Research Journal of Applied Sciences, Engineering and Technology 11(1): 1-9, 2015
DOI: 10.19026/rjaset.11.1668
ISSN: 2040-7459; e-ISSN: 2040-7467

Submitted: October 12, 2014 Accepted: November 3, 2014 Published: September 05, 2015

Research Article

Theory of Breakdown of an Arbitrary Gas-dynamic Discontinuity-the Methods of the Riemann Problem Solution

[1]Pavel Viktorovich Bulat, [1,2]Konstantin Nikolaevich Volkov, [3]Mikhail Vladimirovich Silnikov and [3]Mikhail Viktorovich Chernyshev
[1]University ITMO, Kronverksky Pr., 49, Saint-Petersburg, 197101, Russia
[2]Kigston University, London
[3]Saint-Petersburg State Politechnical University, 29 Politekhnicheskaya Str., Saint-Petersburg 195251, Russia

Abstract: We have considered the modern theory of breakdown of an arbitrary gas-dynamic discontinuity for the space-time dimension equal to two. We consider the Riemann problem of the breakdown of one-dimensional discontinuity of parameters of non-stationary gas flow in application to construction of numerical methods like the Godunov method. The problem is solved as accurate stated and as rough stated (Osher-Solomon difference scheme used in the numerical methods of shock-cupturing): the intensities are determined (static pressure relations) and the flow velocity step on the sides of formed discontinuities and waves, then the other parameters are calculated in all flow areas. We give the classification of the difference schemes using the Riemann problem solution. We compared the results of model flows by means of accurate and rough solutions.

Keywords: Computational gas dynamics, contact discontinuity, discontinuity breakdown scheme, Riemann wave, shock-wave

INTRODUCTION

The classic presentation of the solution of the Riemann problem of breakdown of an arbitrary discontinuity of parameters of the flow for one dimensional waves and discontinuities is given in 2001 in the work of (Igra, 2001). Irrespectively of this work, the complete study of the regions of existence of different alternatives of the discontinuity one-dimensional breakdown in 2000 was carried out by conducted (Uskov, 2000). The problem is met in different technical appendices, for example, a discontinuity arises on the wall at a shock-wave reflection from it (Arkhipova and Uskov, 2013). So, there is a discontinuity of velocity equal to the flow rate following the shock-wave, as on the wall there is a stagnation of flow up to the zero velocity. In this case, similarly to some others, it is convenient to use a concept of dimensionless velocity function of the wave intensity describing a velocity jump on the wave (Arkhipova, 2012). If the wave distributes onto medium at rest, the given parameter is equal to the velocity of the concurrent flow following the wave.

Great attention was paid to the theory of breakdown of an arbitrary discontinuity as a result of development of the numerical methods such as the Godunov method (Godunov, 1959; Godunov *et al*., 1976), where a jump of parameters at the border of difference cells is considered as a discontinuity. The exact solutions reduced to a system of transcendental equations solvable by iteration methods are received. In a number of cases, for example at explosion modeling in real time (Gelfand *et al*., 2001; Silnikov and Mikhaylin, 2014), essential reduction of computation time is necessary. Therefore, the actual solution are approximate like the Osher-Solomon difference scheme (Osher and Solomon, 1982) using the fact that far from shock-waves discontinuities at the cell border, as a rule, are weak. One knows that the isentropic polar of compression and the shock polar at the intensity equal to one have the second order of contact, that allows to receive a simple and one-valued approximate analytical solution for the flow parameters following the outgoing discontinuity. A special speed gain in computation the approximate solutions give in case of the numerical methods of the heightened order of accuracy (Gelfand *et al*., 2001; Gelfand and Silnikov, 2002).

In such a way, the problem of study of breakdown of an arbitrary discontinuity remains actual in both accurate setting and approximate setting. It is important to know the limits of application of approximate solution methods. This study objective is determination of such limits. The problem is solved by means of comparison of the calculation results received by numerical methods in the accurate and approximate

Corresponding Author: Pavel Viktorovich Bulat, University ITMO, Kronverksky Pr., 49, Saint-Petersburg, 197101, Russia

settings. Also we give the direct computation of an error of the approximate Osher-Solomon solution using a model of isentropic compression waves. For evaluation of the dynamic pressure following a simple wave and a shock-wave, the velocity function concept is introduced. Here is given the value of wave special intensity when a simple wave and a shock-wave create the same pressure, i.e., errors of approximate methods are minimal for this case.

MATERIALS AND METHODS

Solution of one-dimensional non-stationary problem of discontinuity breakdown in terms of the velocity function: With the help of idea of dimensionless function and velocity function of the wave intensity, it is possible to write down the universal expression for determination of the gas flow velocity following the progressive wave:

$$\hat{u} = u_0 + \chi_W a_0 U_W (J) \tag{1}$$

Here u_0 is velocity of the medium at rest, a_0-sonic speed in the medium at rest, χ_w-index of the wave direction. For the wave co-directional with the outgoing flow of gas, $\chi_w = 1$ and for oncoming waves $\chi_w = 1$. The velocity function for shock-waves has the appearance:

$$U_D(J) = \frac{1-\varepsilon}{\sqrt{1+\varepsilon}} \frac{J-1}{\sqrt{J+\varepsilon}} \tag{2}$$

where, $\varepsilon = (\gamma - 1)/(\gamma + 1)$, γ-gas adiabatic index. For the Riemann waves (compression waves and depression waves) the relation specifying the velocity function follows from the medium isoentropy:

$$U_R(J) = \frac{1-\varepsilon}{\varepsilon}\left(J^{1/k} - 1\right), k = 2\gamma/(\gamma - 1) \tag{3}$$

If in the result of breakdown of an arbitrary discontinuity the centered depression Riemann wave travelling to area *1* and the shock-wave traveling to area 4 (Fig. 1a) form, the conditions at the contact discontinuity can be written as follows:

$$u_1 - a_1 U_R(J_1, \varepsilon_1) = u_4 + a_4 U(J_2, \varepsilon_2) \tag{4}$$

If, vice versa, a depression wave spreads onto area 4 and a shock-wave travels to area 1, the condition of velocity equality on the contact discontinuity is written as follows:

$$u_1 - a_1 U_D(J_1, \varepsilon_1) = u_4 + a_4 U_R(J_2, \varepsilon_2) \tag{5}$$

Suppose that from the point of discontinuity two shock-waves outgo (Fig. 1b). Then the condition of

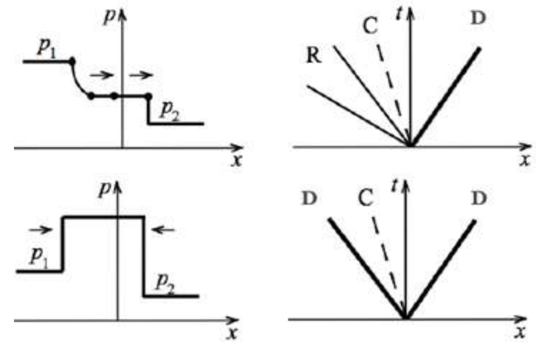


Fig. 1: Riemann problem of breakdown of an arbitrary discontinuity; (a): On the top-outgoing discontinuities; (b): On the bottom-outgoing discontinuities
D: Shock-wave; R: Riemann wave; C: Contact discontinuity; p: Pressure; t: Time

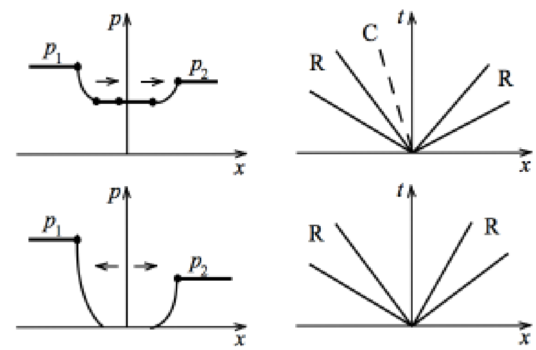


Fig. 2: Riemann problem of breakdown of an arbitrary discontinuity with outgoing waves-Riemann waves

equality of velocity on the contact discontinuity can be written as follows:

$$u_1 - a_1 U_D(J_1, \varepsilon_1) = u_4 + a_4 U_D(J_2, \varepsilon_2) \tag{6}$$

In theory, there can be two shock-wave configurations with outgoing Riemann waves (Fig. 2). There are just a few technical appendices for such cases, if any, but at plotting of difference schemes of shock-capturing they are necessary to be taken into account.

The equation system including the condition of equality of static pressures on the discontinuity sides and one of the Eq. (4)-(6), dependently on the outgoing wave types is closed relatively to the intensities of outgoing waves and is solved numerically. For solutions of this system of equations you need to specify the type of waves formed as a result of breakdown of an arbitrary discontinuity. Determination of regions of existence of the solution with different outgoing waves is the major objective of solution of the breakdown of an arbitrary discontinuity problem.

Solution of one-dimensional Riemann problem in the Godunov scheme: The conditions of equality of the flow velocities and pressures on sides of the contact discontinuity:

$$u_1 = u_2, p_1 = p_2$$

Link the wave intensities $\vec{D}_1\left(\vec{R}_1\right)$ and $\vec{D}_2\left(\vec{R}_2\right)$ with differentials $[u]_1$ and $[u]_2$ of the flow velocity on their sides.

At $J_1>1$ or $J_2>1$ differentials of velocity on the formed shock-waves are related with their intensities:

$$[u]_1 = -(1-\varepsilon)a_1\left(\sqrt{\frac{J_1+\varepsilon}{1+\varepsilon}} - \sqrt{\frac{1+\varepsilon}{J_1+\varepsilon}}\right)$$
$$[u]_2 = (1-\varepsilon)a_2\left(\sqrt{\frac{J_2+\varepsilon}{1+\varepsilon}} - \sqrt{\frac{1+\varepsilon}{J_2+\varepsilon}}\right) \quad (7)$$

Velocity change in the isentropic depression waves is specified with relations:

$$[u]_1 = \frac{2a_1}{\gamma-1}\left(1-J_1^{(\gamma-1)/2\gamma}\right)$$
$$[u]_2 = -\frac{2a_2}{\gamma-1}\left(1-J_2^{(\gamma-1)/2\gamma}\right) \quad (8)$$

If we know the velocity differences, you may specify the inverse relations J_i on $[u]_i$ for shock-waves:

$$J_1 = 1+\frac{\gamma[u]_1}{2(1-\varepsilon)a_1^2}\left([u]_1 - \sqrt{[u]_1^2 + 4(1-\varepsilon)^2 a_1^2}\right)$$
$$J_2 = 1+\frac{\gamma[u]_2}{2(1-\varepsilon)a_2^2}\left([u]_2 - \sqrt{[u]_2^2 + 4(1-\varepsilon)^2 a_2^2}\right) \quad (9)$$

And Riemann waves:

$$J_1 = \left(1-(\gamma-1)[u]_1/2a_1\right)^{2\gamma/(\gamma-1)}$$
$$J_2 = \left(1+(\gamma-1)[u]_2/2a_2\right)^{2\gamma/(\gamma-1)} \quad (10)$$

Relations E_1 and E_2 of gas densities before and after shock-waves are specified as follows (Rankin-Hugoniot adiabat):

$$E_1 = (1+\varepsilon J_1)/(J_1+\varepsilon), E_2 = (1+\varepsilon J_2)/(J_2+\varepsilon), \quad (11)$$

And for isentropic Riemann waves (Poisson adiabat):

$$E_1 = J_1^{-1/\gamma}, E_2 = J_2^{-1/\gamma} \quad (12)$$

Compared to the problem of interaction of inclined supersonic stationary flows, the system (7-12) always has the single solution received numerically.

Analytical Osher-Solomon solution for weak waves: As a rule, differences of the flow parameters on the borders of computation cells in difference methods are not big and solution discontinuities in the Riemann problem can be considered as weak. The difference Osher-Solomon scheme uses this fact and substitutes accurate setting of the Riemann problem for approximate (Kulikovsky *et al*., 2001), where formulae (8), (10) and (12) link the flow intencities, velocities and densities both on depression waves and shock waves. The problem of discontinuity breakdown in approximate setting is solved analytically:

$$J_1 = \left[\frac{(\gamma-1)(u_1-u_2)+2(a_1+a_2)}{2(a_1+a_2/I)}\right]^{2\gamma/(\gamma-1)}$$
$$J_2 = \left[\frac{(\gamma-1)(u_1-u_2)+2(a_1+a_2)}{2(a_1 I+a_2)}\right]^{2\gamma/(\gamma-1)} \quad (13)$$

$$[u]_1 = \frac{2a_2(1-I)+(\gamma-1)I(u_2-u_1)}{a_1 I+a_2}\cdot\frac{a_{j-1}}{\gamma-1}$$
$$[u]_2 = \frac{2a_1(1-I)+(\gamma-1)I(u_1-u_2)}{a_1 I+a_2}\cdot\frac{a_2}{\gamma-1} \quad (14)$$

where, $I = (p_2/p_1)^{(\gamma-1)/2\gamma}$.

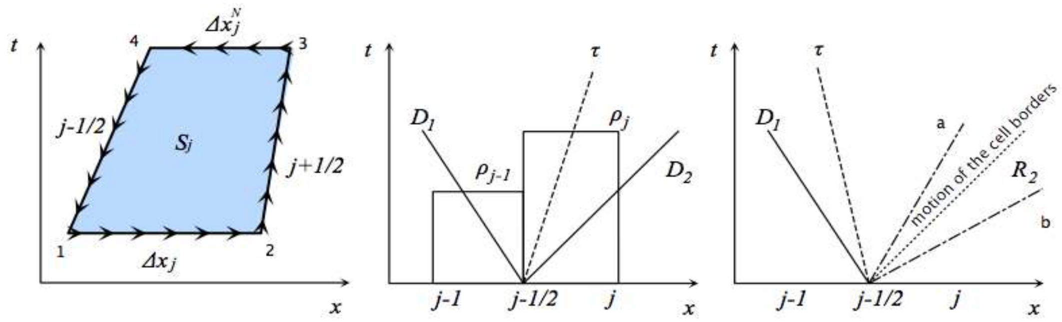
Numerical algorithm of the Godunov method: For solution of the hyperbolic system of quasilinear equations in partial derivatives written in the divergence form:

$$\frac{\partial\vec{Q}}{\partial t} + \frac{\partial\vec{F}}{\partial x} = \vec{H}$$

The primitive scheme of the first order for determination of the velocity vector $\vec{Q}_j$ in difference cell j at a new point of time t + Δt (Fig. 3a) looks as follows:

$$\Delta x_j^N \cdot \vec{Q}_j(t+\Delta t) = \Delta x_j \cdot \vec{Q}_j(t) + \left(\vec{F}_{j-1/2} - V_{j-1/2}\cdot\vec{Q}_{j-1/2}\right)$$
$$\Delta t - \left(\vec{F}_{j+1/2} - V_{j+1/2}\cdot\vec{Q}_{j+1/2}\right)\Delta t + S_j\cdot\vec{H}_j(t)$$

Here Δx_j and Δx_j^N are dimensions of cell j before and after integration step Δt, $V_{j-1/2}$ and $V_{j+1/2}$ are velocities of its border travel, $S_j = (\Delta x_j + \Delta x_j^N)\cdot\Delta t/2$ and $\vec{Q}_j(t)$ and $\vec{H}_j(t)$ are known vectors of conservative variables and source components at the initial time. The values of conservative variables ($\vec{Q}_{j-1/2}$ and $\vec{Q}_{j+1/2}$) and their flows ($\vec{F}_{j-1/2}$ and $\vec{F}_{j+1/2}$) through the cell borders need to be specified.



(a) (b) (c)

Fig. 3: Godunov method validation; (a): Difference cell; (b): Discontinuity of parameters on the cell border; (c): Breakdown of discontinuity with outgoing shock-wave D_1 and Riemann wave R_2

The major issue when constructing difference schemes of computation is a desire to heighten the approximation order and at the same time to provide a monotonic numerical solution in case of strong and weak discontinuities. In his work Godunov (Godunov, 1959) demonstrates that the monotonic difference scheme cannot have the approximation order higher than the first one. The way out the necessity of monotonic solution and heightening the approximation order is suggested in the work of (Kolgan, 1972), which sense is in creation of non-linear mechanisms providing continuous transition from the non-monotonic scheme of the second order of approximation with central differences to the monotonic scheme of the first order with one-sided differences in the grid nodes. Difference schemes with the heightened order of approximation are used in nodes with smooth numerical solution and in points where solution has discontinuities, monotonic difference schemes of low accuracy are used. The Godunov scheme has approximation viscosity, therefore, for computation of strong discontinuities there is no need to introduce artificial viscosity. When computing weak discontinuity like a depression wave, an approximation error gets high enough, which becomes apparent in their strong smearing (the lower the Courant number, the stronger smearing).

In the classic Godunov scheme values $\vec{Q}_{j-1/2}$, $\vec{F}_{j-1/2} = \vec{F}(\vec{Q}_{j-1/2})$ and similar ones are specified from the solution of the problem of breakdown the flow parameters discontinuity ($\vec{Q}_{j-1}$ and $\vec{Q}_j$, Fig. 3b) on the cell binderies. Any space coordinate can be considered as time.

After the Riemann problem is solved in accurate or approximate setting, you can calculate the velocities of all waves and discontinuity. Velocities W_1 and W_2 of movement of shock-waves $\overleftarrow{D}_1$ and $\overrightarrow{D}_2$ is specified by dependences:

$$W_1 = u_{j-1} - a_{j-1}\sqrt{(J_1+\varepsilon)/(1+\varepsilon)}$$

$$W_2 = u_j + a_j\sqrt{(J_2+\varepsilon)/(1+\varepsilon)}$$

The velocity of the contact discontinuity distribution is easily calculated (W_τ) and during formation of depression waves $\overleftarrow{R}_1$ and $\overleftarrow{R}_2$-their front (W_{1a} and W_{2a}) and rare (W_{1b} and W_{2b}) fronts (Fig. 3c):

$$W_\tau = u_1 = u_3, W_{1a} = u_{j-1} - a_{j-1}, W_{2a} = u_j + a_j,$$
$$W_{1b} = u_1 - a_1 = u_{j-1} + [u]_1 - a_{j-1} \cdot J_1^{(\gamma-1)/2\gamma},$$
$$W_{2b} = u_2 + a_2 = u_j + [u]_2 - a_j \cdot J_2^{(\gamma-1)/2\gamma}.$$

Calculated values are compared with the velocity of the cell boarder travel $V_{j-1/2}$. If the border travel velocity is lower or higher the velocity of all waves travel, the properties of a flow on it are the same as in the left ($\vec{Q}_{j-1/2} = \vec{Q}_{j-1}$) or in the right ($\vec{Q}_{j-1/2} = \vec{Q}_j$) cells before discontinuity. If the border velocity is lower than the velocity of one of waves and higher than the velocity of another one, vector $\vec{Q}_{j-1/2}$ is specified by values of physical variables (ρ_1, u_1, p_1) and (ρ_2, u_2, p_2), correspondingly.

If the border trajectory lies inside the wave characteristics fan $\overleftarrow{R}_1$ or $\overrightarrow{R}_2$, the flow target properties depends on the velocity value.

In wave $\overleftarrow{R}_1$:

$$u_{j-1/2} = (1-\varepsilon)(V_{j-1/2} + a_{j-1}) + \varepsilon u_{j-1},$$
$$a_{j-1/2} = \varepsilon\left(\frac{2a_{j-1}}{\gamma-1} - u_{j-1} - V_{j-1/2}\right),$$
$$\rho_{j-1/2} = \rho_{j-1} \cdot \left(\frac{a_{j-1/2}}{a_{j-1}}\right)^{2/(\gamma-1)},$$
$$p_{j-1/2} = p_{j-1} \cdot \left(\frac{a_{j-1/2}}{a_{j-1}}\right)^{2\gamma/(\gamma-1)},$$

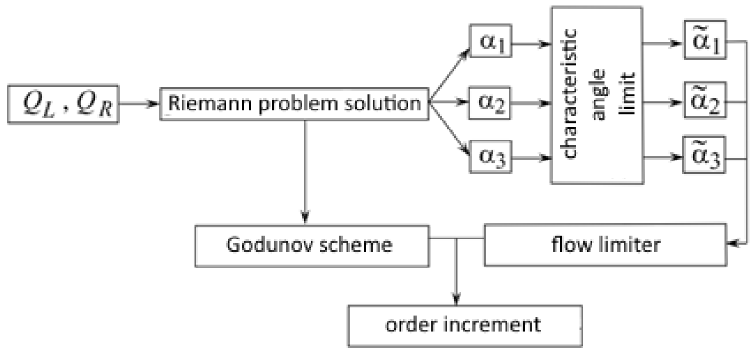


Fig. 4: Typical realization of the Godunov scheme with usage of monotonic limiters of flows

In wave $\vec{R}_2$:

$$u_{j-1/2} = (1-\varepsilon)(V_{j-1/2} - a_j) + \varepsilon u_j,$$

$$a_{j-1/2} = \varepsilon\left(\frac{2a_j}{\gamma-1} - u_j + V_{j-1/2}\right),$$

$$\rho_{j-1/2} = \rho_j \cdot \left(\frac{a_{j-1/2}}{a_j}\right)^{2/(\gamma-1)},$$

$$p_{j-1/2} = p_j \cdot \left(\frac{a_{j-1/2}}{a_j}\right)^{2\gamma/(\gamma-1)}.$$

The last relations are a consequence of the condition the Riemann invariant conservation in isentropic wave:

$$\vec{R}_1 : u + \frac{2a}{\gamma-1} = u_{j-1/2} + \frac{2a_{j-1/2}}{\gamma-1} = u_{j-1} + \frac{2a_{j-1}}{\gamma-1}$$

$$\vec{R}_2 : u - \frac{2a}{\gamma-1} = u_{j-1/2} - \frac{2a_{j-1/2}}{\gamma-1} = u_j - \frac{2a_j}{\gamma-1}$$

They are reduced from the Mendeleev-Clapeyron equation, the Laplace-Poisson adiabat and the equality of velocities of cell borders and one of the straight characteristics of relevant depression waves:

$$V_{j-1/2} = u_{j-1/2} - a_{j-1/2} \text{ or } V_{j-1/2} = u_{j-1/2} + a_{j-1/2}$$

In such a way, the general scheme of construction of the numerical method based on the Godunov scheme can be presented as follows:

- Extrapolation of unknowns for determination of the flow state on its edges according to the values given at the centre (reconstruction). In practice, they use piecewise constant distribution (Godunov scheme), piecewise linear distribution (Van Leer scheme) and piecewise parabolic (Chakravarty-Osher scheme) distribution of parameters of a flow within the limits of the cell and different flow limiters.
- The Riemann problem solution for each edge of the control volume with allowance for local directions of the flow (in direction of the normal to the control volume edge). The accurate solution of the Riemann problem is considered as unprofitable enough from the point of view of computation, therefore approximate approaches are widely used, for example, the Roe scheme or Osher scheme.
- **Realization of the time step (evolution):** The general in all methods of the similar class is usage of diverse monotonic flow limiters (Fig. 4) with switchers dependent on local properties of the solution (α_i is i^{th} characteristic). The majority of limiters have discrete switches like max $\{f_1, f_2\}$, that results to discontinuity of the first derivative and to the accuracy lowering (usage of absolute values of control functions has the same sense and results in the same consequences), in this connection the smooth limiters are applied also.

RESULTS AND DISCUSSION

Velocity function analysis: Velocity function of the Riemann wave U_R (*J*) and shock-wave U_D (*J*) have for *J = 1* the contact order not lower than the second and for γ = 5/3-the third one. At γ<5/3 the concurrent velocity following the Riemann compression wave is always lower than the velocity following shock-wave of the equal intensity. The higher the wave intensity, the bigger the difference. At γ>5/3 the situation changes. Velocity functions of U_R (*J*) and of shock-wave U_D (*J*) have another cross point $U_R(J_x) = U_D(J_x)$. Dependence J_x on adiabatic index is shown in Fig. 5.

If $J>J_x$, the concurrent velocity after the Riemann compression wave is lower than after the shock-wave equal to the intensity. If $1<J<J_x$, the concurrent velocity is lower after the shock-wave. The special intensity J_x has defined applicative value. In a number of technical applications, e.g., in hardening of metal surface, they use throwing of particles by waves. For J_x

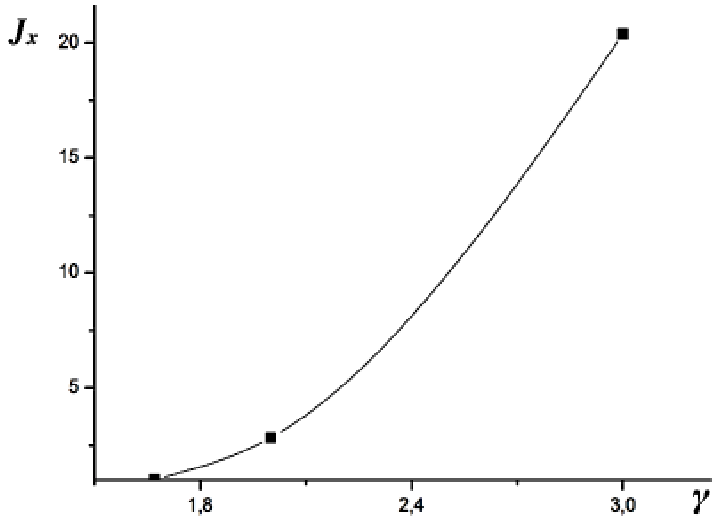


Fig. 5: Dependence on the adiabatic index of special intensity J_x, when the velocity functions of the shock-wave and the Riemann wave are equal

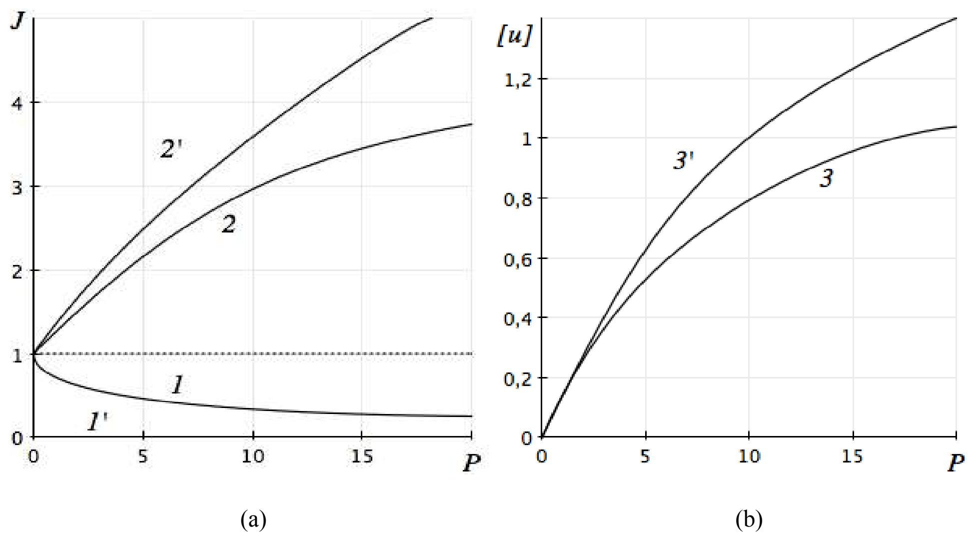


Fig. 6: Accurate approximated-analytic solution of the Riemann problem

the least value of energy cost per unit of increase of the particle kinetic energy is achieved.

Analysis of the Osher-Solomon solution for weak waves: Figure 6a and b shows the values of intensities of waves J_1 and J_2 (curves 1 and 2-accurate solution, 1' и 2' -the Osher-Solomon solution) and the flow velocity step $[u]=[u]_1=[u]_2$ (curves 3 and 3') on the waves formed in the shock tube at pressure discontinuity breakdown *P* of two gases originally at rest with the same temperature and adiabatic index $\gamma = 1.4$.

One can see that for big initial differential static pressures (P>5) inaccuracy of approximate solution gets noticeable. In the area of weak discontinuities, the coincidence is all sufficient. As the solution of the problem of breakdown of an arbitrary discontinuity in the course of numerical computation is made a lot of times, usage of an approximate model is considered reasonable.

Solution of the sod test problem by the Godunov method in ac-curate setting and in Osher-Solomon approximation: Modeling problems act the part of testing area for new methodological concepts and evaluation of accuracy of the results received with the help of built on their base software tools. For computation testing, we use the problem of computation of any progressive wave evolution and different configurations of the Riemann problem. The problem of evolution of progressing waves serves for control of the step of the solution reconstruction, the Riemann problem-for control of the evolution step. The

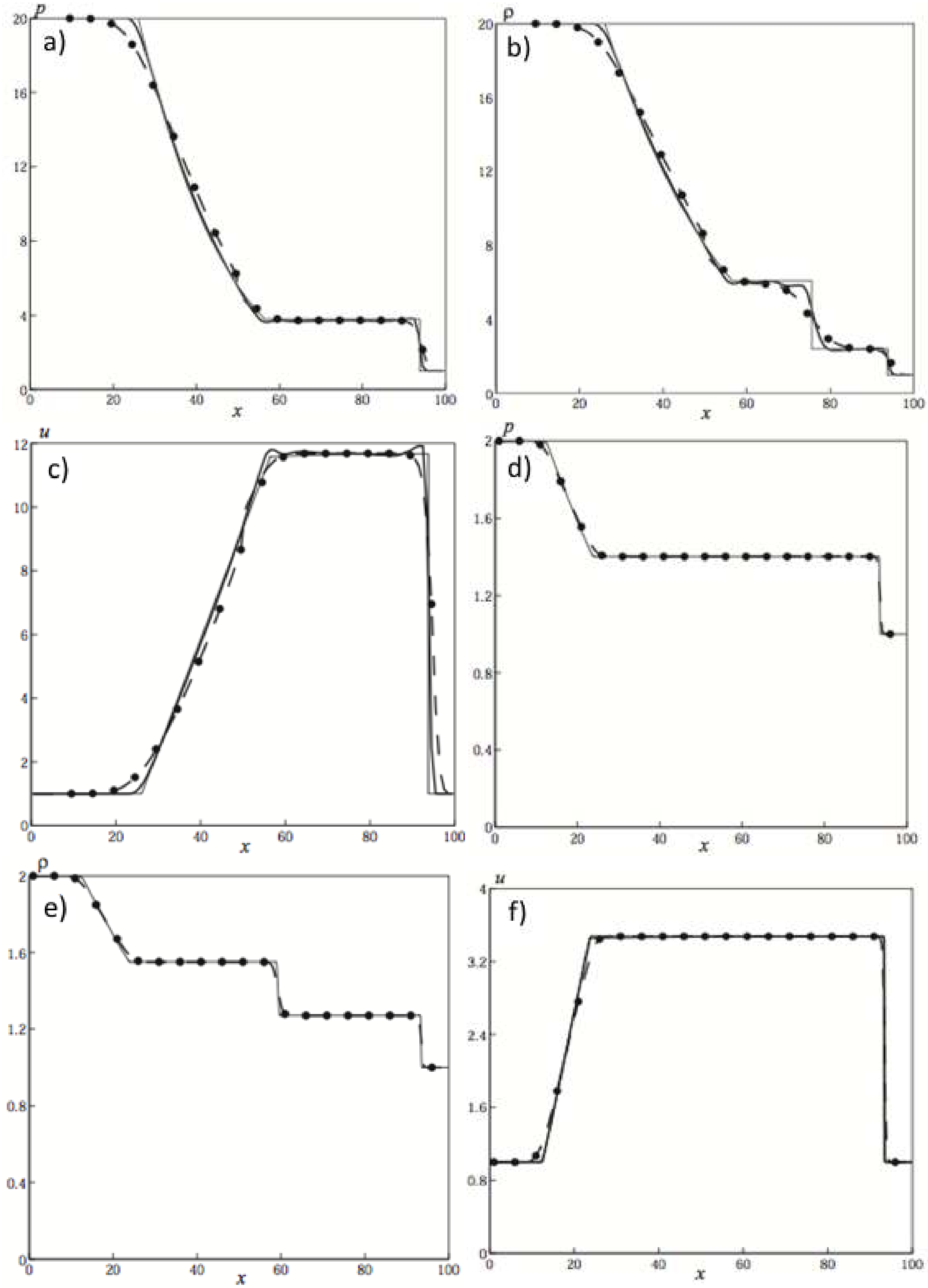


Fig. 7: The sod test problem; (a) to (c): Subsonic flow; (d) to (f): Supersonic flow

computation results (Fig. 7) allow to judge about monotony and accuracy of the numeric method.

The generated shock-wave pattern meets two different solutions of the Sod problem (Sod, 1978). In the first case (Fig. 7a to c) a subsonic flow occurs and in the second case a supersonic flow occurs. Air $\gamma = 1.4$ is taken as working environment. The given solutions meet a small space of time from the moment of start of the discontinuity breakdown, when in a numerical solution we can see the largest deviations from the accurate solution. As against the classic setting of the Sod problem where the initial conditions are given in relative variables, this work uses dimensional values.

Various difference schemes show almost the same results. The solid line corresponds the problem accurate solution, the dotted line corresponds computation under the Godunov scheme, the heavy line ~ -computation under the MUSCL scheme of the 3rd order, circles-the Chakravarty-Osher scheme. As a whole, the Chakravarty-Osher scheme gives more precise solution than other schemes. At the same time, the Godunov scheme requires 4.2 times as much as the estimated time as compared with the schemes based on an approximate solution of the Riemann problem.

The advantages of the high order accuracy schemes are well noticeable at consideration of profiles of contact discontinuity and shockwave.

Nonmonotonicity of the numerical profile received with the help of the usual scheme of the second order which appears near the discontinuity occurs. On the other hand, the profiles of viscosity and pressure received based on the schemes of the high order, are monotonic.

The accuracy order of difference schemes at availability of discontinuity of solutions and their derivatives, as a rule, does not correspond the classic order of approximation on the Taylor development on smooth solutions. For evaluation of accuracy of difference schemes the method based on experimental determination of convergence of numerical computations to the accurate solution of the source problem is applied.

CONCLUSION

The studying of breakdown of an arbitrary discontinuity is topical already many years. The necessity of its solution arises in the numerical methods using the schemes like the Godunov one. The requirement of optimal combination of acceptable accuracy of approximating with high speed computation demands further study of approximate methods of the solution of the problem of discontinuity breakdown, for example, the Osher-Solomon scheme, where for computation of weak shock-waves the relations for isentropic compression waves are used. The computation has shown that the approximate method can be used for the intensity of shock-waves lower that five, which is executed more often on the borders of difference cells in typical cases.

The development of new computation algorithms for the regions of existence of different solutions is actual.

In a range of technical applications (flow of an acute edge of an airfoil profile, shock-wave reflection from an obstacle, shock-wave processes in jets) it is necessary to solve the problem of discontinuity breakdown in accurate setting, with no simplification. On the other hand, for realization of the numerical methods operating the solution of the Riemann problem and, for example, the Godunov method, it is more rationally to apply approximate solutions. The considered approximate solution Osher-Solomon ensures, as the test computation shows, monotonicity of the difference scheme and acceptable accuracy. At the same time, there is a necessity of accurate determination of applicability of approximate solutions. For this purpose, it is convenient to use the velocity function of the breakdown intensity, which is equal to concurrent speed following a shock-wave spreading in the medium at rest. The work specified the special intensity of waves when the velocity functions of a shock-wave and a simple wave are identical, i.e., the error of approximate methods is minimum. Direct computation of application of the Osher-Solomon scheme has shown that it can be used up to the intensity of discontinuity equal to five.

ACKNOWLEDGMENT

This study was prepared as part of the "1000 laboratories" program with the support of Saint-Petersburg National Research University of Information Technologies, Mechanics and Optics (University ITMO) and with the financial support of the Ministry of Education and Science of the Russian Federation (the Agreement No. 14.575.21.0057).

REFERENCES

Arkhipova, L.P., 2012. Velocity function of the intensity of a one-dimensional non-stationary wave and its analysis for compression waves. Herald Samara State Aerospace Univ., 3(34): 57-62.

Arkhipova, L.P. and V.N. Uskov, 2013. Universal solution of the problem of the one-dimensional progressing waves reflection from a solid wall and its analysis for compression waves. Herald Saint-Petersburg Univ., 1(2): 77-81.

Gelfand, B.E. and M.V. Silnikov, 2002. The selection of the effective blast reduction method when detonating explosives. J. Phys. IV, 12(7): 371-374.

Gelfand, B.E., M.V. Silnikov, A.I. Mikhailin and A.V. Orlov, 2001. Attenuation of blast overpressures from liquid in an elastic shell. Combust. Explo. Shock, 37(5): 607-612.

Godunov, S.K., 1959. A difference method for numerical calculation of discontinuous solutions of the equations of hydrodynamics. Mat. Sb. (N.S.), 47(89): 271-306.

Godunov, S.K., A.V. Zabrodin, M.I. Ivanov, A.N. Krayko and G.P. Prokopov, 1976. Numerical Solution of Multidimensional Problems of Gas Dynamics. Izdatel'stvo Nauka, Moscow, pp: 400. (In Russian)

Igra, O., 2001. One-dimensional Interactions. Handbook of Shock Waves. V2. Shock Wave Interactions and Propagation, San Diego, pp: 1-64.

Kolgan, V.P., 1972. Application of the principle of minimal 13 derivative values to plotting of finite-size schemes for computation of discontinuity solution in gas-dynamics. Scientist`s Notes CAGI, 3(6): 68-72.

Kulikovsky, A.G., N.V. Pogorelov and A.Y. Semenov, 2001. Mathematical Aspects of Numerical Solution of Hyperbolic Systems. Chapman and Hall/CRC Press, Boca Raton, pp: 540.

Osher, S. and F. Solomon, 1982. Upwind difference schemes for hyperbolic conserva-tion laws. Math. Comput., 38(158): 339-374.

Silnikov, M.V. and A.I. Mikhaylin, 2014. Protection of flying vehicles against blast loads. Acta Astronaut., 97: 30-37.

Sod, G., 1978. A survey of several finite difference methods for systems of nonlinear hyperbolic conservation laws. J. Comput. Phys., 27(1): 715-736.

Uskov, V.N., 2000. Progressing One-dimensional Waves. SPb.: Publ. H. BGTU Voenmekh, Saint-Petersburg, Russia, pp: 220.