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Research Article

Time Truncated Group Chain Sampling Strategy for Pareto Distribution of the 2nd Kind

Abdur Razzaque Mughal, Zakiyah Zain and Nazrina Aziz
Department of Mathematics and Statistics, School of Quantitative Sciences, UUM College of Arts and Sciences, Universiti Utara Malaysia, UUM Sintok 06010, Kedah, Malaysia

Abstract: The main goal of this study is to propose a chain Group Acceptance Sampling Plan (GASP) when the life time of a product follows the Pareto distribution of the 2nd kind. The design parameters such as the minimum group size and operating characteristic values are obtained by satisfying the producer's and consumer's risks at the specified quality level, acceptance number and the test termination time. A classical example is considered in order to compare the proposed plan with some existing plans.

Keywords: Consumer's risk, group chain sampling, operating characteristic values, pareto distribution of the2nd kind, producer's risk

INTRODUCTION

Acceptance sampling is a useful technique for quality assurance assertion. It is recommended when sample size is essential towards excessive testing for product quality. When an acceptance sampling plan is considered, the lot is either accepted or rejected along with affiliated producer's and consumer's risks. In a life testing plan, suppose $n = (r \times g)$ products are allocated in time truncated test and the number of failure are recorded at a prefix time, a. If the failed products are less than the specified acceptance number, then the same submitted lot will be acceptable for consumer use. In chain sampling technique, the criteria for accepting and rejecting the submitted lot depend on the information of the inspection of immediately preceding samples. The use of chain sampling plan is usually suggested when an extremely high quality product is needed. In established chain sampling plans, a single product is tested but in practice, testers are accessible that can contain more than one product. Thus, the researchers can use the group chain sampling plan to monitor the multiple products at the same time.

Dodge (1955) developed a chain sampling plan and also discussed the importance of this plan. Many investigators introduced acceptance sampling plans based on truncated life test for different distributions: Epstein (1954), Goode and Kao (1961), Baklizi (2003), Tsai and Wu (2006), Balakrishnan *et al*. (2007), Aslam *et al*. (2010a), (2010b), Mughal *et al*. (2011), Mughal and Aslam (2011), Mughal (2011), Aslam *et al*. (2011) and Mughal and Ismail (2013). Recently, Ramaswamy and Jayasri (2014) designed time truncated chain sampling plans for Generalized Rayleigh distribution. As in truncated life testing, the Pareto distribution of the 2nd kind is generally used for fitting failure time distributions and to the best of our understanding, none has constructed the group chain sampling for this distribution. The intention of this research is to attain the optimal sample sizes and operating characteristic values considering a choice of various plan parameters.

METHODOLOGY OF GROUP CHAIN SAMPLING

Dodge (1955) proposed a chain sampling plan (ChSP-1) using the cumulative information of various random samples to overcome the drawback of the single acceptance sampling plan. In chain sampling, it is interesting to note that the current lot under assessment can also be accepted if one defective product is recorded in the sample provided that no other defective product is found in the subsequent lots. The proposed plan is applied in the following steps:

- For each lot, find the optimal number of g groups and allocate r items to each group such that the required sample size is $n = r \times g$.
- Accept the lot when $d = 0$ and reject the lot if $d > 1$.
- Accept the lot if $d = 1$ and continue the inspection if no defectives are found in the preceding i lots.

The group chain sampling plan is characterized by the designed parameters g and i. We are interested in searching the optimal group size required for in the case of Pareto distribution of the 2nd kind. The probability α

Corresponding Author: Zakiyah Zain, Department of Mathematics and Statistics, School of Quantitative Sciences, UUM College of Arts and Sciences, Universiti Utara Malaysia, UUM Sintok 06010, Kedah, Malaysia

Table 1: Number of optimal groups required for the proposed plan for the Pareto distribution of the 2nd kind with λ = 2

			a					
β	*r*	*i*	0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.10	2	1	2	2	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.05	2	1	2	2	2	2	1	1
	3	2	2	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.01	2	1	3	2	2	2	2	2
	3	2	2	2	2	1	1	1
	4	3	2	1	1	1	1	1
	5	4	1	1	1	1	1	1

Table 2: Number of optimal groups required for the proposed plan for the Pareto distribution of the 2nd kind with λ = 3

			a					
β	*r*	*i*	0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.10	2	1	2	2	2	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.05	2	1	2	2	2	2	2	1
	3	2	2	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.01	2	1	3	3	2	2	2	2
	3	2	2	2	2	2	1	1
	4	3	2	2	1	1	1	1
	5	4	2	1	1	1	1	1

Table 3: Number of optimal groups required for the proposed plan for the Pareto distribution of the 2nd kind with λ = 4

			a					
β	*r*	*i*	0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	1	2	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.10	2	1	2	2	2	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.05	2	1	2	2	2	2	2	1
	3	2	2	2	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.01	2	1	3	3	2	2	2	1
	3	2	2	2	2	2	1	1
	4	3	2	2	2	1	1	1
	5	4	2	1	1	1	1	1

and β denote the producer's and consumer's risks, respectively. In literature, the consumer risk is often considered the consumer's confidence level. If the confidence level is α, then the consumer's risk can be written as $\beta = 1 - \alpha$. We will determine the optimal group sizes such that the consumer's risk does not exceed a specified value β. According to Pareto (1897), the Cumulative Distribution Function (CDF) of a Pareto distribution of the 2nd kind can be written as:

$$F(t;\sigma,\lambda) = 1 - \left(1 + \frac{t}{\sigma}\right)^{-\lambda} \quad t > 0, \sigma > 0, \lambda > 0 \tag{1}$$

where, σ and λ are scale and shape parameters respectively. The probability of lot acceptance in the case of group chain sampling plan is given by:

$$P_a(p) = (1-p)^{(r*g)} + (r*g)(p)(1-p)^{(r*g)-1}(1-p)^{(r*g)*i} \tag{2}$$

where, p is the probability of failure of a product during the test termination time, t_0, which is a multiple of the specified mean life, μ_0 and the pre-assumed constant 'a'. Then p can be estimated by:

$$p = F(t;\sigma,\lambda) = 1 - \left[1 + \frac{a}{(\lambda-1)(\mu/\mu_0)}\right]^{-\lambda} \tag{3}$$

In Table 1 to 3, we present the optimal values of groups g when satisfying the Eq. (2) for β = 0.25,0.10,0.05,0.01; a = 0.7, 0.8, 1.0, 1.2, 1.5, 2.0; r = 2(1)5 and i = 1(1)4. Once the optimal group sizes are found, one may be interested to find the probability of lot acceptance when the desired quality level of a product is needed. For a fixed r and i, the operating characteristic values as a function of (μ/μ_0) are shown in Table 4 to 6.

Description of tables and applications in industry: Suppose μ and μ_0 are the true and specified average life of a product, respectively. Then it is acceptable for consumer use when $\mu > \mu_0$. Assume that the life time distribution is Pareto distribution of the 2nd kind and the experimenter is interested in knowing that the true average life of a product is at least 1000 h with confidence level of 0.99. Furthermore, he has the capacity to install more than one product on a tester. If the experimenter would like to select the test termination time for a = 0.70, number of testers r = 3, i = 2 and λ = 2, then the required optimal group size is g = 2, as shown in Table 1. The designed parameters of the proposed group chain sampling are (a, r, i, g) = (0.7, 3, 2, 2). Thus, the practitioner needs to select a random sample of size six products from the lot and put three products to each of the two groups. The lot will be accepted if not more than one failure is observed within 700 h and no defective product is found in the next two samples. For the same design parameters, the probability of lot acceptance increases from 0.0274 to 0.6000 when the mean ratio increases from 2 to 12, as depicted in Table 4.

Table 4: Operating characteristics values having $i = 2$, $r = 3$ for Pareto distribution of the 2nd kind with $\lambda = 2$

β	g	*a*	2	4	6	8	10	12
0.25	1	0.7	0.1763	0.4426	0.6174	0.7256	0.7950	0.8415
	1	0.8	0.1396	0.3845	0.5616	0.6778	0.7551	0.8083
	1	1.0	0.0903	0.2925	0.4641	0.5889	0.6778	0.7419
	1	1.2	0.0609	0.2256	0.3845	0.5105	0.6059	0.6778
	1	1.5	0.0351	0.1566	0.2925	0.4124	0.5105	0.5889
	1	2.0	0.0157	0.0903	0.1911	0.2925	0.3845	0.4641
0.10	1	0.7	0.1763	0.4426	0.6174	0.7256	0.7950	0.8415
	1	0.8	0.1396	0.3845	0.5616	0.6778	0.7551	0.8083
	1	1.0	0.0903	0.2925	0.4641	0.5889	0.6778	0.7419
	1	1.2	0.0609	0.2256	0.3845	0.5105	0.6059	0.6778
	1	1.5	0.0351	0.1566	0.2925	0.4124	0.5105	0.5889
	1	2.0	0.0157	0.0903	0.1911	0.2925	0.3845	0.4641
0.05	2	0.7	0.0274	0.1513	0.2939	0.4190	0.5201	0.6000
	1	0.8	0.1396	0.3845	0.5616	0.6778	0.7551	0.8083
	1	1.0	0.0903	0.2925	0.4641	0.5889	0.6778	0.7419
	1	1.2	0.0609	0.2256	0.3845	0.5105	0.6059	0.6778
	1	1.5	0.0351	0.1566	0.2925	0.4124	0.5105	0.5889
	1	2.0	0.0157	0.0903	0.1911	0.2925	0.3845	0.4641
0.01	2	0.7	0.0274	0.1513	0.2939	0.4190	0.5201	0.6000
	2	0.8	0.0177	0.1159	0.2415	0.3594	0.4596	0.5419
	2	1.0	0.0077	0.0698	0.1657	0.2663	0.3594	0.4411
	1	1.2	0.0609	0.2256	0.3845	0.5105	0.6059	0.6778
	1	1.5	0.0351	0.1566	0.2925	0.4124	0.5105	0.5889
	1	2.0	0.0157	0.0903	0.1911	0.2925	0.3845	0.4641

Table 5: Operating characteristics values having $i = 2$, $r = 3$ for Pareto distribution of the 2nd kind with $\lambda = 3$

β	g	*a*	2	4	6	8	10	12
0.25	1	0.7	0.2582	0.5592	0.7207	0.8096	0.8626	0.8964
	1	0.8	0.2097	0.4998	0.6716	0.7713	0.8325	0.8724
	1	1.0	0.1411	0.3993	0.5804	0.6959	0.7713	0.8223
	1	1.2	0.0973	0.3202	0.4998	0.6247	0.7107	0.7713
	1	1.5	0.0578	0.2325	0.3993	0.5287	0.6247	0.6959
	1	2.0	0.0261	0.1411	0.2772	0.3393	0.4998	0.5804
0.10	1	0.7	0.2582	0.5592	0.7207	0.8096	0.8626	0.8964
	1	0.8	0.2097	0.4998	0.6716	0.7713	0.8325	0.8724
	1	1.0	0.1411	0.3993	0.5804	0.6959	0.7713	0.8223
	1	1.2	0.0973	0.3202	0.4998	0.6247	0.7107	0.7713
	1	1.5	0.0578	0.2325	0.3993	0.5287	0.6247	0.6959
	1	2.0	0.0261	0.1411	0.2772	0.3393	0.4998	0.5804
0.05	2	0.7	0.0555	0.2395	0.4125	0.5441	0.6401	0.7104
	1	0.8	0.2097	0.4998	0.6716	0.7713	0.8325	0.8724
	1	1.0	0.1411	0.3993	0.5804	0.6959	0.7713	0.8223
	1	1.2	0.0973	0.3202	0.4998	0.6247	0.7107	0.7713
	1	1.5	0.0578	0.2325	0.3993	0.5287	0.6247	0.6959
	1	2.0	0.0261	0.1411	0.2772	0.3393	0.4998	0.5804
0.01	2	0.7	0.0555	0.2395	0.4125	0.5441	0.6401	0.7104
	2	0.8	0.0378	0.1914	0.3522	0.4834	0.5837	0.6598
	2	1.0	0.0180	0.1244	0.2584	0.3811	0.4834	0.5658
	2	1.2	0.0089	0.0825	0.1914	0.3013	0.3996	0.4834
	1	1.5	0.0578	0.2325	0.3993	0.5287	0.6247	0.6959
	1	2.0	0.0261	0.1411	0.2772	0.3393	0.4998	0.5804

Table 6: Operating characteristics values having $i = 2$, $r = 3$ for Pareto distribution of the 2nd kind with $\lambda = 4$

β	g	*a*	2	4	6	8	10	12
0.25	1	0.7	0.2974	0.6056	0.7579	0.8381	0.8846	0.9138
	1	0.8	0.2442	0.5475	0.7125	0.8038	0.8583	0.8931
	1	1.0	0.1672	0.4461	0.6261	0.7351	0.8038	0.8494
	1	1.2	0.1167	0.3637	0.5475	0.6685	0.7488	0.8038
	1	1.5	0.0701	0.2693	0.4461	0.35760	0.6685	0.7351
	1	2.0	0.0319	0.1672	0.3179	0.4461	0.5475	0.6261
0.10	1	0.7	0.2974	0.6056	0.7579	0.8381	0.8846	0.9138
	1	0.8	0.2442	0.5475	0.7125	0.8038	0.8583	0.8931
	1	1.0	0.1672	0.4461	0.6261	0.7351	0.8038	0.8494
	1	1.2	0.1167	0.3637	0.5475	0.6685	0.7488	0.8038
	1	1.5	0.0701	0.2693	0.4461	0.35760	0.6685	0.7351
	1	2.0	0.0319	0.1672	0.3179	0.4461	0.5475	0.6261
0.05	2	0.7	0.0719	0.2823	0.4637	0.5939	0.6851	0.7500
	2	0.8	0.0501	0.2294	0.4020	0.5346	0.6319	0.7034
	1	1.0	0.1672	0.4461	0.6261	0.7351	0.8038	0.8494
	1	1.2	0.1167	0.3637	0.5475	0.6685	0.7488	0.8038
	1	1.5	0.0701	0.2693	0.4461	0.35760	0.6685	0.7351
	1	2.0	0.0319	0.1672	0.3179	0.4461	0.5475	0.6261
0.01	2	0.7	0.0719	0.2823	0.4637	0.5939	0.6851	0.7500

Table 6: Continue

2	0.8	0.0501	0.2294	0.4020	0.5346	0.6319	0.7034
2	1.0	0.0248	0.1536	0.3027	0.4317	0.5346	0.6148
2	1.2	0.0126	0.1044	0.2294	0.3486	0.4506	0.5346
1	1.5	0.0701	0.2693	0.4461	0.35760	0.6685	0.7351
1	2.0	0.0319	0.1672	0.3179	0.4461	0.5475	0.6261

COMPARISON AND CONCLUSION

A comparative study was accomplished regarding sample sizes n of the proposed plan and those from the existing plan developed by Ramaswamy and Jayasri (2014). We consider the earlier example for this purpose. The proposed plan needs 6 $(n = r \times g)$ products while the existing plan requires 24 products to attain a similar conclusion about the submitted lot. Hence, the proposed plan meets the economic criteria in life testing because cost, time, energy and labor of the experiment are directly involved with the number of products being tested. We recommend using the proposed plan in order to save the cost and time of the experiment to reach the same decision as the established plans. Our proposed plan can be used to test the lifetime of many electronic components such as the transportation electronics system, wireless devices, global positioning systems and computer aided and integrated manufacturing systems. It can be further explored for many other lifetime distributions, as well as other quality and reliability characteristics.

GLOSSARY OF SYMBOLS

g : Number of groups
r : Number of testers
n : Sample size
d : Number of defective products
i : Allowable acceptance number
α : Producer's risk (Probability of rejecting a good lot)
β : Consumer's risk (Probability of accepting a bad lot)
λ : Shape parameter of Pareto distribution of the 2nd kind
t_0 : Test termination time
μ_0 : Specified average life of a product
μ : True average life of a product
$P(p)$: Lot Acceptance Probability
(μ/μ_0) : Mean ratio

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