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Research Article

Efficient Linear Processing for Non-regenerative MIMO Relay Network with Perfect CSI

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Abstract: In this study, two hop non-regenerative Multiple Input Multiple Output (MIMO) relay network is considered. Each node of the network is equipped with multiple antennas. We apply an efficient linear processing at the relay node only. We utilize Minimum Mean Square Error (MMSE) as the relay receive beamforming and Per-Antenna-Signal to Leakage plus Noise Ratio (PA-SLNR) maximization criteria using Fukunaga Koontz Transform (FKT) is considered to generate relay downlink precoding to mitigate the interference due to the leakage signal. The relay acquires perfect channel state information of the channel between source to relay and relay to the destination. Monte Carlo simulation confirms better performance in terms of ergodic capacity offered by the proposed scheme.

Keywords: Ergodic capacity, non-regenerative relay, per antenna signal to leakage plus noise, precoding

INTRODUCTION

The fundamental concept of transmitting information signal over wireless channel using three node communication network was presented by Van Der Meulen (1971). An appropriate condition for upper and lower bounds on the capacity were derived. Multiple Input and Multiple Output (MIMO) systems validate the link reliability and improvement in the ergodic capacity. The deployment of relay node between the transmitter and the receiver helps to transport information signal with extended range and coverage for point-to-point or multi point MIMO systems. Due to cooperative nature, relays are significantly attracted by the next generation and Long Term Evolution (LTE) networks and considered as a key component (Chandra *et al*., 2011). The most familiar and broadly studied relays are non-regenerative or Amplify-and-Forward (AF) and regenerative or Decode-and-Forward (DF) relays. The function of regenerative relay is to decode the received information signal then re-encoded forward the signal to user/destination node, while, in non-regenerative relay some linear processing is applied on the received signal then forwarded to destination (Chandra *et al*., 2011; Chen, 2013). The non-regenerative relay is widely studied because of implementation simplicity and low complexity. In literature there are various configurations of relay based MIMO networks. In this study, we will consider a non-regenerative MIMO relay operating in half-duplex mode.

Various relay processing techniques have been found in literature to improve network performance by controlling transmission line impairments in wireless systems. Precoding schemes are considered a solution for sinking interference level among multiple streams (Gharan *et al*., 2011; Fawaz *et al*., 2011; Lee *et al*., 2010; Zhang *et al*., 2011; Chen *et al*., 2008; Firag *et al*., 2009; Bolcskei *et al*., 2006; Mohammadi *et al*., 2010; Wang *et al*., 2012; Jin *et al*., 2010; Xu *et al*., 2010; Muñoz-Medina *et al*., 2007; Vouyioukas, 2013).

The authors (Shi *et al*., 2007) proposed three different relay beamforming techniques for increasing ergodic capacity in multi-relay MIMO network. Matched filter and regularized zero forcing based relay beamforming has proposed for dual-hop multi-relay MIMO network to control the interference (Zhang *et al*., 2010). Where, matched filter was used as the relay receive beamforming for controlling noise at the relay node, and regularized zero forcing technique proposed as the relay transmit precoder to control interference among different data streams.

The magnitude of interference among multiple data streams is controlled by the regulating factor in second hop of the network. If the value of this parameter is very small or equal to zero then the RZF precoding technique approaches to zero-forcing precoding and if the value of regulating factor is too large then the equivalent channel matrix will depart from its diagonal matrix, which causes more power expenditure and interference across the data streams. The work presented by the authors (Zhang *et al*., 2010) MF-RZF relay beamforming has shown better ergodic capacity performance in multi-relay network scenario as compared with the relay precoding schemes proposed by Shi *et al*. (2007). The authors (Wang *et al*., 2012) proposed a linear relay beamforming scheme based on MMSE-RZF for dual-hop MIMO network, where MMSE is used as relay

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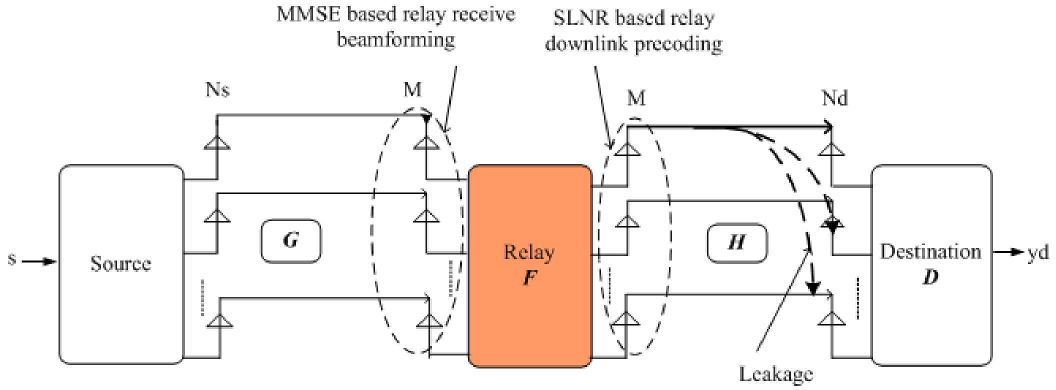

Fig. 1: Schematic diagram of dual-hop MIMO relay network

receive beamforming and RZF is applied as relay broadcast precoding.

The concept of leakage based precoding technique that controls interference caused by leakage signal from the desired signal is not used as the relay downlink precoding along with MMSE relay receive beamforming. The idea of leakage signal is the desired signal leaks a signal that causes interference with other antennas of the same user equipped with multiple antennas. The purpose of leakage based criteria using FKT is to generate vector by vector precoding to maximize element wise signal to leakage plus noise ratio. Pictorial demonstration of the leakage signal is shown Fig. 1.

In this study, we propose (MMSE-PA-SLNR-FKT) linear processing at the relay node. Here Minimum Mean Square Error (MMSE) based beamforming is used at the relay receiving side and Fukunaga Koontz Transforms (FKT) based PA-SLNR maximization technique is exploited as the relay down link precoding. The down-link relay precoding can control the interference caused by leakage from the desired signal for the intended antenna, that causes interference to other antennas and vice versa. We implement this scheme for dual-hop single non-regenerative MIMO relay network.

In this study, the vectors and matrices are symbolized as lower and uppercase boldface letters respectively. The notation $(G)_{i,l}$ represents i^{th} rows and l^{th} entries of matrix G, the trace of matrix is denoted by tr, G^H is matrix Hermitian and I_{N_s} is $N_s \times N_s$ identity matrix, $\|x\|$ is Euclidean norm of a vector x. $E\,(.)$ represents the expectation operation.

SYSTEM MODEL

We consider single user amplify-and-forward relay dual-hop MIMO network as shown in Fig. 1 with single source, single relay and single user/destination node equipped with multiple antennas. The number of antennas at source and user nodes is represented as N_s and N_d, respectively, where M denotes the number relay receive and transmit antennas.

The complex channel matrix between source and relay link is denoted by $G \in C^{M\times N_s}$ with $(m,n)^{th}$ entries. The element $h_{m,n}$ represents the channel gain from source antennas $n = 1, \dots, N_s$ to the relay receive antennas $m = 1, \dots, M$. Here $H \in C^{N_d\times M}$ is the complex channel between relay and user/destination nodes with $(j,i)^{th}$ entries. The element $h_{j.i}$ denotes the channel gain from the relay transmit antenna i to the user receive antenna j, thus $i = 1, \dots, M$ and $j = 1, \dots, N_d$. The channels between source to relay and relay to user are assumed as independent and identically distributed (i.i.d) (Primak and Kontorovich, 2011). Here, n_r denotes zero mean and variance additive Gaussian complex noise vector at the relay node that is $n_r \in CN\,(0, \sigma_r^2 I_M)$ and noise covariance is, $E\,\{n_r n_r^H\} = \sigma_r^2 I_M$ and noise variance at relay is denoted by σ_r^2. Noise covariance and noise variance at the user are shown by $E\,\{n_d n_d^H\} = \sigma_d^2 I_M$ and σ_d^2, respectively. The direct link between the source and the user nodes is ignored due to long distance fading and other impairments. The transmission of information in dual-hop network takes place into two phases.

In first phase, the source node transmits the modulated signal vector x as $N_s \times 1$ to the relay node with transmit signal covariance matrix, $R_x = E\,\{xx^H\}$ where, P_s is the source power, I_{N_s} the identity matrix of $N_s \times N_s$ size. When channel knowledge is unknown to the source node then uniform power allocation strategy is adopted for signal transmission from source to the relay as $R_x = \frac{P}{N_s} I_{N_s}$. The signal received at relay node is given as:

$$y_1 = Gx + n_r \tag{1}$$

Here x is the source transmit signal vector, y_1 is the signal vector received by the relay node, n_r is complex Gaussian noise vector at the relay node and $G \in C^{M\times N_s}$ is complex channel matrix between source and relay whose complex gain from i^{th} transmit antenna of source to j^{th} receive antenna of relay is denoted with $g_{j,i}$. When G is random matrix and ergodic, we assume its elements are i.i.d complex Gaussian with zero mean and unit variance having uniform phase and Rayleigh magnitude.

In second phase, a linear processing is applied on the received signal at relay and then the relay forwards the processed signal to the user that can be expressed as:

$$\hat{y}_1 = F y_1 \tag{2}$$

Here F is the relay beamforming matrix. A self-power constraint is assumed at the relay node that is independent of source power P_s, which satisfies the condition $E\,\{\hat{y}_1^H \hat{y}_1\} \leq P_r$, where P_r is the relay transmit power. Therefore the power constraint condition of the relay transmit signal $\hat{y}_1$ can be derived as:

$$\zeta\,(\hat{y}_1) = \left\{F\left(\frac{P_s}{N_s} G G^H + \sigma_r^2 I_M\right)\right\} \leq P_r \tag{3}$$

From (3) the power control factor ρ can be generated as:

$$\rho = \sqrt{\frac{P_r}{\zeta(\hat{y}_1)}} = \sqrt{\frac{P_r}{\left\{F\left(\frac{P_s}{N_s} G G^H + \sigma_r^2 I_M\right)\right\}}} \tag{4}$$

Finally signal received at the designation is given as:

$$y_d = \rho H F G x + H F n_r + n_d \tag{5}$$

MMSE BASED RELAY RECEIVE BEAMFORMING DESIGN

In this section, we use Minimum Mean Square Error (MMSE) as the relay receives beamforming. In this antenna array signals received from multiple antennas elements are weighted and combined to produce an output signal. The beamforming weights can be complex, with both phase and amplitude. When interference is present, the received signals can be combined to increase the output SNR this is done by these weights. In output signal, when these weights are used to minimize mean square error then this technique is termed as MMSE beamforming (Bölcskei *et al*., 2006):

$$F_{MMSE} = \left(G^H G + \sigma_r^2 I_{N_s}\right)^{-1} G^H \tag{6}$$

where,

G^H : The Hermitian of the channel matrix G, the complex channel between source and relay

σ_r^2 : The noise variance of white Gaussian noise

I_{N_s} : Identity matrix of the same size as the number of transmit antennas

PA-SLNR-FKT relay downlink precoding: In this section, we use leakage based per antenna signal to leakage plus noise ratio precoding as relay downlink precoder using FKT. The FKT based PA-SLNR downlink precoding reduces computational load of the precoder controls the interference among multiple antennas. The pictorial view of leakage signal from desired signal is illustrated in second hop of the given network in Fig. 1.

The leakage signal is the source of interference with other antennas of same user equipped with multiple antennas that degrades the system performance. The purpose of this design is to control interference caused by the leakage from intended signals and to maximize per antenna SLNR that improves the ergodic capacity gain performance. The Per Antenna Signal to Leakage plus Noise Ratio (PA-SLNR) for single user system is written as the ratio of desired received signal power to leakage signal plus noise power at user end that is given by:

$$SLNR^j = \frac{\left\|h^j f^j\right\|_F^2}{\sum_{\substack{j=1 \\ k\neq j}}^{N_d} \left\|h^j f^k\right\|_F^2 + \sigma_d^{2^j}} \tag{7}$$

The numerator in Eq. (7) shows desired signal and denominator has summation of total leakage power plus noise power at the j^{th} receive antennas.

Here h^j shows the complex row vector and $\sigma_d^{2^j}$ represents noise variance of the j^{th} antenna elements at the user end, where, $j = 1, \ldots, N_d$ and $\sigma_d^{2^j} = \sigma_1^2, \ldots, \sigma_d^2$. A vector by vector relay downlink precoder for each antenna is symbolized by $F^j = f^1, \ldots, f^{N_d}$ and is generated for each receive antenna independently to maximize per antenna signal to leakage plus noise ratio.

Therefore the PA-SLNR maximization criterion based precoder is built as:

$$F = \arg \max_{F^j \in C^{M\times 1}} SLNR^j\,(F^j)$$

or,

$$F = \arg \max_{F^j \in C^{M\times 1}} \frac{f^{j^H} (h^{j^H} h^j) f^j}{f^{j^H} (H^{j^H} H^j + \sigma_d^{2^j} I_M) f^j}$$

Subject to:

$$\zeta\,(\hat{y}_1) \leq P_r \tag{8}$$

After multiplying minimum mean square error based relay receive beamforming (6) with (8), we get MMSE-PA-SLNR linear processing at the relay node as:

$$F = \arg \max_{F^j \in C^{M\times 1}} \frac{f^{j^H} (h^{j^H} h^j) f^j}{f^{j^H} (H^{j^H} H^j + \sigma_d^{2^j} I_M) f^j} \times F_{MMSE} \tag{9}$$

where, F_{MMSE} is the MMSE beamformer at the relay receiving side.

Precoding optimization problem: The optimization problem of (8) is mapped by using Fukunaga Koontz Transform (FKT). The transform has been broadly utilized in several signal processing systems. The details of FKT can found in Saeid *et al*. (2012) and Saand *et al*. (2014). The precoding optimization problem of (8) is mapped as follows:

$$B_1 = {h^j}^H h^j \tag{10}$$

$$B_2 = {H^j}^H H^j + {\sigma_d^2}^j I_M \tag{11}$$

Here B_1 and B_2 the sum of the two covariance matrices is given as:

$$B = B_1 + B_2 \tag{12}$$

The positive semi definite and symmetric auto correlation matrices $B_1 = X_1 + X_1$ and $B_2 = X_2 + X_2$ of the two data matrices X_1 and X_2 from two classes, the addition of these both auto correlation matrices is still positive semi definite and symmetric. The auto correlation matrix B is factorized by applying singular value decomposition:

$$B = B_1 + B_2 = [U \Lambda V^T] \tag{13}$$

Here, Λ represents diagonal matrix with non-zero singular values. The set of eigenvectors corresponding to eigenvalues is $U \in R^{\Lambda \times 1}$ is the unitary matrix and r is the rank of matrix.

When auto-correlation matrix whitened by FKT transform by using FKT factor $P = U\Lambda^{-\frac{1}{2}}$ then the sum of covariance matrices $B_1 + B_2$ becomes:

$$P^T BP = P^T(B_1 + B_2)P = \tilde{B}_1 + \tilde{B}_2 = I \tag{14}$$

where, $\tilde{B}_1 = P^T B_1 P$, $\tilde{B}_2 = P^T B_2 P$ and $I \in R^{r \times r}$ is identity matrix. v is the eigenvector of $\tilde{B}_1$ having eigenvalue σ_1 that is $\tilde{B}_1 v = \sigma_1 v$, since $\tilde{B}_1 = I - \tilde{B}_2$ so it can be rewritten as:

$$\left(I - \tilde{B}_2\right)v = \sigma_1 v \tag{15}$$

$$\tilde{B}_2 v = (1 - \sigma_1)v \tag{16}$$

It is clear from (16) that $\tilde{B}_2$ has same eigenvector as $\tilde{B}_1$ and the corresponding eigenvalue of $\tilde{B}_1$ is the weakest eigenvector of $\tilde{B}_2$ and vice versa (Saeid *et al*., 2012). The FKT factor for each receive antenna can be calculated by using $P = U\Lambda^{-\frac{1}{2}}$ factor, this generates two shared Eigen-space matrices $\tilde{B}_1$ and $\tilde{B}_2$.

The eigenvectors of unitary matrix U are multiplied by FKT factor $\mathrm{P} = U\Lambda^{-\frac{1}{2}}$ correspond to the best eigenvalue of the diagonal matrix Λ of transformed antenna covariance matrix, or the eigenvector that is corresponding to the least eigenvalue of the transformed leakage plus noise covariance matrix (Saeid *et al*., 2012). It is noted that for number of $N_d + 1$ receive antennas the FKT factor is only one time computed, which reduces the computational load.

The corresponding received signal at the user end is given as:

$$y_d = \sum_{\substack{j=1 \\ k \neq j}}^{N_d} \rho\, HFGx + \sum_{\substack{j=1 \\ k \neq j}}^{N_d} H\,Fx_i \\ + HFn_r + n_d \tag{17}$$

The first term of (17) at the right hand side represents desired signal forwarded by the relay node and the second term shows interference plus noise signal, whereas x_i represents inference signal. An equivalent complex channel of both channels is considered as in Zhang *et al*. (2010), is $G_{sd} = \rho HFG$ that comes from (17). The QR decomposition of the equivalent channel is given as (Zhang *et al*., 2010):

$$G_{sd} = QRD \tag{18}$$

The (18) gives Q and R as output of QR decomposition, Q is unitary matrix and R is upper right triangular matrix. The destination filter based on QRD is set as:

$$W = (Q_{sd})^{\mathrm{H}} \tag{19}$$

Consequently, the effective SINR for j^{th} data streams with MMSE-PA-SLNR relay precoding is given by:

$$SINR^j = \frac{\left(\frac{P_s}{N_d}\right) r_{j,j}^2}{\left\|(\rho(Q_{sd})^H H f^j)_j\right\|^2 {\sigma_r^2}^n + {\sigma_d^2}^j} \tag{20}$$

In (20) term $r_{j,j}^2$ has j^{th} diagonal entries of the upper triangular matrix. Finally the network ergodic capacity is given as:

$$C_{erg} = E_{\{G,H\}} \\ \left\{\frac{1}{2}\sum_{\substack{j=1 \\ k \neq j}}^{N_d} \log_2\left(1 + \frac{\left(\frac{P_s}{N_d}\right) r_{j,j}^2}{\left\|(\rho(Q_{sd})^H H f^j)_j\right\|^2 {\sigma_r^2}^n + {\sigma_d^2}^j}\right)\right\} \tag{21}$$

In (21) C_{erg} represents the ergodic capacity. The upper bound capacity for MIMO relay networks (Bolcskei *et al*., 2006) is given as:

$$C_{upper} = E_{\{G\}}\left\{\frac{1}{2}\log\left|I_{N_s} + \frac{P_s}{N_s \sigma_r^2} G^H G\right|\right\} \tag{22}$$

In (22) C_{upper} denotes upper bound capacity, the factor $1/2$ represents that the signal is transmitted into two time slots and $|H|$ shows determinant.

NUMERICAL RESULTS

The system performance is measured in terms of ergodic capacity given in (21). The Monte Carlo (MC) simulations are performed to compare the performance

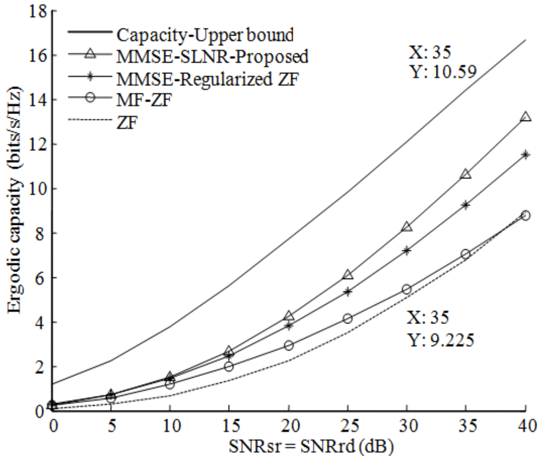

Fig. 2: Performance comparison of MMSE-PA-SLNR-FKT and MMSE-RZF precodings under system configuration of $N_s = M = N_d = 4$

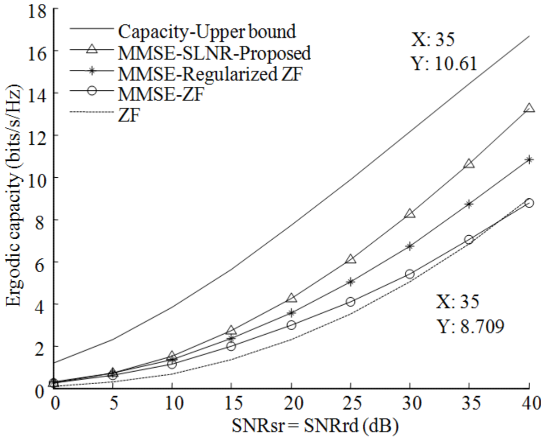

Fig. 3: Performance comparison of MMSE-PA-SLNR-FKT and MMSE-RZF precodings under system configuration of $N_s = M = N_d = 4$ and $\alpha = 0.5$

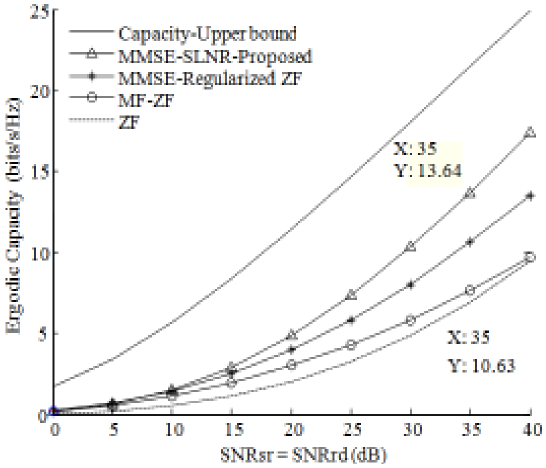

Fig. 4: Performance comparison of MMSE-PA-SLNR-FKT and MMSE-RZF precodings under system configuration of $N_s = M = N_d = 6$ and $\alpha = 0.5$

of proposed MMSE-PA-SLNR-FKT precoding scheme with MMSE-RZF (Wang *et al*., 2012), MMSE-ZF and ZF-ZF schemes. A dual-hop single non-regenerative relay based MIMO network is considered. Following system parameters are taken for simulations in this setup. The number of antennas at each node is set as 4 and 6. The noise variance at relay node and user nodes is selected as:

$$\sigma_r^2 = \sigma_d^2 = 1$$

We will assume that there is independent Rayleigh fading at each antenna, so that the columns of the channels are linearly independent and the channel elements are complex independent and identically-distributed zero-mean Gaussian random variables. The noise is assumed to be additive zero-mean i.i.d. Gaussian as well. We assume perfect CSI of both the channels at the relay node. The upper bound capacity is selected as a baseline (Bolcskei *et al*., 2006).

In Fig. 2 the ergodic capacity performance behavior of MMSE-PA-SLNR-FKT relay precoding scheme is compared with MMSE-RZF. The performance curves in Fig. 2 Illustrates that at $SNR_{sr} = SNR_{rd}\,35\,dB$, 1.365 bits/sec/Hz capacity gain is achieved with proposed schemes over the MMSE-RZF scheme, when the regularization factor for RZF is set at high value of $\alpha = 1$.

In Fig. 3 the performance curves for ergodic capacity illustrates that at $SNR_{sr} = SNR_{rd} = 35\,dB$ and $\alpha = 0.5$, 1.901 bits/sec/Hz capacity gain is achieved. It is noted that, the performance of MMSE-RZF scheme decreases by decreasing the value of regularization factor. If the value of α is much decreased then MMSE-RZF approaches to the MMSE-ZF.

In Fig. 4 the simulation results exhibit the performance of proposed technique with 6 antennas at each node. At $SNR_{sr} = SNR_{rd} = 35\,dB$ and $\alpha = 0.5$, 3.01 capacity gain is achieved. It is noted that, the performance of proposed scheme is again better with increased number of antennas at each node.

In summary, It has been validated through Monte Carlo simulations that by applying MMSE as relay receive beamforming and leakage based relay precoding at the relay downlink gives better performance and the use of FKT transform in relay downlink precoding optimization reduces computational load.

CONCLUSION

A linear processing based on MMSE-PA-SLNR-FKT has been derived for the non-regenerative MIMO relay. MMSE is exploited as the relay receiving beamforming and PA-SLNR maximization criteria based on FKT as the relay downlink precoder. Besides of conventional relay precoding techniques which control the noise and interference among multiple data streams at the relay and user nodes, the proposed precoding scheme controls the noise and interference at the relay receiver side and cancels interference to the other antennas of same user caused by the leakage signal. In terms of ergodic capacity the numerical results demonstrate the improved performance of proposed scheme bases on MMSE and PA-SLNR-FKT schemes. The proposed scheme will be applied in dual-hop multi relay MIMO network as future work.

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