

Research Article

Reduction of Losses at the Start-up of DC Motor Using the Voltage Amplitude Variation Law

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Abstract: This study is concerned for exploring the possibilities of reducing losses in electric drives of heavy start-up conditions. Under the latter refers to electric drives with large quantities of the total moment of inertia of the rotating parts of the electric drives and machinery, in the presence of the moment of inertia at the starting process.

Keywords: DC motor, moment of inertia, startup losses, variation law

INTRODUCTION

In Abuzalata *et al.* (2011) the relationships for calculating the start-up losses in a separately excited DC motor is derived using the dynamic equations of the electric drive, One result of this study is concluded that among other factors which affect on the start-up losses is the variation of the armature voltage, formed by an electric power source during start-up process. In particular, it is shown that, if the form of the voltage is as steps with equal values and durations to power losses in the electric drive when it starts without load is:

$$\Delta A_{st} = J \frac{\omega_0^2}{2K} \quad (1)$$

where,

J : The total moment of inertia of all rotating parts

ω_0 : No-load speed

K : The number of steps of the armature voltage

Equation (1) shows that the losses in the armature circuit of the motor are inversely proportional to the number of steps of the armature voltage. It is assumed that the acceleration in each step should be continued until reaching the steady state speed. For this type of voltage law relationships are achieved for determining the energy losses and for the start-up of the electric drive with load (Chilikin, 1965; Chilikin and Sandler, 1981; Andreev and Sabinin, 1963). However, to minimize start-up losses, the step voltage control law is not the best.

The aim of this study is to reduce the start-up losses to show the advantages of other laws of voltage

variations. In particular, the greatest attention is paid to a law of linear rising voltage with the limitation by its nominal value (Andreev and Sabinin, 1963; Keys, 1985). The starting up losses of a separately excited DC motor will be studied.

METHODOLOGY

The dynamic equations of the electric drive represented by:

$$(T_m s + 1)\omega(t) = \frac{V_a}{C_e} - \frac{R_a}{C_e C_m} T_L \quad (2)$$

$$M_m = J \frac{d\omega}{dt} - T_L \quad (3)$$

where, Eq. (2) is the equation of electrical equilibrium in the rotor and (3) is the equation of mechanical equilibrium. In these relations:

T_m : Electromechanical time constant -[s]

$\omega(t)$: The angular velocity of rotor -rad/s

V_a : The applied armature voltage-[V]

R_a : Resistance of the armature -[Ohm]

C_e, C_m : Coefficients determined from structural data of the motor

T_m : Torque developed by motor-[N.m]

T_L : The is moment of the (load)[N.m]

J : The total moment of inertia of all moving parts and electric machinery-[kg/m²]

$S = d/dt$: The symbol of differentiation

Linear rising law with limited of nominal final value can be represented as:

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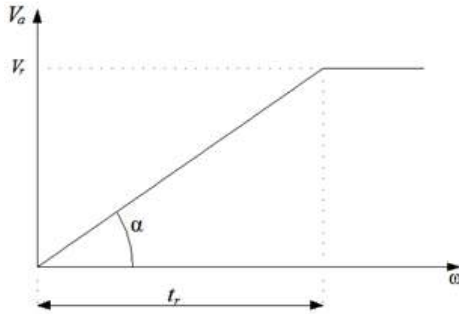


Fig. 1: The terminal armature voltage

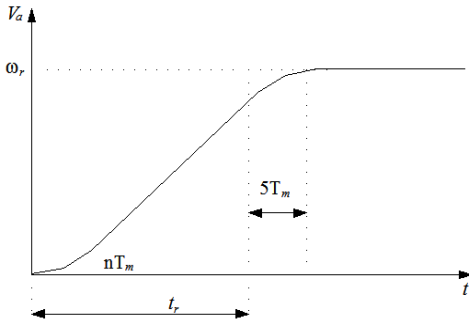


Fig. 2: Chart of the voltage changing

$$V_a = \begin{cases} K_r t & V_a < V_r \\ V_r & V_a = V_r \end{cases} \quad (4)$$

Relation (4) corresponds to the graph in Fig. 1: where, $K_r = tg\alpha = V_n/t_r$ Slope (0, 1, 2, 3, 4...), $t_r = nT_m$ (t_r - voltage rising time)

$$K_r = tg\alpha = \frac{V_n}{t_r}$$

$$V_a = \begin{cases} K_r t & V_a < V_n \\ V_n & V_a = V_n \end{cases}$$

Using Laplace transformation of the dynamic equation of start up without load, the result is:

$$(T_m s + 1)\omega(s) = \frac{V_a}{C_e}, \text{ Or } \omega(s) = \frac{V_a}{C_e(T_m s + 1)} \quad (5)$$

For the linear rising part vs. time of the voltage:

$$V_a(s) = \frac{V_n}{nT_m s^2} = \frac{K_r}{s^2}$$

Then Eq. (5) becomes:

$$\omega(s) = \frac{K_r}{C_e(T_m s + 1)s^2} = \frac{K_r}{C_e s^2} - \frac{K_r T_m}{C_e s(T_m s + 1)} \quad (6)$$

In the time domain the Eq. (6) is represented as:

$$\omega(t) = \frac{K_r}{C_e} \left[t - T_m \left(1 - e^{-\frac{t}{T_m}} \right) \right] = \frac{V_n}{C_e n T_m} t - \frac{V_n}{C_e n} + \frac{V_n}{C_e n} e^{-\frac{t}{T_m}} =$$

$$= \frac{\omega_0}{n T_m} t - \frac{\omega_0}{n} + \frac{\omega_0}{n} e^{-\frac{t}{T_m}} \quad (7)$$

Assuming that the transient process comes to an end when ($e^{-\frac{t}{T_m}} \approx 0$) for time $t = 5T_m$, let $t_r \gg 5T_m$. Let $t_r = 20T_m$, as shown in Fig. 2, during $5T_m$, the component $\frac{\omega_0}{n} e^{-\frac{t}{T_m}}$ tends to zero and Eq. (7) remain as:

$$\frac{\omega_0}{n} e^{-\frac{t}{T_m}}$$

$$\omega(t) = \frac{\omega_0}{n T_m} t - \frac{\omega_0}{n} \text{ if } t = t_s, t = nT_m \text{ then}$$

$$\omega(t) = \omega_0 - \frac{\omega_0}{n}$$

Or, in other words, the speed is different from the ideal no load speed with the value $\Delta\omega = \omega_0/n$, but at the end of the linear ramp voltage the transient process of the speed continues, then for the part of the transient process when $t_r < t < \infty$ we have the following equation:

$$\omega(t) = \omega_0 - \frac{\omega_0}{n} + \frac{\omega_0}{n} \left(1 - e^{-\frac{t}{T_m}} \right) = \omega_0 - \frac{\omega_0}{n} e^{-\frac{t}{T_m}} \quad (8)$$

During $t = t_r + 5T_m$ the transient process is over and speed $\omega(t) = \omega_0$.

Thus, if the time of linear ramp voltage $t_r = nT_m$, then the transient process of the speed is $t_{tp} = nT_m + 5T_m$ ($n + 5$) T_m from the achieved velocity law, we find the behavior of the armature current vs. time.

Using the equation of mechanical equilibrium $C_m I_a = J d\omega/dt$; then $I_a = J/C_m d\omega/dt$; by differentiating the Eq. (7) the equation below is achieved:

$$\frac{d\omega}{dt} = \frac{\omega_0}{n T_m} - \frac{\omega_0}{n T_m} e^{-\frac{t}{T_m}} = \frac{\omega_0}{n T_m} \left(1 - e^{-\frac{t}{T_m}} \right)$$

For the armature current in this part we have:

$$I_a = \frac{J}{C_m} \frac{\omega_0}{n T_m} \left(1 - e^{-\frac{t}{T_m}} \right) \quad (9)$$

After a period of time equals $5T_m$ at the origin the $e^{-\frac{t}{T_m}} \approx 0$ and the Eq. (9) becomes:

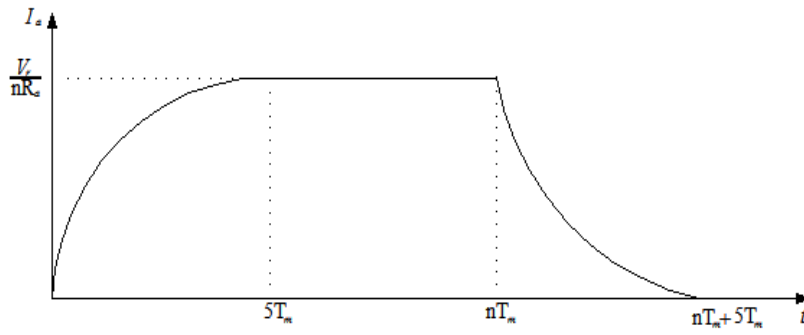


Fig. 3: The plot of the starting current

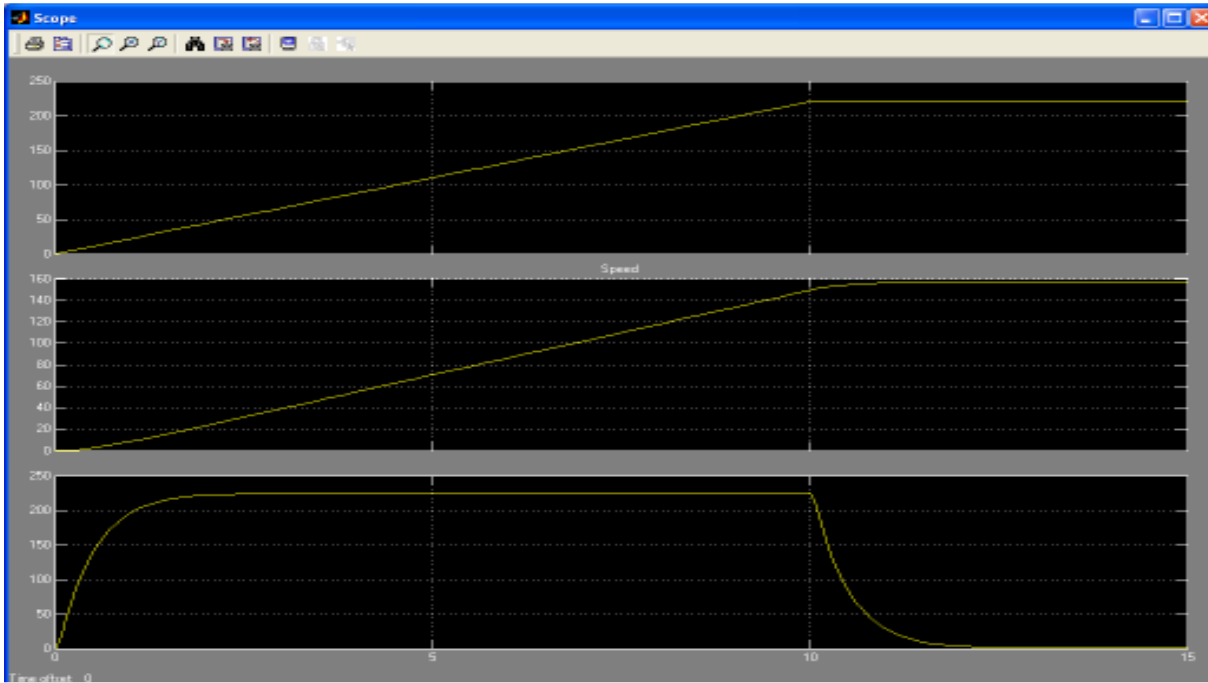


Fig. 4: Charts of voltage, speed and armature current simulation results

$$I_a = \frac{J}{C_m} \frac{\omega_0}{nT_m} = \frac{J}{C_m} \frac{\omega_0}{n} \frac{JR_a}{C_c C_m} = \frac{V_n}{nR_a} \quad (10)$$

The third region; $t > nT_m$ $I_a = \frac{V_n}{nR_a} e^{-\frac{t}{T_m}}$

For the latter part of the transient process:

$$I_a = \frac{J}{C_m} \frac{\omega_0}{nT_m} e^{-\frac{t}{T_m}} = \frac{V_n}{nR_a} e^{-\frac{t}{T_m}}$$

Thus, for the armature current, as well as for the speed, we can distinguish three regions of the transient process:

The first region; $0 \leq t \leq 5T_m$ $I_a = \frac{V_n}{nR_a} \left(1 - e^{-\frac{t}{T_m}}\right)$

The second region; $5T_m < t \leq nT_m$ $I_a = \frac{V_n}{nR_a}$

The corresponding current schedule is shown in Fig. 3. Charts show voltage, speed and armature current for the transient process which is presented in Fig. 4. Starting losses will also consist of three components; In the first period:

$$\Delta A_{e1} = \int_0^{5T_m} I^2 R_a dt = \left(\frac{V_n}{nR_a}\right)^2 \int_0^{5T_m} R_a \left(1 - e^{-\frac{t}{T_m}}\right)^2 dt$$

In the second period:

$$\Delta A_{e2} = \int_{5T_m}^{nT_m} \left(\frac{V_n}{nR_a}\right)^2 R_a dt = \left(\frac{V_n}{nR_a}\right)^2 R_a (nT_m - 5T_m)$$

Table 1: Comparison, energy levels of the control voltage and time ramp voltage

Number of the voltage steps	3	4	5	6	7	8
n- for linear law	10	15	20	25	30	35
The ratio of losses $\Delta A_{step} / \Delta A_{linear}$	1.85	2.009	2.1	2.17	2.217	2.25

In the third period:

$$\Delta A_{e3} = \int_0^{5T_m} \left(\frac{V_n}{nR_a} \right)^2 R_a e^{-\frac{2t}{T_m}} dt$$

It is useful to sum the first two components, so:

$$\begin{aligned} \Delta A_{e1} + \Delta A_{e3} &= \left(\frac{V_n}{nR_a} \right)^2 R_a \int_0^{5T_m} \left[\left(1 - e^{-\frac{t}{T_m}} \right)^2 + e^{-\frac{2t}{T_m}} \right] dt = \\ &= \left(\frac{V_n}{nR_a} \right)^2 R_a \int_0^{5T_m} \left(1 - 2e^{-\frac{t}{T_m}} + e^{-\frac{2t}{T_m}} + e^{-\frac{2t}{T_m}} \right) dt = \\ &= \left(\frac{V_n}{nR_a} \right)^2 R_a \left[t \Big|_0^{5T_m} + 2T_m e^{-\frac{t}{T_m}} \Big|_0^{5T_m} - 2 \frac{T_m}{2} e^{-\frac{2t}{T_m}} \Big|_0^{5T_m} \right] \\ &= \left(\frac{V_n}{nR_a} \right)^2 R_a (5T_m - 2T_m + T_m) = \left(\frac{V_n}{nR_a} \right)^2 R_a 4T_m \\ \Delta A_{e1} + \Delta A_{e2} &= \left(\frac{V_n}{nR_a} \right)^2 R_a 4T_m \end{aligned}$$

Then the summation of all starting losses is:

$$\begin{aligned} \Delta A &= \Delta A_{e1} + \Delta A_{e2} + \Delta A_{e3} = \left(\frac{V_n}{nR_a} \right)^2 R_a [4T_m + nT_m - 5T_m] = \\ &= \left(\frac{V_n}{nR_a} \right)^2 R_a [nT_m - T_m] = \left(\frac{V_n}{nR_a} \right)^2 R_a T_m [n - 1] = \\ &= \frac{V_n^2}{n^2 R_a} \frac{J R_a}{C_e C_m} [n - 1] = \frac{J \omega_0^2}{n^2} [n - 1] \end{aligned}$$

$$\text{Finally } \Delta A_S = \frac{J \omega_0^2}{n^2} (n - 1) \quad (11)$$

$$\text{If } n \gg 1, \text{ then } \Delta A \approx J \omega_0^2 / n \quad (12)$$

where $n = t_r / T_m$;

where n-number of T_m , that fit into the length of time of the armature voltage ramp.

From these expressions, in addition to Eq. (11) and (12), it is very important the equation (10). It allows selecting the time of the ramp of the control action depending on the desired acceleration current (see DC section in Fig. 3). If at the start up the current exceeds the nominal start up current, say ten times, then by selecting $t_r = nT_m$ any value can be limited. For one of the above selected motors the starting rush current

equals $20I_n$. If it is necessary to starting current is equal I_n , you should install $t_r = 20T_m$.

It is interesting the losses comparison between a linear start-up and a step-laws method.

If the last option $\Delta A_{st} = J \frac{\omega_0^2}{2K}$, then the linear growth $\Delta A_{st} = \frac{J \omega_0^2}{n^2} [n - 1]$. How is the difference between these losses? By comparing them with each other is necessary for consideration of ways to set the same start time of transient process. As previously noted, starting with a step function $t_{ip} = 5T_m * K$, where K- the number of start-up stages of the armature voltage. When the ramp of armature voltage is limited with the nominal voltage:

$$t_{ip} = nT_m + 5T_m$$

It is useful to compare, when $K \geq 2$ and assume $K = 2$. Then:

$$t_{ip} = 5T_m * 2 = nT_m + 5T_m \quad (13)$$

From Eq. (13) if $n = 5$, then $\Delta A_{st} = \frac{J \omega_0^2}{4}$ at step start-up $\Delta A_{st} = \frac{J \omega_0^2 * 4}{25} = \frac{J \omega_0^2}{6.25}$ and ramp voltage. This shows that the efficiency of the second method $\frac{J \omega_0^2}{4} \div \frac{J \omega_0^2}{6.25} = 1.56$ times the step in starting.

For comparison, a larger number of energy levels of the control voltage and the corresponding time ramp voltage are made in the Table 1.

The table shows that, compared with step-start performance increases linearly with an increase in the number of stages and consequently the rise time. But it hardly makes sense to take the number of steps above start four to eight. The equivalent rise time is it (15 - 35) T_m and inrush current is equal (1.2 - 0.6) I_r . Therefore we can say that when you start the motor with a voltage ramp the starting losses are about 2 times lower than that for step change.

Motor starting in the presence of the moment of Inertia:

Electric drive starting conditions deteriorate in the presence of the electric moment of inertia. We assume that such a reactive torque that comes with the beginning of the movement and that for the sake of simplicity $M_L = const$ (Andreev and Sabinin, 1963).

Then the dynamic equation of the electric drive of the type:

$$(T_m s + 1) \omega(t) = \frac{V_a - \frac{R_a}{C_m} M_L}{C_e}, \text{ Can be converted to}$$

$$(T_m s + 1)\omega(t) = \frac{V_a - I_a R_a}{C_e} \text{ And:}$$

$$(T_m s + 1)\omega(t) = \frac{V_a - \Delta V_L}{C_e} \quad (14)$$

The difference $(V_a - \Delta V_L)$ is equal to the armature voltage, which determines the rate of motor acceleration and its rate at steady state. In particular, if the voltage rise to a value, the rate is established not on the level V_r and where, then expression (7) for speed at the site will change linearly increasing voltage to the form:

$$\omega(t) = \frac{\omega_c}{nT_m} t - \frac{\omega_c}{n} + \frac{\omega_c}{n} e^{-\frac{t}{T_m}} \quad (15)$$

Strictly speaking, the solution (15) did not accurately reflect the behavior of a variable $\omega(t)$, since the latter should not start from the origin and have some

delay on the time axis is equal to the rise time of the armature current to a value $I_a = I_L$. But in this case significantly increases the difficulties of the analytical solutions (15). Therefore, trying to stay within the framework of analytical solutions, we adopt an approximate solution in the form of (15) and the error estimate assumptions simulation on a PC, knowing that it is small enough. The equation of mechanical equilibrium at a constant load torque becomes:

$$C_m I_a = J \frac{d\omega}{dt} + C_m I_L \quad (16)$$

$$I_a = \frac{J}{C_m} \frac{d\omega}{dt} + I_L \text{ or we get } \frac{d\omega}{dt} \text{ from} \quad (17)$$

$$\frac{d\omega}{dt} = \frac{\omega_L}{nT_m} - \frac{\omega_L}{nT_m} e^{-\frac{t}{T_m}} = \frac{\omega_L}{nT_m} \left(1 - e^{-\frac{t}{T_m}}\right) \quad (18)$$

For the armature current view of (18) we have:

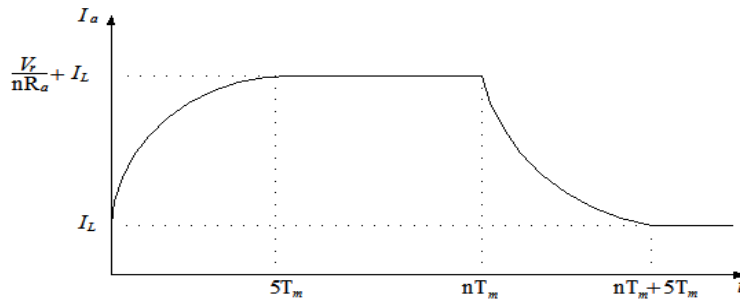


Fig. 5: starter armature current

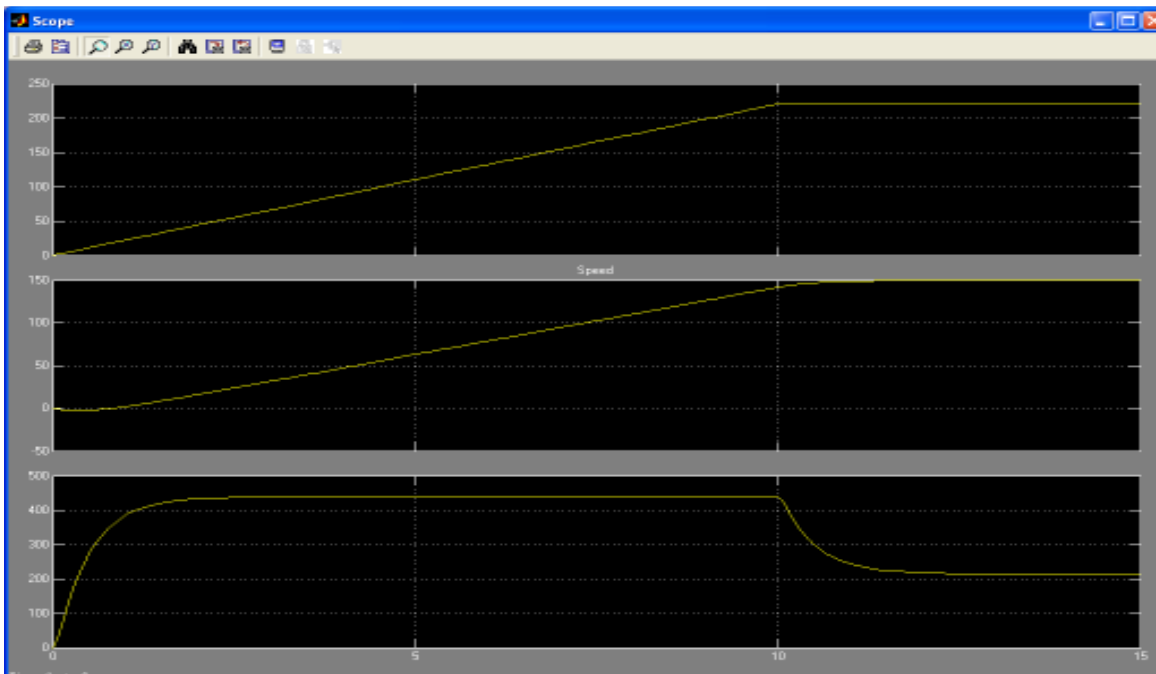


Fig. 6: Charts of voltage, speed and armature current simulation result

$$I_a = \frac{J}{C_m} \frac{\omega_L}{nT_m} \left(1 - e^{-\frac{t}{T_m}}\right) + I_L$$

Substituting in this expression instead of:

$$\omega_c = \frac{U_c}{C_m}$$

And $T_m = \frac{JR_a}{C_e C_m}$ we obtain $I_a = \frac{V_L}{nR_a} \left(1 - e^{-\frac{t}{T_m}}\right) + I_L$

For the latter part of the transition process, when we have the following changes in the current law:

$$I_a = \frac{V_L}{nR_a} e^{-\frac{t}{T_m}} + I_L$$

The corresponding plot is shown in Fig. 5 the current charts conduct voltage, speed and armature current for the transition process is presented in Fig. 6.

RESULTS AND DISCUSSION

To calculate the energy losses during acceleration the engine is useful to consider separately three regions of acceleration and then summarize them in the form of

$$\Delta A_{est} = \sum_{i=1}^3 \Delta A_{ei}$$

First period:

$$\begin{aligned} \Delta A_{e1} &= \int_0^{5T_m} I_a^2 R_a dt = \int_0^{5T_m} \left[\frac{V_L}{nR_a} \left(1 - e^{-\frac{t}{T_m}}\right) + I_L \right]^2 R_a dt = \\ &= \int_0^{5T_m} \left[\frac{V_L^2}{n^2 R_a^2} - 2 \frac{V_L^2}{n^2 R_a^2} e^{-\frac{t}{T_m}} + \frac{V_L^2}{n^2 R_a^2} e^{-\frac{2t}{T_m}} \right. \\ &\quad \left. + 2V \frac{V_L I_L}{nR_a} - 2 \frac{V_L I_L}{nR_a} e^{-\frac{t}{T_m}} + I_L^2 \right] R_a dt \\ &= \frac{V_L^2 R_a}{n^2 R_a^2} t \Big|_0^{5T_m} - 2 \frac{V_L^2 R_a}{n^2 R_a^2} \left[-T_m e^{-\frac{t}{T_m}} \Big|_0^{5T_m} + \frac{V_L^2}{n^2 R_a^2} \right. \\ &\quad \left. - \frac{T_m}{2} e^{-\frac{2t}{T_m}} \Big|_0^{5T_m} + \frac{2V_L I_L R_a}{nR_a} t \Big|_0^{5T_m} - \right. \\ &\quad \left. - 2 \frac{V_L I_L R_a}{nR_a} \left[-T_m e^{-\frac{t}{T_m}} \Big|_0^{5T_m} + I_L^2 R_a t \Big|_0^{5T_m} \right] = \right. \\ &= \frac{V_L^2 5T_m}{n^2 R_a} - 2 \frac{V_L^2 T_m}{n^2 R_a} + \frac{V_L^2 T_m}{n^2 R_a * 2} + \frac{2V_L I_L 5T_m}{n} \\ &\quad - 2 \frac{V_L I_L T_m}{n} + I_L^2 R_a 5T_m \\ &= \frac{3.5V_L^2 T_m}{n^2 R_a} + \frac{8V_L I_L T_m}{n} + I_L^2 R_a 5T_m \end{aligned}$$

Second period:

$$\begin{aligned} \Delta A_{e2} &= \int_{5T_m}^{nT_m} \left[\frac{V_L}{nR_a} + I_L \right]^2 R_a dt = \\ &= \int_{5T_m}^{nT_m} \left[\frac{V_L^2}{n^2 R_a^2} R_a + \frac{2V_L I_L}{nR_a} R_a + I_L^2 R_a \right] dt = \\ &= \frac{V_L^2 R_a}{n^2 R_a^2} t \Big|_{5T_m}^{nT_m} + \frac{2V_L I_L}{nR_a} R_a t \Big|_{5T_m}^{nT_m} + I_L^2 R_a t \Big|_{5T_m}^{nT_m} = \\ &= \frac{V_L^2}{n^2 R_a} (nT_m - 5T_m) + \frac{2V_L I_L}{n} (nT_m - 5T_m) + I_L^2 R_a (nT_m - 5T_m) \end{aligned}$$

The third period:

$$\begin{aligned} \Delta A_{e3} &= \int_{nT_m}^{nT_m+5T_m} \left[\frac{V_L}{nR_a} e^{-\frac{t}{T_m}} + I_L \right]^2 R_a dt = \\ &= \int_0^{5T_m} \left[\frac{V_L^2}{n^2 R_a^2} e^{-\frac{2t}{T_m}} R_a + \frac{2V_L I_L}{nR_a} R_a + I_L^2 R_a \right] dt = \\ &= \frac{V_L^2}{n^2 R_a} \left[-\frac{T_m}{2} e^{-\frac{2t}{T_m}} \Big|_0^{5T_m} + \frac{2V_L I_L}{n} 5T_m + I_L^2 R_a 5T_m \right. \\ &= \frac{V_L^2 T_m}{2n^2 R_a} + \frac{10V_L I_L T_m}{n} + I_L^2 R_a 5T_m \\ \Delta A_e &= \sum_{i=1}^3 \Delta A_{ei} = \frac{3.5V_L^2 T_m}{n^2 R_a} + \frac{V_L^2 (nT_m - 5T_m)}{n^2 R_a} \\ &\quad + \frac{V_L^2 T_m}{2n^2 R_a} + \frac{8V_L I_L T_m}{n} + \frac{2V_L I_L (nT_m - 5T_m)}{n} + \\ &\quad + \frac{10V_L I_L T_m}{n} + 5I_L^2 R_a T_m + I_L^2 R_a (nT_m - 5T_m) + 5I_L^2 R_a T_m = \\ &= \frac{V_L^2 T_m (n-1)}{n^2 R_a} + \frac{2V_L I_L T_m (n+4)}{n} + I_L^2 R_a T_m (n+5) = \\ &= \frac{V_L^2 T_m (3.5+n-5+0.5)}{n^2 R_a} + \frac{2V_L I_L (2nT_m - 10T_m + 10T_m + 8T_m)}{n} \\ &\quad + I_L^2 R_a (5T_m + nT_m - 5T_m + 5T_m) = \\ &= \frac{V_L^2 T_m (n-1)}{n^2 R_a} + \frac{2V_L I_L T_m (n+4)}{n} + I_L^2 R_a T_m (n+5) = \end{aligned}$$

Then

$$\Delta A_{est} = \frac{J\omega_L^2 (n-1)}{n^2} + \frac{2\omega_L M_L (n+4)}{n} + I_L^2 R_a t_{mm} \quad (19)$$

In order to have an understanding of the components of losses in (19), we take a numerical example: P = 50 kW, J = 20 kgm², Ir = 220a, R_a = 0.05 ohm, C_e = C_m = 1.4, T_m = 0, 51s, I_c = 200A, M_c = 280 N.ω_L = 154.9 rad/s.

Let n = 15 Then

$$\Delta A_{e1} = \frac{20 * 154,9^2 (14)}{15^2} = 29859 \text{ Joules [j];}$$

$$\Delta A_{e2} = \frac{2 * 154,9 * 280 (19)}{15} = 109875 \text{ j;}$$

$$\Delta A_{e3} = 200^2 * 0,05 * 0,51 * 20 = 20400 \text{ j;}$$

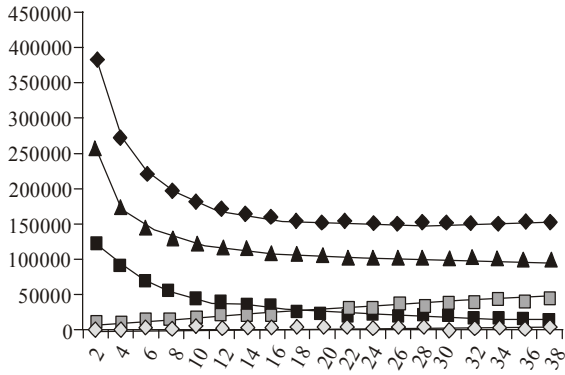


Fig. 7: Plotted to determine the optimum energy losses when the load torque, equal to half the nominal value

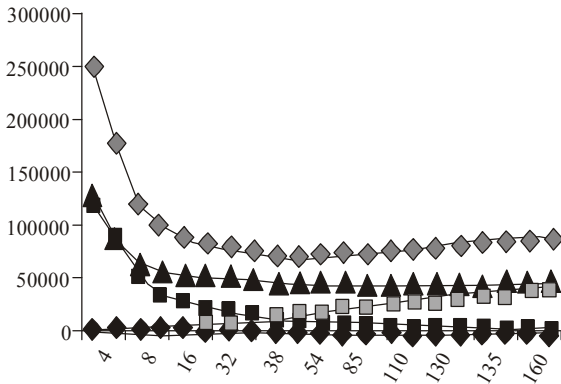


Fig. 8: Plotted to determine the decreasing values of load torque, the optimum energy loss shifts toward larger "n"

We investigate (19) on the optimum (minimum losses) for "n". Increase the length of the linear growth of the armature voltage up to n = 20. Then:

$$\Delta A_{e1} = \frac{20 * 154,9^2 (19)}{20^2} = 22794 \text{ j};$$

$$\Delta A_{e2} = \frac{2 * 154,9 * 280 (24)}{20} = 104093 \text{ j};$$

$$\Delta A_{e3} = 200^2 * 0,05 * 0,51 * 25 = 25500 \text{ j};$$

We see that as n increases the first component of losses (19) decreases, decreases slightly and the second component and the third $I_c^2 R_{\alpha} (n + 5) T_m$ is growing as increasing the acceleration time.

It follows that the function (19) has an extremum on (n). Find the value of (n) providing for (19) minimum:

$$\frac{d\Delta A_c}{dn} = \frac{J\omega_L^2 n^2 - 2nJ\omega_L (n-1)}{n^4} + \frac{2\omega_L M_L n - 2\omega_L M_L (n+4)}{n^2} + I_L^2 R_a T_m = 0$$

$$= J\omega_L^2 n^2 - 2J\omega_L n^2 + 2J\omega_L n + 2\omega_L M_L n^3 - 2\omega_L M_L n^3 + 8\omega_L M_L n^2 + I_L^2 R_a T_m n^4 = 0$$

$$= -J\omega_L n + 2J\omega_L + 8\omega_L M_L n + I_L^2 R_a T_m n^3 = 0$$

Finally $I_L^2 R_a T_m n^3 + n(8\omega_L M_L - J\omega_L) + 2J\omega_L = 0$;

This equation can be found analytically or by selection of "n". We use the latter:

Let n = 24,

$$\text{then } \Delta A_{e1} = \frac{20 * 154,9^2 * 23}{24^2} = 19161 \text{ j};$$

$$\Delta A_{e2} = \frac{2 * 154,9 * 280 * 28}{24} = 101201 \text{ j};$$

$$\Delta A_{e3} = 200^2 * 0,05 * 0,51 * 29 = 29580 \text{ j};$$

n = 26

$$\Delta A_{e1} = \frac{20 * 154,9^2 * 25}{26^2} = 17747$$

$$\Delta A_{e2} = \frac{2 * 154,9 * 280 * 30}{26} = 100089 \text{ j};$$

$$\Delta A_{e3} = 200^2 * 0,05 * 0,51 * 31 = 31620 \text{ j};$$

n = 28

$$\Delta A_{e1} = \frac{20 * 154,9^2 * 27}{28^2} = 16526 \text{ j}$$

$$\Delta A_{e2} = \frac{2 * 154,9 * 280 * 32}{28} = 99136 \text{ j};$$

$$\Delta A_{e3} = 200^2 * 0,05 * 0,51 * 33 = 33660 \text{ j};$$

n = 30

$$\Delta A_{e1} = \frac{20 * 154,9^2 * 29}{30^2} = 15462$$

$$\Delta A_{e2} = \frac{2 * 154,9 * 280 * 34}{30} = 98309 \text{ j};$$

$$\Delta A_{e3} = 200^2 * 0,05 * 0,51 * 35 = 35700 \text{ j};$$

That is, when n = 26, $\sum \Delta A = 149456 \text{ j}$ with n = 28 $\sum \Delta A = 149322 \text{ j}$, with n = 30 $\sum \Delta A = 149471 \text{ j}$.

Additionally in Fig. 7 plotted to determine the optimum energy-loss program using excel, at the moment the load is equal to the nominal value. From it also shows that the minimum loss occurs when "n" = 28.

From the graph in Fig. 8 shows that with decreasing values of load torque, the optimum energy loss shifts toward larger "n" (n opt = 48).

CONCLUSION

- One of the important factors affecting the value of start-up losses is the variation of armature voltage, formed by an electric transformer.
- A relation (10), allowing to select parameters of the law ramp voltage to provide the selected value of the current acceleration.

- Linear law of increase effective armature voltage step, since approximately 2-times reduced start-up losses.

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