

Research Article

Time Truncated Generalized Chain Sampling Plan for Pareto Distribution of the 2nd Kind

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Abstract: The main purpose of this study is to intend a generalized chain sampling plan for Pareto distribution of the 2nd kind. The designed parameters such as the minimum sample size and operating characteristic values are found by satisfying the consumer's risk at the specified quality levels in terms of average. The results are explained by an example.

Keywords: Chain sampling, consumer's risk, operating characteristic values, pareto distribution of the 2nd kind, producer's risk

INTRODUCTION

Acceptance sampling plan plays a vital role for increasing the quality of a submitted product by the vendor. The main goal of all the companies in the new era is to produce high quality products. The high quality products lead to the high chance of acceptance. In a life testing, sample is being selected randomly from a submitted lot and conduct an experimental test to observe the damaged product. If the number of damaged products is not more than the pre-assumed acceptance number then the submitted lot will be accepted and sent to the consumer's use. There are always two risks related to an acceptance sampling. The probability of rejecting the good lot is called producer's risk and the probability of accepting the bad lot is called the consumer's risk. They are being represented by α and β respectively. An acceptance sampling plan should be developed in such a way that, these two risks would become less than the required values.

A major issue always related with a life testing is to find the optimal sample size from the submitted lot. The criteria for acceptance and non-acceptance of a submitted lot partially depend on the information taken from the immediate preceded lot. This phenomenon is being adopted by Chain Sampling Plan (ChSP-1). Dodge (1955), proposed a concept of chain sampling inspection plan. He used the cumulative results by taking various samples to overcome the disadvantages of the single acceptance sampling plan. The ChSP-1 is more efficient than the single acceptance sampling plan in terms of required sample size. Many researchers proposed various acceptance sampling plans based on

truncated life test for different distributions, Epstein (1954), Goode and Kao (1961), Baklizi (2003), Tsai and Wu (2006), Aslam *et al.* (2010a, 2010b, 2011), Mughal *et al.* (2011), Mughal and Aslam (2011), Mughal (2011), Mughal and Ismail (2013), Ramaswamy and Jayasri (2014) and Mughal *et al.* (2015a, 2015b, 2015c, 2015d).

In life testing, the Pareto distribution of the 2nd kind is commonly used for fitting failure time distributions and the best of our consideration none has developed generalized chain sampling for this distribution. The objective of this research is to generalize the chain sampling plan and find the design parameters of the proposed plan considering a selection of various pre-specified plan parameters.

List of symbols:

- α : Producer's risk (Probability of rejecting a good lot)
- β : Consumer's risk (Probability of accepting a bad lot)
- n : Sample size
- c : Acceptance number
- i : Acceptance criteria
- d : Number of defective products.
- σ : Scale parameter of Pareto distribution of the 2nd kind
- λ : Shape parameter of Pareto distribution of the 2nd kind
- t_0 : Test termination time
- μ_0 : Specified average life of a product
- μ : True average life of a product
- $P(p)$: Lot Acceptance Probability
- (μ/μ_0) : Mean ratio

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METHODOLOGY

The Chain sampling Plan is characterized by the three parameters n, c and i . In usual chain sampling plan, the current lot under inspection can be accepted if one failure unit is observed in the sample, provided that no other failure units are found in the samples from the instantly preceding lots. But we have generalized usually chain sampling plan and now the experimenter can use the c and i values of his choice. The proposed plan is practiced in the following steps:

- For each lot, randomly select a sample of n units and test each unit to the specified requirements.
- Accept the lot when $d < c$ and reject the lot if $d > c$.
- If $d = c$ then continue the inspection and accept the lot if no defectives are observed in the immediately preceding lots.

The cumulative distribution function (CDF) of a Pareto distribution of the 2nd kind, according to Pareto (1897) can be written in this form:

$$F(t, \sigma, \lambda) = 1 - \left(1 + \frac{t}{\sigma}\right)^{-\lambda} \quad t > 0, \sigma > 0, \lambda > 0 \quad (1)$$

where, σ and λ are scale and shape parameters respectively. The probability of lot acceptance in the case of proposed plan is given by:

$$P_a(p) = \sum_{j=0}^{d_j < c} \binom{n}{j} p^j (i-p)^{n-j} + \binom{n}{c} p^c (1-p)^{n-c} (1-p)^{ni} \quad (2)$$

where, p is the probability of failure of a product in the experiment. If $t_0 = a\mu_0$, then p can be estimated by:

$$p = F(t; \sigma, \lambda) = 1 - \left[1 + \frac{a}{(\lambda-1)(\mu/\mu_0)}\right]^{-\lambda} \quad (3)$$

In Table 1 to 4, we present the minimum values of sample size n when satisfying the Eq. (2) for $\beta = 0.25, 0.10, 0.05, 0.01$ and $a = 0.7, 0.8, 1.0, 1.2, 1.5, 2$. Once the minimum sample size is found, one may be interested to find the probability of lot acceptance when the preferred quality standard of a product is needed.

Table 1: Minimum sample size for the proposed plan for the Pareto distribution of the 2nd kind with $\lambda = 2$

β	c	i	a						
			0.7	0.8	1.0	1.2	1.5	2.0	
0.25	1	1	2	2	2	2	2	2	1
	1	2	2	2	2	1	1	1	1
	1	3	2	2	2	1	1	1	1
	1	4	2	2	2	1	1	1	1
0.25	2	1	4	3	3	3	3	3	2
	2	2	4	3	3	3	3	3	2
	2	3	4	3	3	3	3	3	2
	2	4	4	3	3	3	3	3	2
0.25	3	1	5	5	5	4	4	4	4
	3	2	5	5	5	4	4	4	4
	3	3	5	5	5	4	4	4	4
	3	4	5	5	5	4	4	4	4
0.25	4	1	7	7	6	6	5	5	5
	4	2	7	7	6	6	5	5	5
	4	3	7	7	6	6	5	5	5
	4	4	7	7	6	6	5	5	5

Table 2: Minimum sample size for the proposed plan for the Pareto distribution of the 2nd kind with $\lambda = 2$

β	c	i	a						
			0.7	0.8	1.0	1.2	1.5	2.0	
0.10	1	1	3	3	2	2	2	2	2
	1	2	3	2	2	2	2	2	2
	1	3	3	2	2	2	2	2	2
	1	4	3	2	2	2	2	2	2
0.10	2	1	5	4	4	4	3	3	3
	2	2	5	4	4	4	3	3	3
	2	3	5	4	4	4	3	3	3
	2	4	5	4	4	4	3	3	3
0.10	3	1	7	6	6	5	5	4	4
	3	2	7	6	6	5	5	4	4
	3	3	7	6	6	5	5	4	4
	3	4	7	6	6	5	5	4	4
0.10	4	1	9	8	7	7	6	5	5
	4	2	9	8	7	7	6	5	5
	4	3	9	8	7	7	6	5	5
	4	4	9	8	7	7	6	5	5

Table 3: Minimum sample size for the proposed plan for the Pareto distribution of the 2nd kind with $\lambda = 2$

β	c	i	a					
			0.7	0.8	1.0	1.2	1.5	2.0
0.05	1	1	4	3	3	3	2	2
	1	2	3	3	3	2	2	2
	1	3	3	3	3	2	2	2
	1	4	3	3	3	2	2	2
0.05	2	1	6	5	5	4	4	3
	2	2	6	5	5	4	4	3
	2	3	6	5	5	4	4	3
	2	4	6	5	5	4	4	3
0.05	3	1	8	7	6	6	5	5
	3	2	8	7	6	6	5	5
	3	3	8	7	6	6	5	5
	3	4	8	7	6	6	5	5
0.05	4	1	10	9	8	7	7	6
	4	2	10	9	8	7	7	6
	4	3	10	9	8	7	7	6
	4	4	10	9	8	7	7	6

Table 4: Minimum sample size for the proposed plan for the Pareto distribution of the 2nd kind with $\lambda = 2$

β	c	i	a					
			0.7	0.8	1.0	1.2	1.5	2.0
0.01	1	1	5	4	4	3	3	3
	1	2	5	4	4	3	3	3
	1	3	5	4	4	3	3	3
	1	4	5	4	4	3	3	3
0.01	2	1	7	7	6	5	5	4
	2	2	7	7	6	5	5	4
	2	3	7	7	6	5	5	4
	2	4	7	7	6	5	5	4
0.01	3	1	10	9	8	7	6	6
	3	2	10	9	8	7	6	6
	3	3	10	9	8	7	6	6
	3	4	10	9	8	7	6	6
0.01	4	1	12	11	9	9	8	7
	4	2	12	11	9	9	8	7
	4	3	12	11	9	9	8	7
	4	4	12	11	9	9	8	7

Table 5: Operating characteristics values having $c = 2, i = 2$ for Pareto distribution of the 2nd kind with $\lambda = 2$

β	n	a	2	4	6	8	10	12
0.25	4	0.7	0.3919	0.7124	0.8481	0.9114	0.9442	0.9627
	3	0.8	0.5218	0.7988	0.9001	0.9439	0.9655	0.9774
	3	1.0	0.4202	0.7216	0.8506	0.9117	0.9439	0.9622
	3	1.2	0.3401	0.6487	0.7988	0.8758	0.9185	0.9439
	3	1.5	0.2508	0.5511	0.7216	0.8183	0.8785	0.9117
	2	2.0	0.4397	0.7034	0.8278	0.8921	0.9283	0.9501
0.10	5	0.7	0.2551	0.5902	0.7635	0.8539	0.9042	0.9341
	4	0.8	0.3297	0.6565	0.8095	0.8852	0.9261	0.9498
	4	1.0	0.2347	0.5542	0.7316	0.8289	0.8852	0.9196
	4	1.2	0.1688	0.4662	0.6565	0.7704	0.8405	0.8852
	3	1.5	0.2508	0.5511	0.7216	0.8183	0.8785	0.9117
	3	2.0	0.1564	0.4202	0.6035	0.7216	0.7988	0.8506
0.05	6	0.7	0.1622	0.4807	0.6774	0.7904	0.8576	0.8994
	5	0.8	0.2009	0.5250	0.7118	0.8156	0.8761	0.9133
	5	1.0	0.1258	0.4133	0.6133	0.7375	0.8156	0.8663
	4	1.2	0.1688	0.4662	0.6565	0.7704	0.8405	0.8852
	4	1.5	0.1052	0.3594	0.5542	0.6841	0.7704	0.8289
	3	2.0	0.1564	0.4202	0.6035	0.7216	0.7988	0.8506
0.01	7	0.7	0.1012	0.3867	0.5946	0.7248	0.8069	0.8602
	7	0.8	0.0695	0.3197	0.5274	0.6673	0.7600	0.8224
	6	1.0	0.0655	0.3022	0.5056	0.6463	0.7415	0.8067
	5	1.2	0.0801	0.3246	0.5250	0.6615	0.7531	0.8156
	5	1.5	0.0420	0.2263	0.4133	0.5568	0.6615	0.7375
	4	2.0	0.0508	0.2347	0.4152	0.5542	0.6565	0.7316

For a fixed c and i , the operating characteristic values as a function of (μ/μ_0) are shown in Table 5.

Illustrative example and comparison: A comparative study is accomplished regarding sample sizes n of the proposed plan and the existing plan developed by Ramaswamy and Jayasri (2014). Suppose μ and μ_0 are the true and specified average life of a product and it is acceptable only when $\mu > \mu_0$. If an experimenter is interested in knowing that the true average life of a product is at least 1000 h with confidence level of 0.99 and the life time of product follow Pareto distribution of the 2nd kind. He would like to select $a = 0.70$, $c = 1$, $i = 4$ and $\lambda = 2$ then the required minimum sample size is $n = 5$, as shown in Table 4. The proposed plan requires 5 products while the existing plan needs 24 products to achieve a similar inference about the submitted lot.

CONCLUSION

In this study, generalized chain sampling plan for the truncated life test is proposed in the case of Pareto distribution of the 2nd kind. It is assumed that the shape parameter is known and the minimum sample size is found when the other designed parameters are specified. From Table 5 we can see that the probability of acceptance increases when (μ/μ_0) increases and it reaches the maximum value 1 when (μ/μ_0) is greater than 12. Hence, the proposed plan meet the economic conditions because cost, time, energy and labor in the life testing are directly involved with the number of products being inspected. We suggest using the proposed plan in order to save all the manufacturing factors of the experiment to reach the same conclusion as the ordinary plans. It can be further calculated for many other lifetime distributions, as well as other quality and reliability parameters.

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