p=F(t)=1-\left[1+\left(\frac{a\Gamma\left(\alpha-\frac{1}{\delta}\right)\Gamma\left(1+\frac{1}{\delta}\right)}{\left(\frac{\mu}{\mu_0}\right)\Gamma(\alpha)}\right)^{\delta}\right]^{-\alpha} \quad (9)$$

where, (μ/μ_0) is a ratio of true average life and the specified average life. We will consider $\lambda = 0$, $a = \delta = 2$, under these assumptions, the (2) and (5) are:

$$F(t)=1-\left[1+(t/\beta)^2\right]^{-2} \quad (10)$$

$$\mu=\beta\frac{\Gamma(3/2)\Gamma(3/2)}{\Gamma(2)}+0=\beta(\pi/4) \quad (11)$$

$$p=F(t_0)=F(a\mu_0)=1-\left[1+\left(\frac{\pi a\mu_0}{4\mu}\right)^2\right]^{-2} \quad (12)$$

According to Govindaraju and Ganesalingam (1997), the probability of lot acceptance for the resubmitted lots with m-1 resubmissions is given by (21):

$$P_a(p) = 1 - (1 - L(p))^m \tag{13}$$

Therefore, the lot acceptance probability proposed plan can be given by (14):

$$P_a(p) = 1 - \left[1 - \left\{\sum_{i=0}^{c} \binom{rg}{i} p^i (1-p)^{rg-1}\right\}\right]^m \tag{14}$$

Table 1: Optimal group size for the proposed plan when a = 2, δ = 2, m = 2

			a					
β	*r*	*c*	0.7	0.8	1.0	1.2	1.5	2.0
	4	2	3	3	2	2	1	1
	5	3	3	3	2	2	2	2
	6	4	3	3	2	2	2	2
0.25	7	5	3	3	2	2	2	2
	8	6	3	3	2	2	2	2
	9	7	3	3	2	2	2	2
	10	8	3	3	2	2	2	2
	4	2	3	3	2	2	2	1
	5	3	4	3	2	2	2	1
	6	4	4	3	2	2	2	2
0.10	7	5	4	3	2	2	2	2
	8	6	4	3	2	2	2	2
	9	7	4	3	2	2	2	2
	10	8	4	3	2	2	2	2
	4	2	4	3	3	2	2	2
	5	3	4	3	3	2	2	2
	6	4	4	3	3	2	2	2
0.05	7	5	4	3	3	2	2	2
	8	6	4	3	3	2	2	2
	9	7	4	3	3	2	2	2
	10	8	4	3	3	2	2	2
	4	2	5	4	3	2	2	2
	5	3	5	4	3	2	2	2
	6	4	5	4	3	2	2	2
0.01	7	5	5	4	3	2	2	2
	8	6	5	4	3	2	2	2
	9	7	5	4	3	2	2	2
	10	8	5	4	3	2	2	2

Table 2: Optimal group size for the proposed plan when a = 2, δ = 2, m = 3

			a					
β	*r*	*c*	0.7	0.8	1.0	1.2	1.5	2.0
	4	2	3	3	2	2	2	1
	5	3	3	3	2	2	2	1
	6	4	3	3	2	2	2	1
0.25	7	5	3	3	2	2	2	2
	8	6	3	3	2	2	2	2
	9	7	3	3	2	2	2	2
	10	8	3	3	2	2	2	2
	4	2	4	3	2	2	2	2
	5	3	4	3	2	2	2	2
	6	4	4	3	2	2	2	2
0.10	7	5	4	3	2	2	2	2
	8	6	4	3	2	2	2	2
	9	7	4	3	2	2	2	2
	10	8	4	3	2	2	2	2
	4	2	4	4	3	2	2	2
	5	3	4	4	3	2	2	2
	6	4	4	4	3	2	2	2
0.05	7	5	4	4	3	2	2	2
	8	6	4	4	3	2	2	2
	9	7	4	4	3	2	2	2
	10	8	4	4	3	2	2	2
	4	2	5	4	3	3	2	2
	5	3	5	4	3	3	2	2
	6	4	5	4	3	3	2	2
0.01	7	5	5	4	3	3	2	2
	8	6	5	4	3	3	2	2
	9	7	5	4	3	3	2	2
	10	8	5	4	3	3	2	2

There are two parameters c and g in this proposed plan for the resubmitted lot, given the group size r and the pre-assumed truncated life test time $t_0 = a\ \mu_0$, in terms of a multiple of expected lifetime average, μ_0. When the true average μ, is greater than or equal to the expected one, the product is called acceptable, otherwise not acceptable. Both consumer and producer want an acceptance sampling plan to make the inference fulfill their specified assumptions. Consumer wants the lot acceptance probability less than β and producer requires the lot acceptance probability of at least $1-\alpha$. The main objective of an acceptance sampling plan is to minimize the producer's risk and the consumer's risk. The purpose of this study is to searchfor the optimal values of group size at the pre-defined consumer's risk so that the following inequality (15) should satisfy:

$$1-\left[1-\left\{\sum_{i=0}^{c}\binom{rg}{i}p^{i}(1-p)^{rg-1}\right\}\right]^{m} \leq \beta \tag{15}$$

We found the optimal group size for generalized Pareto distribution for specified values of acceptance number (c = 2, 3,….8), various values of number of testers (r = 4, 5,…10) and different values of termination ratios (a = 0.7, 0.8, 1.0, 1.2, 1.5, 2.0). The optimal group sizes for the generalized Pareto distribution for given values of the shape parameter are presented in Table 1 and 2. If the shape parameters are not known, they may be calculated through maximum likelihood estimation to enable the use of the proposed plans. From Table 1 and 2, the number of required groups increases as the consumer's risk decreases and decreases as the test termination time increases. The probability of lot acceptance is another important issue in acceptance sampling for producer's point of view. The probability of lot acceptance can be found by using the proposed plan OC function (16):

$$L(p)=1-\left[1-\left\{\sum_{i=0}^{c}\binom{rg}{i}p^{i}(1-p)^{rg-1}\right\}\right]^{m} \tag{16}$$

In Table 3 and 4, we note that for fixed values of group g, the probability of lot acceptance increases and the mean ratio also increases from 2 to 12. In acceptance sampling based on truncated life test, the mean ratios are used as quality parameter and usually calculated at the producer's confidence level. These mean ratios are obtained such that the following inequality (17) should satisfy:

$$1-\left[1-\left\{\sum_{i=0}^{c}\binom{rg}{i}p^{i}(1-p)^{rg-1}\right\}\right]^{m} \geq 1-\alpha \tag{17}$$

The minimum mean ratios for generalized Pareto distribution are placed in Table 5 and 6. From these tables, it is clear that as the test termination ratio decreases, the minimum mean ratios also decrease for the same values of number of testers, r and acceptance number c.

Table 3: OC values of the proposed plan for a = 2, δ = 2, m = 2, r = 4, c =2

β	g	a	2	4	6	8	10	12
0.25	3	0.7	0.7775	0.9911	0.9991	0.9998	1.0000	1.0000
	3	0.8	0.6508	0.9821	0.9980	0.9996	0.9999	1.0000
	2	1.0	0.6716	0.9829	0.9981	0.9996	0.9999	1.0000
	2	1.2	0.4652	0.9578	0.9947	0.9989	0.9997	0.9999
	1	1.5	0.7536	0.9867	0.9985	0.9997	0.9999	1.0000
	1	2.0	0.4857	0.9476	0.9927	0.9985	0.9996	0.9998
0.10	3	0.7	0.7775	0.9911	0.9991	0.9998	1.0000	1.0000
	3	0.8	0.6508	0.9821	0.9980	0.9996	0.9999	1.0000
	2	1.0	0.6716	0.9829	0.9981	0.9996	0.9999	1.0000
	2	1.2	0.4652	0.9578	0.9947	0.9989	0.9997	0.9999
	2	1.5	0.2139	0.8852	0.9829	0.9963	0.9989	0.9996
	1	2.0	0.4857	0.9476	0.9927	0.9985	0.9996	0.9998
0.05	4	0.7	0.6183	0.9797	0.9977	0.9996	0.9999	1.0000
	3	0.8	0.6508	0.9821	0.9980	0.9996	0.9999	1.0000
	3	1.0	0.3821	0.9461	0.9932	0.9986	0.9996	0.9999
	2	1.2	0.4652	0.9578	0.9947	0.9989	0.9997	0.9999
	2	1.5	0.2139	0.8852	0.9829	0.9963	0.9989	0.9996
	2	2.0	0.0373	0.6716	0.9312	0.9829	0.9947	0.9981
0.01	5	0.7	0.4672	0.9631	0.9956	0.9991	0.9998	0.9999
	4	0.8	0.4547	0.9605	0.9952	0.9990	0.9997	0.9999
	3	1.0	0.3821	0.9461	0.9932	0.9986	0.9996	0.9999
	2	1.2	0.4652	0.9578	0.9947	0.9989	0.9997	0.9999
	2	1.5	0.2139	0.8852	0.9829	0.9963	0.9989	0.9996
	2	2.0	0.0373	0.6716	0.9312	0.9829	0.9947	0.9981

Table 4: Optimal group size for the proposed plan when a = 2, δ = 2, m = 3, r = 4, c = 2

β	g	a	2	4	6	8	10	12
0.25	3	0.7	0.7775	0.9911	0.9991	0.9998	1.0000	1.0000
	3	0.8	0.6508	0.9821	0.9980	0.9996	0.9999	1.0000
	2	1.0	0.6716	0.9829	0.9981	0.9996	0.9999	1.0000
	2	1.2	0.4652	0.9578	0.9947	0.9989	0.9997	0.9999
	2	1.5	0.2139	0.8852	0.9829	0.9963	0.9989	0.9996
	1	2.0	0.4857	0.9476	0.9927	0.9985	0.9996	0.9998
0.10	4	0.7	0.6183	0.9797	0.9977	0.9996	0.9999	1.0000
	3	0.8	0.6508	0.9821	0.9980	0.9996	0.9999	1.0000
	2	1.0	0.6716	0.9829	0.9981	0.9996	0.9999	1.0000
	2	1.2	0.4652	0.9578	0.9947	0.9989	0.9997	0.9999
	2	1.5	0.2139	0.8852	0.9829	0.9963	0.9989	0.9996
	2	2.0	0.0373	0.6716	0.9312	0.9829	0.9947	0.9981
0.05	4	0.7	0.6183	0.9797	0.9977	0.9996	0.9999	1.0000
	4	0.8	0.4547	0.9605	0.9952	0.9990	0.9997	0.9999
	3	1.0	0.3821	0.9461	0.9932	0.9986	0.9996	0.9999
	2	1.2	0.4652	0.9578	0.9947	0.9989	0.9997	0.9999
	2	1.5	0.2139	0.8852	0.9829	0.9963	0.9989	0.9996
	2	2.0	0.0373	0.6716	0.9312	0.9829	0.9947	0.9981
0.01	5	0.7	0.4672	0.9631	0.9956	0.9991	0.9998	0.9999
	4	0.8	0.4547	0.9605	0.9952	0.9990	0.9997	0.9999
	3	1.0	0.3821	0.9461	0.9932	0.9986	0.9996	0.9999
	3	1.2	0.1788	0.8786	0.9821	0.9961	0.9989	0.9996
	2	1.5	0.2139	0.8852	0.9829	0.9963	0.9989	0.9996
	2	2.0	0.0373	0.6716	0.9312	0.9829	0.9947	0.9981

Table 5: The minimum mean ratios for the proposed plan when a = 2, δ = 2, m = 2 and producer's risk 5%

			a					
β	r	c	0.7	0.8	1.0	1.2	1.5	2.0
0.25	4	2	2.0	2.2	2.2	2.6	2.0	2.7
	5	3	1.8	2.0	2.0	2.4	3.0	4.0
	6	4	1.7	1.9	1.8	2.2	2.8	3.7
	7	5	1.6	1.8	1.7	2.1	2.6	3.2
	8	6	1.5	1.7	1.7	2.0	2.5	3.5
	9	7	1.5	1.7	1.6	2.0	2.5	3.3
	10	8	1.5	1.7	1.6	1.9	2.4	3.2
0.10	4	2	2.0	2.2	2.2	2.6	3.3	2.7
	5	3	2.1	2.0	2.0	2.4	3.0	2.3
	6	4	2.0	1.9	1.8	2.2	2.8	3.7
	7	5	1.9	1.8	1.7	2.1	2.6	3.2
	8	6	1.8	1.7	1.7	2.0	2.5	3.5
	9	7	1.8	1.7	1.6	2.0	2.5	3.3
	10	8	1.7	1.7	1.6	1.9	2.4	3.2
0.05	4	2	2.3	2.2	2.8	2.6	3.3	4.4
	5	3	2.1	2.0	2.5	2.4	3.0	4.0
	6	4	2.0	1.9	2.4	2.2	2.8	3.7
	7	5	1.9	1.8	2.3	2.1	2.6	3.2
	8	6	1.8	1.7	2.2	2.0	2.5	3.5
	9	7	1.8	1.7	2.1	2.0	2.5	3.3
	10	8	1.7	1.7	2.1	1.9	2.4	3.2
0.01	4	2	2.6	2.6	2.8	2.6	3.3	4.4
	5	3	2.4	2.4	2.5	2.4	3.0	4.0
	6	4	2.2	2.2	2.4	2.2	2.8	3.7
	7	5	2.1	2.2	2.3	2.1	2.6	3.2
	8	6	2.1	2.1	2.2	2.0	2.5	3.5
	9	7	2.0	2.0	2.1	2.0	2.5	3.3
	10	8	2.0	2.0	2.1	1.9	2.4	3.2

ILLUSTRATIVE EXAMPLE

If an experimenter would like to establish are submitted group acceptance sampling plan and he has the facility to accommodate more than one product for inspection. Suppose that lifetime of the products follows the generalized Pareto distribution with shape parameter a = 2, δ = 2. The experimenter wants to adopt the proposed group sampling plan having r = 4, c = 2, β = 0.25 for resubmitted lot with m = 2. Let the test termination time schedule ratio be a = 0.7 and specified average life, μ_0, of product is 1000 h for this experiment. From Table 1, performing the original inspection by selecting a sample of size 12 and distributing 4 products to each tester, accept the lots if the number of failures from 3 groups is at most 2 at the end of the experiment time of 7000 h. If the number of failures from 3 groups are larger than 2, the product is not accepted and apply the proposed re-submitted acceptance sampling plan for the 2nd time. Accept the

Table 6: The minimum mean ratios for the proposed plan when $a = 2$, $\delta = 2$, m = 3 and producer's risk 5%

			a					
β	r	c	0.7	0.8	1.0	1.2	1.5	2.0
	4	2	1.7	1.9	1.9	2.3	2.8	2.3
	5	3	1.5	1.8	1.7	2.1	2.6	2.0
	6	4	1.5	1.7	1.6	2.0	2.5	1.8
0.25	7	5	1.4	1.6	1.6	1.9	2.4	3.2
	8	6	1.4	1.6	1.5	1.8	2.3	3.1
	9	7	1.4	1.6	1.5	1.8	2.2	3.0
	10	8	1.3	1.5	1.4	1.7	2.2	2.9
	4	2	2.0	1.9	1.9	2.3	2.8	3.8
	5	3	1.8	1.8	1.7	2.1	2.6	3.5
	6	4	1.7	1.7	1.6	2.0	2.5	3.3
0.10	7	5	1.7	1.6	1.6	1.9	2.4	3.2
	8	6	1.6	1.6	1.5	1.8	2.3	3.1
	9	7	1.6	1.6	1.5	1.8	2.2	3.0
	10	8	1.6	1.5	1.4	1.7	2.2	2.9
	4	2	2.0	2.3	2.4	2.3	2.8	3.8
	5	3	1.8	2.1	2.2	2.1	2.6	3.5
	6	4	1.7	2.0	2.1	2.0	2.5	3.3
0.05	7	5	1.7	1.9	2.0	1.9	2.4	3.2
	8	6	1.6	1.9	2.0	1.8	2.3	3.1
	9	7	1.6	1.9	2.0	1.8	2.2	3.0
	10	8	1.6	1.8	1.9	1.7	2.2	2.9
	4	2	2.2	2.3	2.4	2.9	2.8	3.8
	5	3	2.1	2.1	2.2	2.7	2.6	3.5
0.01	6	4	2.0	2.0	2.1	2.6	2.5	3.3
	7	5	1.9	1.9	2.0	2.5	2.4	3.2
	8	6	1.9	1.9	2.0	2.4	2.3	3.1
	9	7	1.8	1.9	2.0	2.4	2.2	3.0
	10	8	1.8	1.8	1.9	2.3	2.2	2.9

Table 7: Comparisons of sample size 'n' when β = 0.05, c = 4

β	a	Existing Plan. Rao (2009)	Existing Plan. Aslam *et al.* (2010)	Existing Plan. Rao *et al.* (2014)	Proposed plan for the resubmitted lot
0.05	0.7	180	384	42	24
	0.8	99	180	42	18
	1.0	45	60	36	18
	1.2	27	30	36	12
	1.5	18	18	30	12

lot, if the number of failures is not more than 2 from the second sample; otherwise reject the same lot. According to this plan, the probability of lot acceptance when mean ratio is 8, is 0.9998 as obtained from Table 3. The minimum mean ratio for the referenced plan from Table 5 is 2.0. Then producer needs to improve the quality level of the product 2.0 times of 1000 h.

COMPARISON AND CONCLUSION

The proposed plan is an extension from the ordinary group sampling plan under a truncated life test. In this section, comparisons between the proposed plan and the existing plan developed by Rao (2009), Aslam *et al*. (2010c) and Rao *et al*. (2014) are discussed. The design parameters for these group acceptance sampling plans are shown in Table 7 for comparison purpose. The comparison could be generalized to any other cases by a computer program which is available from the authors. From Table 7, it is to be noticed that the proposed plan requires a very small sample size$n$ than theordinary group sampling plans. It is concluded that the proposed plan reduces the sample size as compared with the established plans with the same conditions and also meets the economic criteria in life testing. In life testing, cost and time of the experiment are directly attached with the required sample size. We recommend the industrial practitioners to adopt the proposed plan in order to save the cost, time and energy of the submitted products. This study can also be extended to other lifetime distributions using the cost model and Bayesian approach.

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