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Research Article

Group Complexity for Semigroup of Electroencephalography Signals during Epileptic Seizure

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Abstract: Electroencephalography (EEG) signals during epileptic seizure can be viewed as a semigroup of upper triangular matrices under matrix multiplication. In this study, we will provide a novel algebraic structure for EEG signals during epileptic seizure and then find out the group complexity. In this case, the novel structure of EEG signals during seizure is investigated for potential and Average Potential Differences (APD).

Keywords: Electroencephalography, group complexity, semigroup

INTRODUCTION

Epilepsy is a chronic disorder of the nervous system characterized by seizures which can affects people to suddenly become unconscious, violent and uncontrolled movements of the body (Magiorkinis *et al*., 2010). Seizures are categorized into two major groups, partial and generalized. Partial seizures are those in which the clinical or electroencephalographic evidence recommends that the attacks have a localized onset in the brain (Gastaut, 1970). This kind of seizure involves only a part of the cerebral hemisphere at seizure onset and produces symptoms in corresponding parts of the body or disturbances in some related mental functions. Contrarily, generalized seizures are said to occur if the evidence proposes that the attacks were well spread (Ahmad *et al*., 2012).

Electroencephalography (EEG) is a system to measure electrical activity produced by the firing of neurons in the brain. It functions by recording the instabilities in the potential difference of electrodes connected to the scalp of the patient (Fig. 1), hence indicating the presence of neural activity. Furthermore, the treatment and diagnosis of epilepsy are really aided by the use of EEG signal as a monitoring tool (Niedermeyer and Da Silva, 2005).

The presence of the skull between the outer surface and the cortex tends to introduce far field effects and low-pass filters the signal. In consequence of the far field effects, scalp currents farther from the recording point may also be recorded. This tends to make the signals from different electrodes become correlated, not due to synchronization of the brain areas during a seizure but caused by the mixing effects presented by the skull.

The EEG system reads differences of voltage on the head, relative to a given point. Therefore, if the activity of electrical is to be ascertained, then one shall need to place three electrodes, one on every hemisphere and another in the center, linked to both electrodes. This will give an absolute difference between activities of the hemispheric brain.

The mathematical analysis of EEG signals helps medical professionals by providing an explanation of the brain activity being observed, hence increasing the understanding of the brain function of human. There are several techniques recommended in order to specify the EEG information. One of these, the Fast Fourier Transforms (FFT) occurred as a very powerful tool capable of symbolizing the frequency components of EEG signals, even reaching diagnostic importance (Abarbanel *et al*., 1985; Selvaraj and Sivaprakasam, 2014). However, FFT has some disadvantages that limit its applicability and therefore, other techniques for extracting hidden data from the EEG signals are necessary. In this approach, the theoretical foundation of the elementary components, which include the flattening and algebraic pattern of EEG data during epileptic seizure is developed.

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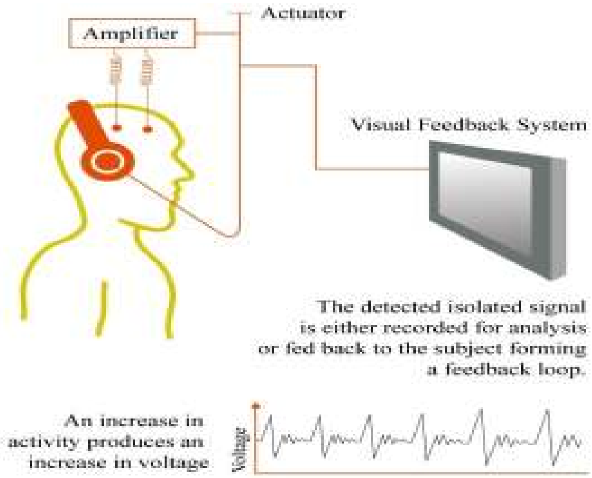
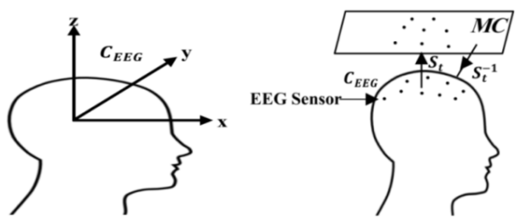


Fig. 1: EEG system (Michel *et al*., 2004)



(a) (b)

Fig. 2: (a): EEG coordinate system; (b): EEG projection

FLATTENING THE EEG

Zakaria and Ahmad (2007) developed a novel method to map high dimensional signal, namely EEG into low dimensional space. The processes of this technique included three essential parts. The first part deals with flattening the EEG data which generally entails transformation of three dimensional spaces into two-dimensional spaces, which involves the position of sensors in a patient head with EEG signal. The second part involves processing EEG signals using Fuzzy c-Means clustering technique. The last part involves finding the optimal number of clusters using analysis of cluster validity.

The coordinate system of EEG signals (Fig. 2a) was defined by (Zakaria, 2008) as follows:

$$C_{EEG} = \left\{\left((x, y, z), e_p\right) : x, y, z, e_p \in \mathbb{R} \text{ and } x^2 + y^2 + z^2 = r^2\right\}$$

where, r is the radius of a patient head. Moreover, a function is defined from C_{EEG} to MC plane as the following: $S_t : C_{EEG} \rightarrow MC$ (Fig. 2b) such that:

$$S_t\left((x, y, z), e_p\right) = \left(\frac{rx + iry}{r + z}, e_p\right)$$

$$= \left(\frac{rx}{r + z}, \frac{rx}{r + z}\right)_{e_p(x,y,z)}$$

where $MC = \{\left((x, y), e_p\right) : x, y, e_p \in \mathbb{R}.\}$

Together, C_{EEG} and MC were designed and proven as two-manifolds (Ahmad *et al*., 2008). S_t is an injective mapping of a conformal structure. Thus, S_t mapping can preserve information in a particularangle and orientation of the surface through the recorded EEG signals. They implemented this technique followed by clustering on real time EEG data obtained from patients who suffer from epileptic seizure.

The signals were digitized at 256 samples per second using Nicolet One EEG software. The average potential difference was calculated from the 256 samples of raw data at every second. Similarly to the position of the electrodes, the EEG signal was also preserved using this technique (Fig. 3). Then, every single second of the particular average potential difference was stored along with the position of the electrode on MC plane.

SEMIGROUP OF EEG SIGNALS DURING EPILEPTIC SEIZURE

Binjadhnan and Ahmad (2010) shown EEG signals during epileptic seizure can be recorded and composed into a set of $(n \times n)$ square matrices. In other words, every single second of the specific average potential difference was kept in a square matrix which contains the position of electrode on MC plane. Thus, MC plan became a set of $(n \times n)$ square matrices defined as following:

$$MC_n(\mathbb{R}) = \left\{\left[\beta_{ij}(z)_t\right]_{n \times n} : i, j \in \mathbb{Z}^+, \beta_{ij}(z)_t \in \mathbb{R}\right\}$$

where, $\beta_{ij}(z)_t$ is an average potential difference reading of EEG signals from a particular ij sensor at time t (Appendix 1).

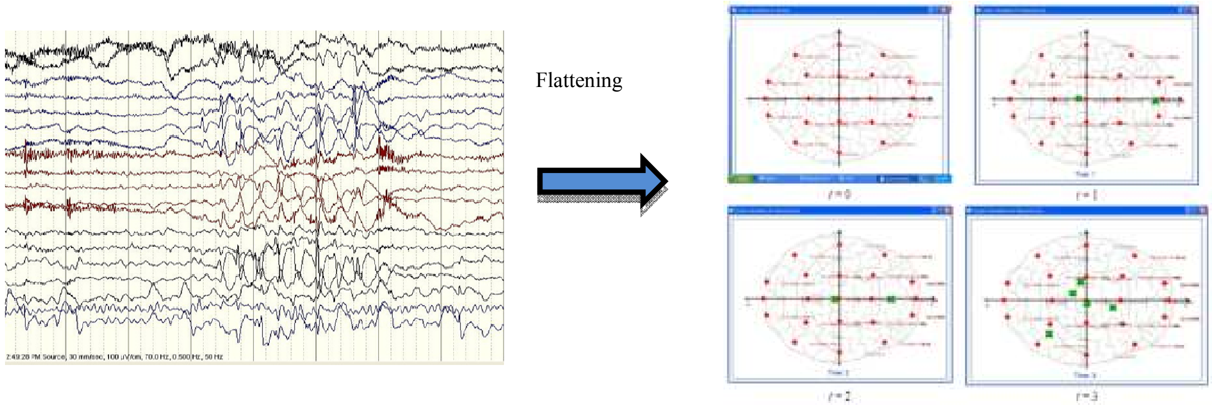


Fig. 3: EEG flattening

$$\begin{pmatrix} \beta_{1,1} & \cdots & \beta_{1,n} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \beta_{n,n} \end{pmatrix}$$

Fig. 4: $\boldsymbol{MC}$ plan transformed into an upper triangular matrix

In addition, they transformed the set $MC_n(\mathbb{R})$to the set of upper triangular matrices $MC_n^{''}(\mathbb{R})$ using QR-real Schur triangularization (Fig. 4) as following:

$$MC_n^{''}(\mathbb{R}) = \left\{ \left[\beta_{ij}(z)_t \right]_{n\times n} : \beta_{ij}(z)_t = 0, \forall 1 \le j < i \le n \, , i, j \in \mathbb{Z}^+ \, , \beta_{ij}(z)_t \in \mathbb{R} \right\}$$

Furthermore, the set $MC_n^{''}(\mathbb{R})$ satisfies all the axioms of a semigroup under matrix multiplication. In short, $MC_n^{''}(\mathbb{R})$ is closed and associative under matrix multiplication.

MATERIALS AND METHODS

In this section, some related definitions and theorems that are used in this study are introduced.

Definition 1 (Putcha, 1988): A subset $\alpha \subseteq \{1,2, \ldots \ldots, n\}$ with the property that for any $i \le j \le k \in \{1,2, \ldots \ldots, n\}$ with $i, k \in \alpha$ we have also $j \in \alpha$ is called$n - interval$. Let A be $n \times n$ matrix over a field F and α an $n - interval$ the restriction $A|_\alpha$ is the matrix $\left(A_{ij}\right)_{i,j\in\alpha}$ with rows and columns indexed by α. Two matrices A and B agree on α if $A_{ij} = B_{ij}$ for all $i, j \in \alpha$. We say that A and B are scalar multiples on α if there exists a non-zero field element $\lambda \in F$ such that $A_{ij} = \lambda B_{ij}$ for all $i, j \in \alpha$, such that A and λB agree on α.

Definition 2 (Almeida *et al.*, 2005): Let $S(n, F)$be a semigroup of all $n \times n$ upper triangular matrices with entries drawn from field F, with usual operation (matrix multiplication). Let $A \in S(n, F)$, the diagonal shape of A is the set $shape(A) = \{i \in Z | \, 1 \le i \le n \, , a_{ii} \ne 0\}$. Hence, two matrices have the same diagonal shape if they have zeros in exactly the same positions on the main diagonal.

Definition 3 (Okniński, 1998): Let $S(n, F)$be a semigroup of all $n \times n$ upper triangular matrices over a field F with usual matrix multiplication. Define the relation Ω on each $S(n, F)$ by $A\Omega B$ if and only if $A = \Omega B$ for some non-zero field element Ω.

Theorem 1 (The prime decomposition theorem) (Krohn and Rhodes, 1965): Every finite semigroup can be expressed as divisor (a homomorphic image of a subsemigroup) of wreath product of finite groups and finite aperiodic semigroups.

Definition 4 (Krohn and Rhodes, 1968): Let S be a semigroup. The group complexity of S is the least number of group factors appearing in any such wreath product decomposition of S.

Definition 5 (Faisal, 2011): The elementary EEG signals are a square matrix of EEG signals reading at time t in terms of one of the following types:

- Diagonal matrix (special case subidentity matrix)
- Unipotent matrix
- Permutation

Definition 6 (Putcha, 1988): Let S be a semigroup, an element $s \in S$ is said to be regular if there is an element $t \in S$ such that $s = sts$ and S is said to be a regular semigroup if every element of S is regular.

Theorem 2 (Barja *et al.*, 2014): Assume that A_t is an upper triangular matrix of EEG signals during epileptic seizure $\left(A_t \in MC_n^{''}(R)\right)$. Then the following are equivalent:

- A_t is regular
- Every row (column) in A_t is a linear combination of rows (columns) in A_t with non-zero diagonal entries
- A_t is $\mathcal{J}$ related to subidentity matrix

The following theorem is a consequence of Tilson and Rhodes which provides the connection through Krohn-Rhodes complexity theory.

Theorem 3 (Rhodes and Tilson, 1968): Let S be a semigroup in which each regular $\mathcal{J}-class$ is a subsemigroup. Then $S/\equiv$ has the same group complexity as S and $(S/\equiv)/\sim$ has group complexity one less than that of S (or zero if S has complexity zero).

NEW SEMIGROUPS OF EEG SIGNALS AND ITS GROUP COMPLEXITY

In this section, our main goal is to define a new semigroup of upper triangular matrices of EEG signals during epileptic seizure and compute its group complexity.

The set of all congruence class of $\sim$ forms a semigroup with zero called the quotient semigroup (factor semigroup) (Howie, 1995). The quotient semigroups of upper triangular matrices of EEG signals during epileptic seizure can be defined as $MC_n^{''}(\mathbb{R})/\sigma$ which call it the projective triangular semigroups of EEG signals and denoted by $PMC_n^{''}(\mathbb{R})$. Furthermore, an element of $PMC_n^{''}(\mathbb{R})$ is denoted by $\overline{A_t}$ which is the σ equivalence class of EEG signals matrix $(A_t \in MC_n^{''}(\mathbb{R}))$.

Let A_t, B_t are EEG signals matrices during epileptic seizure, define a relation $\equiv$ on $MC_n^{''}(\mathbb{R})$ by $A_t \equiv B_t$ if and only if for all regular elements X_t and Y_t in the same $\mathcal{J}-$class, we have $X_tA_tY_t\mathcal{J}\,X_t \Leftrightarrow X_tB_tY_t\mathcal{J}\,X_t$ and if $X_tA_tY_t\,, X_tB_tY_t\;\mathcal{J}\,X_t$ then $X_tA_tY_t = X_tB_tY_t$. Throughout this study, the equivalence class of an element A_t of $MC_n^{''}(\mathbb{R})$ under this relation will denoted by $[A_t]$. In addition, another relation $\sim$ can be defined on $MC_n^{''}(\mathbb{R})$ as by $A_t \sim B_t$ if and only if for every regular element X_t we have $X_tA_t\,\mathcal{R}\,X_t \Leftrightarrow X_tB_t\,\mathcal{R}\,X_t$ and if $X_tA_t, X_tB_t\,\mathcal{R}\,X_t$ then $X_tA_t\,\mathcal{L}\,X_tB_t$. The equivalence class of an element A_t of $MC_n^{''}(\mathbb{R})$ under this relation will denoted by $\langle A_t \rangle$.

Now we consider the previous relations as applied to semigroups $MC_n^{''}(\mathbb{R})$ of upper triangular matrices during epileptic seizure as defined on the following sets:

$$MC_n^{''1}(\mathbb{R}) = MC_n^{''}(\mathbb{R})/\equiv\; = \{[A_t] | A_t \in MC_n^{''}(\mathbb{R})\}$$

$$MC_n^{''2}(\mathbb{R}) = MC_n^{''1}(\mathbb{R})/\sim\; = \{\langle [A_t] \rangle | A_t \in MC_n^{''}(\mathbb{R})\}$$

Define binary relations on $MC_n^{''1}(\mathbb{R})$, $MC_n^{''2}(\mathbb{R})$ by $[A_t][B_t] = [A_tB_t]$ and $\langle [A_t] \rangle \langle [B_t] \rangle = \langle [A_t][B_t] \rangle$ respectively. We need to make sure that these are well-defined relations.

If $[A_t] = [A_t^{\backslash}]$ and $[B_t] = [B_t^{\backslash}]$ then $A_t \equiv A_t^{\backslash}$ and $B_t \equiv B_t^{\backslash}$; as $\equiv$ is a congruence, it follows that $A_tB_t \equiv A_t^{\backslash}B_t^{\backslash}$ and therefore $[A_tB_t] = [A_t^{\backslash}B_t^{\backslash}]$.

Thus the operation is well-defined. Similarly we can show the binary relation on $MC_n^{''2}(\mathbb{R})$ is well-defined.

The following theorems show that the sets $MC_n^{''1}(\mathbb{R})$ and $MC_n^{''2}(\mathbb{R})$ form semigroups with respect to their multiplication (Fig. 5).

Theorem 4: $MC_n^{''1}(\mathbb{R})$ is a semigroup under multiplication.

Proof: Firstly, we have to show $MC_n^{''1}(\mathbb{R})$ is closed with respect to matrix multiplication.

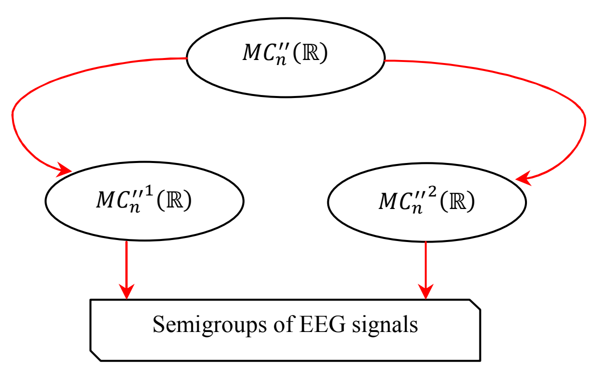


Fig. 5: Algebraic semigroups of EEG signals

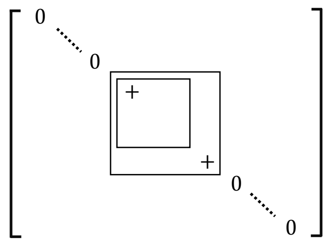


Fig. 6: 1st and 2nd interior squares of $\boldsymbol{A_t}$

pick $[A_t]$, $[B_t] \in MC_n^{''1}(\mathbb{R})$, from observations above we have $[A_t][B_t] = [A_t B_t] \in MC_n^{''1}(\mathbb{R})$.
Thus $MC_n^{''1}(\mathbb{R})$ is closed.
Secondly, we have to show $MC_n^{''1}(\mathbb{R})$ is associative.
pick $[A_t]$,$[B_t]$, $[C_t] \in MC_n^{''1}(\mathbb{R})$, from observations above we have:

$$\begin{aligned}([A_t][B_t])[C_t] &= ([A_t B_t])[C_t]\\ &= ([A_t B_t][C_t])\\ &= ([A_t B_t C_t])\\ &= ([A_t][B_t C_t])\\ &= [A_t]([B_t C_t])\end{aligned}$$

Thus $MC_n^{''1}(\mathbb{R})$ is associative.

Theorem 5: $\boldsymbol{MC_n^{''2}}(\mathbb{R})$ is a semigroup under multiplication.

Proof: Firstly, we have to show $\boldsymbol{MC_n^{''2}}(\mathbb{R})$ is closed with respect to matrix multiplication. pick $\langle[\boldsymbol{A_t}]\rangle, \langle[\boldsymbol{B_t}]\rangle \in \boldsymbol{MC_n^{''2}}(\mathbb{R})$, from previous observations we have:

$$\langle[A_t]\rangle\langle[B_t]\rangle = \langle[A_t][B_t]\rangle \in MC_n^{''2}(\mathbb{R}).$$

Thus, $MC_n^{''2}(\mathbb{R})$ is closed.
Secondly, we have to show $MC_n^{''2}(\mathbb{R})$ is associative.
pick $\langle[A_t]\rangle, \langle[B_t]\rangle, \langle[C_t]\rangle \in MC_n^{''2}(\mathbb{R})$, from observations above we have:

$$\begin{aligned}(\langle[A_t]\rangle\langle[B_t]\rangle)\langle[C_t]\rangle &= (\langle[A_t][B_t]\rangle)\langle[C_t]\rangle\\ &= (\langle[A_t][B_t]\rangle\langle[C_t]\rangle)\\ &= \langle[A_t][B_t][C_t]\rangle\\ &= \langle[A_t]\rangle\langle[B_t][C_t]\rangle\\ &= \langle[A_t]\rangle([B_t][C_t])\end{aligned}$$

Thus $MC_n^{''2}(\mathbb{R})$ is associative.
Given an EEG signal matrix:

$$A_t = \begin{pmatrix} \beta_{11} & \beta_{12} & \beta_{13}\beta_{14}\beta_{15} \\ \vdots & \beta_{22} & \beta_{23}\beta_{24}\beta_{25} \\ & & \beta_{33}\beta_{34} \ \vdots \\ \vdots & & \ddots \ \vdots \\ 0 & \cdots & \cdots \beta_{55} \end{pmatrix} \in MC_n^{''}(\mathbb{R})$$

We associate to it two specific$n-intervals$, called respectively the first and second interior (Fig. 6 for an illustration) squares of A_t:

$$SQ^{I}(A_t) = \{i \in Z \mid p \leq i \leq q \text{ for some } p, q \in shape(A_t)\}$$

$$SQ^{2}(A_t) = \{i \in Z \mid p \leq i \leq q \text{ for some } p, q \in shape(A_t)\}$$

Figure 6 shows the first and second interior squares of an upper triangular matrix of EEG signals during epileptic seizure. The symbol + indicates a non-zero entry, while omitted entries may take any value.

Theorem 6 (Kambites, 2007): For any $A, B \in S(n, F)$ such that $S(n, F)$is a semigroup of all upper triangular matrices over a field F. Then

- $A \equiv B$ if and only if A and B have the same diagonal shape and agree on their first interior square.
- $[A] \sim [B]$ if and only if$A$and Bhave the same diagonal shape and are scalar multiples on their second interior square.

The Fundamental lemma of group complexity is given as follows.

Theorem 7 (Rhodes, 1968): Assume that S and T be semigroups over finite field and suppose there exists a surjective morphism $S \rightarrow T$ which is injective when restricted to each subgroup of S. Then S and T have the same group complexity.

The fundamental lemma is a highly powerful tool for calculating the group complexity of a semigroup (Rhodes and Tilson, 1968). We shall use the same strategy followed in it proof to compute the group complexity of our new semigroup $MC_n^{''2}(\mathbb{R})$.

Theorem 8: Assume that $\mathbb{R}$ be a field of real number and $n \geq 2$. Then the semigroup $MC_n^{''2}(\mathbb{R})$ has the same group complexity as $PMC_{n-1}^{''}(\mathbb{R})$.

Proof: The Fundamental Lemma of Complexity both directly and through the application of result of Rhodes and Tilson (1968) are used to prove this theorem. In other words, we have to show that $MC_n^{''2}(\mathbb{R})$ divides the direct product of $PMC_{n-1}^{''}(\mathbb{R})$ with the 2-element semilattice.

Let $L = \{0,1\}$ be the 2-element semilattice, which can be viewed as a subset of the a field of real number $\mathbb{R}$.

Define a map $f: MC_{n-1}^{''}(\mathbb{R}) \times L \rightarrow MC_n^{''}(\mathbb{R})$ by (Fig. 7):

$$f(A_t, \varepsilon)_{ij} = \begin{cases} \beta_{ij} \text{if } 1 \leq i, j < n \\ \varepsilon \text{ if } i = j = n \\ 0 \text{ otherwise} \end{cases}$$

$$\left(\begin{bmatrix} \beta_{11} & \beta_{12} & \cdots & \cdots & \beta_{1,n-1} \\ & \beta_{22} & \cdots & \cdots & \beta_{2,n-1} \\ & & \ddots & & \vdots \\ & & & \ddots & \vdots \\ & & & & \beta_{n-1,n-1} \end{bmatrix}, \varepsilon\right) \rightarrow \begin{bmatrix} \beta_{11} & \beta_{12} & \cdots & \beta_{1(n-1)} & 0 \\ & \beta_{22} & & \beta_{2(n-1)} & 0 \\ & & \ddots & & \vdots \\ & & & & \\ & & & \beta_{(n-1)(n-1)} & 0 \\ & & & & \varepsilon \end{bmatrix}$$

Fig. 7: The behavior of the map f

We claim that this induces an onto homomorphism $f':PMC''_{n-1}(\mathbb{R}) \times L \rightarrow MC''^{2}_{n}(\mathbb{R})$ given by $f'(\overline{A_t}, \varepsilon) = \langle[f(A_t, \varepsilon)]\rangle$. The map f is a standard faithful representation of the direct product of matrix groups and in particular is a homomorphism. Hence f' is a homomorphism.

Firstly, we show that f' is well-defined. Suppose $A_t, B_t \in MC''_{n-1}(\mathbb{R})$ are such $\overline{A_t} = \overline{B_t}$ in $PMC''_{n-1}(\mathbb{R})$. Then $A_t = \lambda B_t$ for some none-zero field element λ (by definition 3).

Let $\varepsilon \in L$ Then for any $i, j < n$ we have $f(A_t, \varepsilon)_{ij} = \beta 1_{ij} = \lambda \beta 2_{ij} = \lambda f(B_t, \varepsilon)_{ij}$ and moreover $f(A_t, \varepsilon)_{nn} = \varepsilon = f(B_t, \varepsilon)_{nn}$

So, $f(A_t, \varepsilon)$ and $f(B_t, \varepsilon)$ have the same diagonal shape. Furthermore, since their second interior square cannot contain n, therefore they are scalar multiples on the second interior square. Thus, by theorem 6, we have $\langle[f(A_t, \varepsilon)]\rangle = \langle[f(B_t, \varepsilon)]\rangle$ as required.

Next, we have to show f' is subjective.

Let $\langle[A_t]\rangle \in MC''^{2}_{n}(\mathbb{R})$ and define $B_t \in MC''_{n-1}(\mathbb{R})$ by $\beta 2_{ij} = \beta 1_{ij}$ for $1 \le i, j < n$. Now by the same procedure earlier, it is easily to verify that $f(B_t, \beta_{nn})$ has the same diagonal shape as A_t and agrees with A_t on the second interior square, hence by theorem 6, $f'(B_t, \beta_{nn}) = \langle(A_t)\rangle$ as required.

Consequently, $MC''^{2}_{n}(\mathbb{R})$ is a homomorphism image of the direct product of $PMC''_{n-1}(\mathbb{R})$ with an aperiodic semigroup. In other words, $MC''^{2}_{n}(\mathbb{R})$ divides the direct product of $PMC''_{n-1}(\mathbb{R})$. Thus, $MC''^{2}_{n}(\mathbb{R})$ has the same group complexity as $PMC''_{n-1}(\mathbb{R})$ as required.

CONCLUSION

In this study, a novel semigroup of upper triangular matrices of EEG signals during epilepticseizure has been presented. In addition, the group complexity for this semigroup has been computed.

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Appendix 1:

MC_nTriangularization: During epileptic seizure, the EEG signals digitized at 256 samples per second using Nicolet One EEG software by Zakaria (2008). The average potential difference (APD) was calculated from the 256 samples of raw data at every second. Subsequently, every single second of the particular average potential difference was stored in a file which comprises the position of electrode on MC plane. Here, the recorded EEG signals during seizure are composed into a set of square matrices. Then, they are transformed into a set of upper triangular matrices using QR- real Schur triangularization.

The readings in the Table 1 are rephrasing in terms of square matrix (5 × 5) as following:

let$x_1 \le x_2 \le x_3 \le \cdots \le x_{21}, i, j = \{1,2,3,4,5\}$ and define a map β_{ij} as:

$$\beta_{ij} = \begin{cases} (x_{(i-1)5+j}, y_{(i-1)5+j})(i-1)5 + j \le 21 \\ 0\ (i-1)5 + j > 21 \end{cases}$$

the map β_{ij} could be reworded in terms of matrix below:

$$\begin{pmatrix} (x_1, y_1) & (x_2, y_2) & (x_3, y_3) & (x_4, y_4) & (x_5, y_5) \\ (x_6, y_6) & (x_7, y_7) & (x_8, y_8) & (x_9, y_9) & (x_{10}, y_{10}) \\ (x_{11}, y_{11}) & (x_{12}, y_{12}) & (x_{13}, y_{13}) & (x_{14}, y_{14}) & (x_{15}, y_{15}) \\ (x_{16}, y_{16}) & (x_{17}, y_{17}) & (x_{18}, y_{18}) & (x_{19}, y_{19}) & (x_{20}, y_{20}) \\ (x_{21}, y_{21}) & 0 & 0 & 0 & 0 \end{pmatrix}$$

Or

$$\begin{pmatrix} \beta_{11} & \beta_{12} & \beta_{13}\beta_{14}\beta_{15} \\ \beta_{21} & \beta_{22} & \beta_{23}\beta_{24}\beta_{25} \\ \beta_{31} & \beta_{32} & \beta_{33}\beta_{34}\beta_{35} \\ \beta_{41} & \beta_{42} & \beta_{43}\beta_{44}\beta_{45} \\ \beta_{51} & \beta_{52} & \beta_{53}\beta_{54}\beta_{55} \end{pmatrix}$$

Table 1: APD at sensor on a $\boldsymbol{MC}$

Sensor	X	Y	APD
Fpz	x_{21}	y_{21}	z_{21}
Fp1	x_{19}	y_{19}	z_{19}
Fp2	x_{20}	y_{20}	z_{20}
F3	x_{15}	y_{15}	z_{15}
F4	x_{16}	y_{16}	z_{16}
C3	x_9	y_9	z_9
C4	x_{10}	y_{10}	z_{10}
P3	x_6	y_6	z_6
P4	x_7	y_7	z_7
O1	x_2	y_2	z_2
O2	x_3	y_3	z_3
F7	x_{17}	y_{17}	z_{17}
F8	x_{18}	y_{18}	z_{18}
T3	x_{11}	y_{11}	z_{11}
T4	x_{12}	y_{12}	z_{12}
T5	x_4	y_4	z_4
T6	x_5	y_5	z_5
Fz	x_{14}	y_{14}	z_{14}
Cz	x_{13}	y_{13}	z_{13}
Pz	x_8	y_8	z_8
Oz	x_1	y_1	z_1

Table 2: APD at sensor on MC (t = 5) for patient B

Sensor	X	Y	APD
Oz	-8.3	0	0
O1	-7.8938	2.5648	54.85769531
O2	-7.8938	-2.5648	74.65964844
T5	-4.8786	6.7148	101.5934375
T6	-4.8786	-6.7148	178.1446875
P3	-3.6411	3.6411	49.75691406
P4	-3.6411	-3.6411	120.0916797
Pz	-3.438	0	139.3900391
C3	0	3.438	126.7973047
C4	0	-3.438	78.32976563
T3	0	8.3	33.43929688
T4	0	-8.3	94.37691406
Cz	0	0	122.7670313
Fz	3.438	0	36.37917969
F3	3.6411	3.6411	8.262070313
F4	3.6411	-3.6411	0.518320313
F7	4.8786	6.7148	57.02277344
F8	4.8786	-6.7148	88.69984375
Fp1	7.8938	2.5648	237.2856641
Fp2	7.8938	-2.5648	20.25851563
Fpz	8.3	0	0

by replacing the similarity APD for each entry in the above matrix, the corresponding square matrix is created. Consequently, every single second of the certain APD is stored in a square matrix which consists the position of electrode on MC plane. Therefore, MC plane became a set of $(n \times n)$ square matrices (EEG signals) defined as:

$$MC_n(\mathbb{R}) = \left\{ \left[\beta_{ij}(z)_t\right]_{n\times n} : i, j \in \mathbb{Z}^+, \beta_{ij}(z)_t \in \mathbb{R} \right\}$$

where $\beta_{ij}(z)_t$ is APD reading of EEG signals from a particular ij sensor at time t.

Using QR-real Schur technique for triangularize a matrix $\left[\beta_{ij}(z)_t\right]_{n\times n}$ in $MC_n(\mathbb{R})$ we obtain the following EEG signals matrix:

$$\begin{pmatrix} \beta_{11} & \beta_{12} & \beta_{13}\beta_{14}\beta_{15} \\ \vdots & \ddots & \beta_{23}\beta_{24}\beta_{25} \\ & & \beta_{34} \vdots \\ \vdots & & \ddots \vdots \\ 0 & \cdots & \cdots \beta_{55} \end{pmatrix}$$

As example for $MC_n(\mathbb{R})$, here is a sample of APD at sensor on a $\text{MC}(\text{t} = 5)$ for patient B (see, implementation of Zakaria (2008)) (Table 2).

The corresponding matrix of above data given as the following:

$$\begin{pmatrix} 0 & 54.8577 & 74.65965 & 101.5934 & 178.1447 \\ 49.75691 & 120.0917 & 139.39 & 126.7973 & 78.32977 \\ 33.4393 & 94.37691 & 122.767 & 36.37918 & 8.26207 \\ 0.518320 & 57.02277 & 88.69984 & 237.2857 & 20.25852 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Using QR-real Schur technique for triangularize this matrix we obtain the following:

$$\begin{pmatrix} 356.4567 & -7.016946 & -24.52558 & 131.892 & -124.9928 \\ 0 & 127.424 & 21.37413 & -52.62274 & 0 \\ 0 & 0 & -13.6006 & 17.35439 & -106.4489 \\ 0 & 0 & 0 & 9.864296 & 106.5759 \\ 0 & 0 & 0 & 0 & -6.063805 \end{pmatrix}$$

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