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Research Article

Numerical Treatment of Analytic Functions via Mixed Quadrature Rule

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Abstract: A mixed quadrature rule of degree of precision seven by using Birkhoff-Young and Cleanshaw Curtis five point rules is formed. The mixed rule has been tested and found to be more effective than, that of its constituent rules for the numerical evaluation of an analytic function over a line segment in the complex plane. An asymptotic error estimate of the rule has been determined and the rule has been numerically verified.

Keywords: Analytic function, asymptotic error, numerical integration, quadrature rule

INTRODUCTION

There are several rules for the numerical evaluation of real definite integral:

$$\int_{-1}^{1} f(x)dx \tag{1}$$

However, there are only few quadrature rules for evaluating an integral type:

$$I(f) = \int_{L} f(z)dz \tag{2}$$

where, L is a directed line segment from the point $(z_0 - h)$ to $(z_0 + h)$ in the complex plane and f (z) is analytic in certain domain Ω containing the line segment L. Birkhoff and Young (1950) have derived the following interpolatory type of quadrature rule:

$$R_{BY}(f) = \frac{h}{15}\begin{bmatrix} 24f(z_0) + 4\{f(z_0+h) + f(z_0-h)\} \\ -\{f(z_0+ih) + f(z_0-ih)\} \end{bmatrix} \tag{3}$$

Lether (1976) using the transformation z = (z_0 + th), where $t \in [-1, 1]$ transform the integral (2) to the integral:

$$h\int_{-1}^{1} f(z_0 + th)dt \tag{4}$$

and then made an approximation of this integral by applying standard quadrature rule meant for the approximation evaluation of real definite integral (1). Das and Pradhan (1996) have constructed a quadrature rule for the numerical evaluation of the integral (1) from two standard quadrature rule of different type but equal precision, such a rule is termed as mixed quadrature rule. Jena and Dash (2009), Dash and Jena (2008) have constructed some mixed quadrature rules for analytic functions. In this study we desire a mixed quadrature rule of precision seven in the same vein for the approximation of the integral (2).

FORMULATION OF THE RULE

$$R_{CC5}(f) = \frac{h}{15}\begin{bmatrix} f(z_0+h) + f(z_0-h) + \\ 8f\left(z_0 + \frac{h}{\sqrt{2}}\right) + 8f\left(z_0 - \frac{h}{\sqrt{2}}\right) + 12f(z_0) \end{bmatrix} \tag{5}$$

For the construction of the desired mixed quadrature rule of degree of precision seven, we choose the rule (3) and (5) each of precision five. Denoting the truncation error, E_{RBY} (f) and E_{RCC5} (f) due to rule (3) and (5), respectively in approximating the integral (2) we have:

$$I(f) = R_{BY}(f) + E_{RBY}(f) \tag{6}$$

and,

$$I(f) = R_{CC5}(f) + E_{RCC5}(f) \tag{7}$$

where, f is infinitely differentiable since it is assumed to be analytic in certain domain containing the line segment L. So by using Taylor's series expansion the truncation error associated with the quadrature rules under reference can be expressed as:

$$E_{RBY}(f) = \frac{-1}{1890}h^7 f^{vi}(z_0) - \frac{1}{226800}h^9 f^{viii}(z_0)\ldots\ldots\ldots\ldots \tag{8}$$

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Table 1: Mixed quadrature rule R_{BYCC5} (f)

Quadrature rule	Approximation value I_1	Approximation value I_2	Approximation value I_3
R_{BY} (f)	1.682417145154309 i	2.350936031119045 i	0.654389151885734 i
R_{CC5} (f)	1.682967877663091 i	2.350375376931479 i	0.654389405660818 i
R_{BYCC5} (f)	1.682941652305530 i	2.350402074749935 i	0.654389393576290 i
Exact value	1.682941969615793 i	2.350402387287603 i	0.654389393592304 i

$$E_{RCC5}(f) = \frac{2}{105\times 6!}h^7 f^{vi}(z_0) + \frac{1}{45\times 8!}h^9 f^{viii}(z_0) + ... \quad (9)$$

Multiplying ($\frac{1}{20}$) in Eq. (6) and adding it to Eq. (7):

$$I(f) = \frac{1}{21}[20R_{CC5}(f) + R_{BY}(f)] + \frac{1}{21}[20E_{RCC5}(f) + E_{RBY}(f)] \quad (10)$$

where,

$$R_{\text{BYCC5}}(f) = \frac{1}{21}[20R_{CC5}(f) + R_{BY}(f)] \quad (11)$$

$$E_{RBYCC5}(f) = \frac{1}{21}[20E_{RCC5}(f) + E_{RBY}(f)] \quad (12)$$

where, Eq. (11) is the desired mixed quadrature rule of degree of precision seven.

Error analysis: Let f (z) is analytic in the disc:

$$\Omega_R = \{z : |z - z_0| \le R > |h|\}$$

So that the point z_0, (z_0±h), (z_0±ih) all are interior to the disc Ω_R. Now using Taylor's expansion:

$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n$$

where,

$$a_n = \frac{1}{n!} f^n(z_0)$$

In (12), we obtain after simplification:

$$E_{RBYCC5}(f) = \frac{1}{75\times 8!}h^9 f^{viii}(z_0) + ... \quad (13)$$

Thus the mixed quadrature rule R_{BYCC5} (f) is of precision seven.

Theorem 1: If *f* is assumed to be analytic in domain Ω containing *L* then:

$$E_{RBYCC5}(f) = O(h^9)$$

Error comparison: From (8) and (9):

$$|E_{RCC5}(f)| \le |E_{RBY}(f)| \quad (14)$$

And from (13):

$$|E_{RBYCC5}(f)| \le |E_{RCC5}(f)| \quad (15)$$

From (14) and (15):

$$|E_{RBYCC5}(f)| \le |E_{RCC5}(f)| \le |E_{RBY}(f)| \quad (16)$$

NUMERICAL VERIFICATION

Let us approximate the value of the following integrals I_1, I_2, I_3 using $R_{BY}(f)$, $R_{CC5}(f)$, $R_{BYCC5}(f)$ quadrature rule.
where,

$$I_1 = \int_{-i}^{i} e^z \, dz, \quad I_2 = \int_{-i}^{i} \cos z \, dz, \quad I_3 = \int_{\frac{-i}{3}}^{\frac{i}{3}} \cosh z \, dz$$

CONCLUSION

From the Table 1 it is evident that the mixed quadrature rule $R_{BYCC5}(f)$ of degree of precision seven is giving us better result than the constituent rules $R_{BY}(f)$, $R_{CC5}(f)$ each of degree of precision five. Our quadrature rule is more efficient and numerically better convergent to exact result.

REFERENCES

Birkhoff, G. and D. Young, 1950. Numerical quadrature of analytic and harmonic functions. J. Math. Phys., 29: 217.

Das, R.N. and G. Pradhan, 1996. A mixed quadrature rule for approximate evaluation of real definite integrals. Int. J. Math. Educ. Sci., 27: 279-283.

Dash, R.B. and S.R. Jena, 2008. A mixed quadrature of modified Birkhoff-Young using Richardson extrapolation and Gauss-Legendre-4 point transformed rule. Int. J. Appl. Math. Appl., 2: 111-117.

Jena, S.R. and R.B. Dash, 2009. Mixed quadrature of real definite integrals over triangles. Paci-Asian J. Math., 3: 119-124.

Lether, F.G., 1976. On Birkhoff-Young qadrature of analytic functions. J. Comut. Appl. Math., 2: 81.