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Research Article

Study of the Natural Convection of a Newtonian Fluid in a Porous Medium Confined in Portions of Cylinders

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Abstract: By using a bicylindric coordinates and vorticity-stream function formalism, the authors study the natural convection in a porous medium in an enclosure delimited by portions the cylindrical enclosure. After having a dimensionnalise our equations, the space discretization is performed using a finite difference method while a purely implicit schema is adopted for the time discretization. The algebraic systems of equations of the discretization are solved by a Successive under Relaxation method (SUR). The results obtained show the dependence of the heat transfers in various porous median (several of the Darcy number).

Keywords: Bicylindric coordinates, convection, fluid, heat, porous medium

INTRODUCTION

The study of natural convection in porous mediuminan enclosure delimited by portions of cylinders makes possible tobetter understandthe heat transfers in various industrial applications such as proceeds of drying, ingeophysical flow, in the storage of radioactive waste, in crystal growth, etc.

The majority of experimental, analytical and numerical work which treats of heat and mass transfer in porous medium were summarized in the study of Nield and Bejan (1992). By using the Darcy-Boussinesq model, Mota *et al*. (2000) studied the natural convection in an eccentric elliptic porous ring. The results presented provide an alternative approach for optimize the rate of thermal transfer by a suitable choice of the annular form.

Bennacer *et al*. (2002) presented a numerical study of the thermosolutal natural convection in an anisotropic porous rectangular v cavity with the Darcy-Brinkman formulation. They decoupled in this study the effects of the thermal and hydraulic anisotropy in order to be able to analyze more precisely the influence of each anisotropy. A study of quality of the heat insulation with a porous layer has been done by Saada *et al*. (2007), the non Darcian model is considered to describe the flow in the porous medium. Bousri and Bouhadef (2007) brought a contribution to the research of heat and mass transfers in the flows of certain fluids in reactive mediums. They quantified the various mass and heat transfers and they established the variation of concentration and temperature in the reactive porous medium, according to a certain number of parameters such as the Darcy number, the Reynolds number and the modified. Frank-Kamenetskii number.

Sankar *et al*. (2011) studied the natural convection in a vertical ring filled with a saturated porous material. They analyzed the effects of Rayleigh and Darcy number with various lengths of the heat source.

Mathematical approach: The aim of this study is to study natural convection in a porous medium containe delimited by portions of nonconcentric cylinders (Fig. 1). The cylindrical wall θ_1 is maintained at a temperature T_1 while the external cylinder θ_2 is maintained at a temperature below T_2. The two other cylinders (η_1 and η_2) walls located on either sides ofthe symmetry axisare thermally insulated.
To formulate and solve this problem it is assumed that:

- The transfers are two-dimensional and laminar
- All the phenomena are symmetrical
- All fluid properties are taken to be constant, with the exception of the density in the momentum equation. In this equation variations of density obey to the Boussinesq linear law
- In the heat equation the viscous dissipation functions as well as the compression effects are neglected
- The porous medium is considered homogeneous, isotropic and does not undergo space variation of porosity in the enclosure

Taking into account the geometry of our enclosure, we employ abicylindrical system of coordinates (η, θ)

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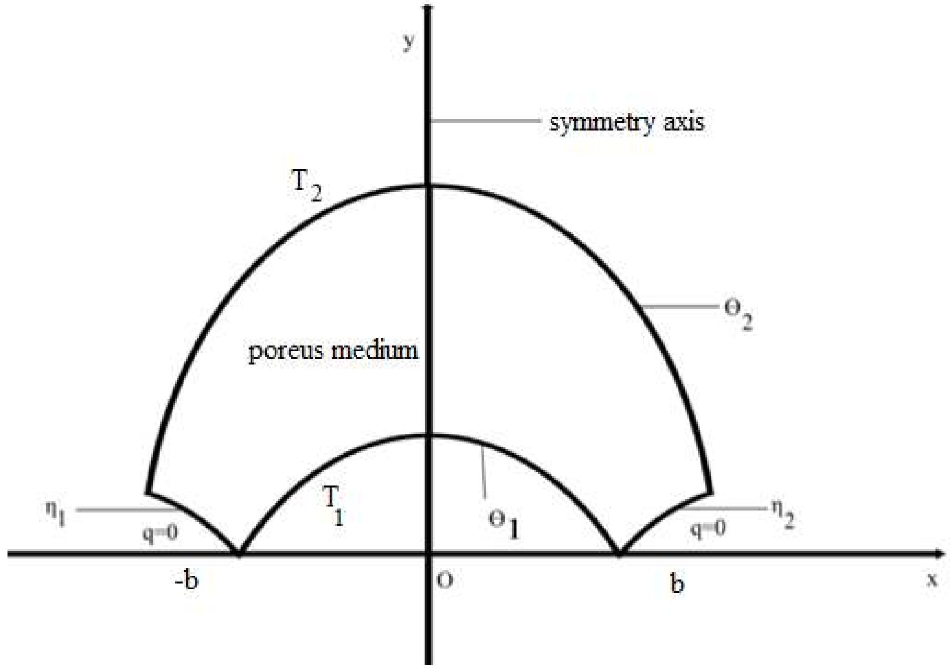

Fig. 1: Geometry of the problem

(Moon and Spencer, 1971) in which the boundaries of our cavity are given by constant coordinates lines. The passage of Cartesian coordinates to the bicylindrical coordinates is carried out using the following relations:

$$x = \frac{b \sinh \eta}{\cosh \eta - \cos \theta}, y = \frac{b \sin \theta}{\cosh \eta - \cos \theta}, z = z \qquad (1)$$

where b is the parameter of torus pole.

By introducing the stream function and the vorticity, transfers equations and the boundary conditions associated are written in the following dimensionless forms:

Stream function equation:

$$\Omega = -\frac{1}{H^2}\left(\frac{\partial^2 \Psi}{\partial \eta^2} + \frac{\partial^2 \Psi}{\partial \theta^2}\right) \qquad (2)$$

Momentum equation:

$$\frac{\partial \Omega}{\partial t} + \frac{U}{H}\frac{\partial \Omega}{\partial \eta} + \frac{V}{H}\frac{\partial \Omega}{\partial \theta} = \frac{Pr}{H^2}\left[\frac{\partial^2 \Omega}{\partial \eta^2} + \frac{\partial^2 \Omega}{\partial \theta^2}\right] - \frac{\Pr}{Da}\Omega - \frac{C}{\sqrt{Da}}\left|\vec{V}\right|\Omega + \frac{Ra * Pr}{H}\left[G_1 \frac{\partial T}{\partial \eta} + G_2 \frac{\partial T}{\partial \theta}\right] \qquad (3)$$

Heat equation:

$$\frac{\partial T}{\partial t} + \frac{U}{H}\frac{\partial T}{\partial \eta} + \frac{V}{H}\frac{\partial T}{\partial \theta} = \frac{1}{H^2}\left[\frac{\partial^2 T}{\partial \eta^2} + \frac{\partial^2 T}{\partial \theta^2}\right] \qquad (4)$$

where:

$$U = \frac{1}{H}\frac{\partial \Psi}{\partial \theta}$$
$$V = -\frac{1}{H}\frac{\partial \Psi}{\partial \eta}$$

The length, velocity, steam function, vorticity and time scale are respectively defined by:

$$b, \frac{\alpha}{b}, \alpha, \frac{\nu}{b^2} \text{ and } \frac{b^2}{\alpha}$$

The associated initial and boundary conditions of the problem considered are:

At $t = 0$

$$U(\eta,\theta) = V(\eta,\theta) = \psi(\eta,\theta) = \Omega(\eta,\theta) = T(\eta,\theta) = 0 \qquad (5)$$

At t>0 the boundary conditions are the following ones:

- On the inner cylinder $(\theta = \theta_1)$:

$$U = V = \left.\frac{\partial \psi}{\partial \theta}\right|_{\theta=\theta_1} = \left.\frac{\partial \psi}{\partial \eta}\right|_{\theta=\theta_1} = 0 \qquad (6)$$

$$\Omega = \left.\frac{-1}{H^2}\frac{\partial^2 \psi}{\partial \theta^2}\right|_{\theta=\theta_1} \qquad (7)$$

$$T = 1 \qquad (8)$$

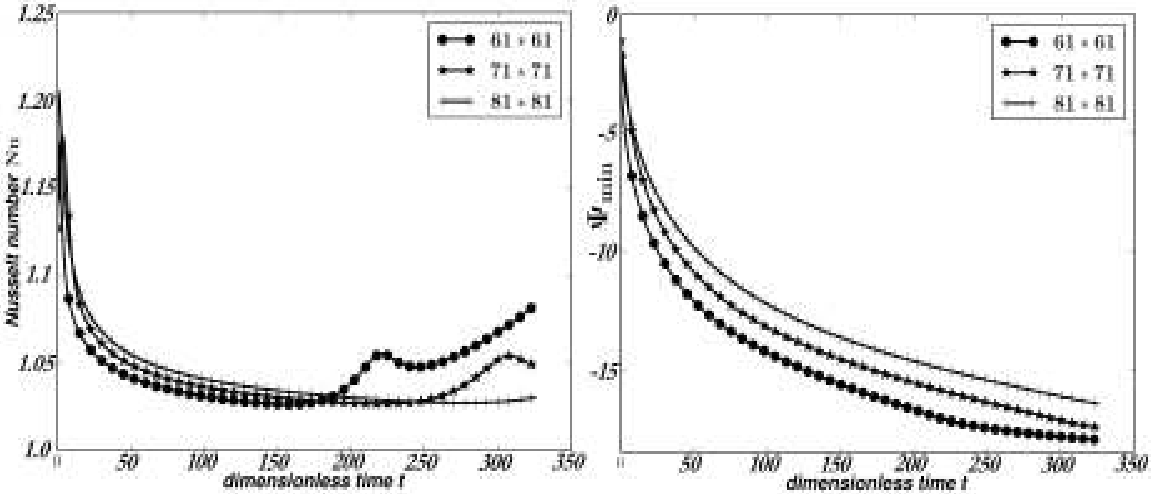

Fig. 2: Variation of the Nusselt number and the minimal of the stream function against

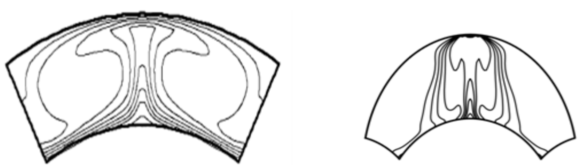

Fig. 3: Isotherms observed by Sarr (left) at $Gr = 0.1.10^7$ and by us (right) at $Ra = 10^7$

- On the outer cylinder $(\theta = \theta_2)$:

$$U = V = \left.\frac{\partial \psi}{\partial \theta}\right|_{\theta=\theta_2} = \left.\frac{\partial \psi}{\partial \eta}\right|_{\theta=\theta_2} = 0 \tag{9}$$

$$\Omega = \frac{-1}{H^2}\left.\frac{\partial^2 \psi}{\partial \theta^2}\right|_{\theta=\theta_2} \tag{10}$$

$$T = 0 \tag{11}$$

- On the walls η_1 and η_2 :

$$U = V = \left.\frac{\partial \psi}{\partial \eta}\right|_{\eta_1,\eta_2} = \left.\frac{\partial \psi}{\partial \theta}\right|_{\eta_1,\eta_2} = \left.\frac{\partial T}{\partial \eta}\right|_{\eta_1,\eta_2} = 0 \tag{12}$$

$$\Omega = \frac{-1}{H^2}\left.\frac{\partial^2 \psi}{\partial \eta^2}\right|_{\eta=\eta_1,\eta_2} \tag{13}$$

$$\left.\frac{\partial T}{\partial \eta}\right|_{\eta=\eta_1;\eta_2} = 0 \tag{14}$$

The local and the average Nusselt number and friction coefficient relative to the lower cylindrical are defined by:

$$Nu = \frac{1}{T - T_P} \qquad \overline{Nu} = \frac{1}{S}\int Nu.ds$$
$$Cf = \frac{2\Pr}{H}.\frac{\partial V}{\partial \theta} \qquad \overline{Cf} = \frac{1}{S}\int Cf.ds \tag{15}$$

T_p = The average temperature inside the enclosure.

Numerical formulation: Equations are integrated numerically using a finite difference method. The spatial discretization is made by a finite difference method. A fully implicit procedure is retaining for treating the temporary derivatives. The resulting algebraic equations were solved by successive under relaxation method (SUR) (Patankar, 1980). The iterative process is repeats until the criterion of convergence below is satisfied:

$$\frac{\sum\left|F_{i,j}^{k+1} - F_{i,j}^{k}\right|}{\sum\left|F_{i,j}^{k+1}\right|} \le \varepsilon_r \tag{16}$$

where k is the incrementing index of the iterative process and ε_r his precision.

The grid size selected is equal to (71*71), with uniform grid spacing in both directions, Fig. 2 shows the influence of the grid system according to the instantaneous average Nusselt number $\overline{Nu}$ on the inner cylinder ($\theta = \theta_1$) and the minimum value of the stream-function ψ_{min}, the Rayleigh number is fixed at $Ra = 5.10^6$. All the results are obtained with Pr = 0.7 and a time step $\Delta t = 10^{-3}$ is retained to carry out all numerical tests. Refining this time step results in minor changes of the transient patterns.

In order to validate our results, we compare them with those of Sarr *et al*. (2001). In Fig. 3 we can see on the right our isotherms obtained for $Ra = 10^7$ and on the left those observed by experimentally by Sarr *et al*. (2001) for a Grashof number $Gr = 0.1.10^7$.

Although the geometries and the boundary conditions are not rigorously identical, the topology of the isotherms is quite similar.

RESULTS AND DISCUSSION

The numerical results presented correspond to a Darcy number (Da) varying from 0.1 to 1, the Rayleigh (Ra) and the Prandtl (Pr) are respectively equal to 10^7 and 0.7.

On Fig. 4 we can see the evolution in time of the thermal field (left) and the stream function (right) for various Da. The porous layer constitutes a strong dynamic resistance to the flow due to the Darcian effects which attenuate the thermal effects.

In the first instance, theis otherms areal most parallel curves and fit the profile of the hot wall (lower wall). The temperature distribution is simply decreasing from the hot wall to the cold wall. But as we progress in time the isotherms become more and more close to the wall of coordinate $\theta = \theta_1$, we assist to the formation of a boundary layer. The same phenomenon has been observed by Thiam *et al*. (2014) in the same cavity with no porous medium. The flow fieldis organized in two symmetrical cell srotatingin opposite directions.

For the Darcy number Da>0.5, the effects of the viscous forces will be dominant and thus the flow rate becomes significant. By consequent, the streamlines present a strong flow pattern with the swirl principal moves towards the cold wall. I can note that the intensity of convective flow becomes stronger than the value of the Darcy number is increasing.

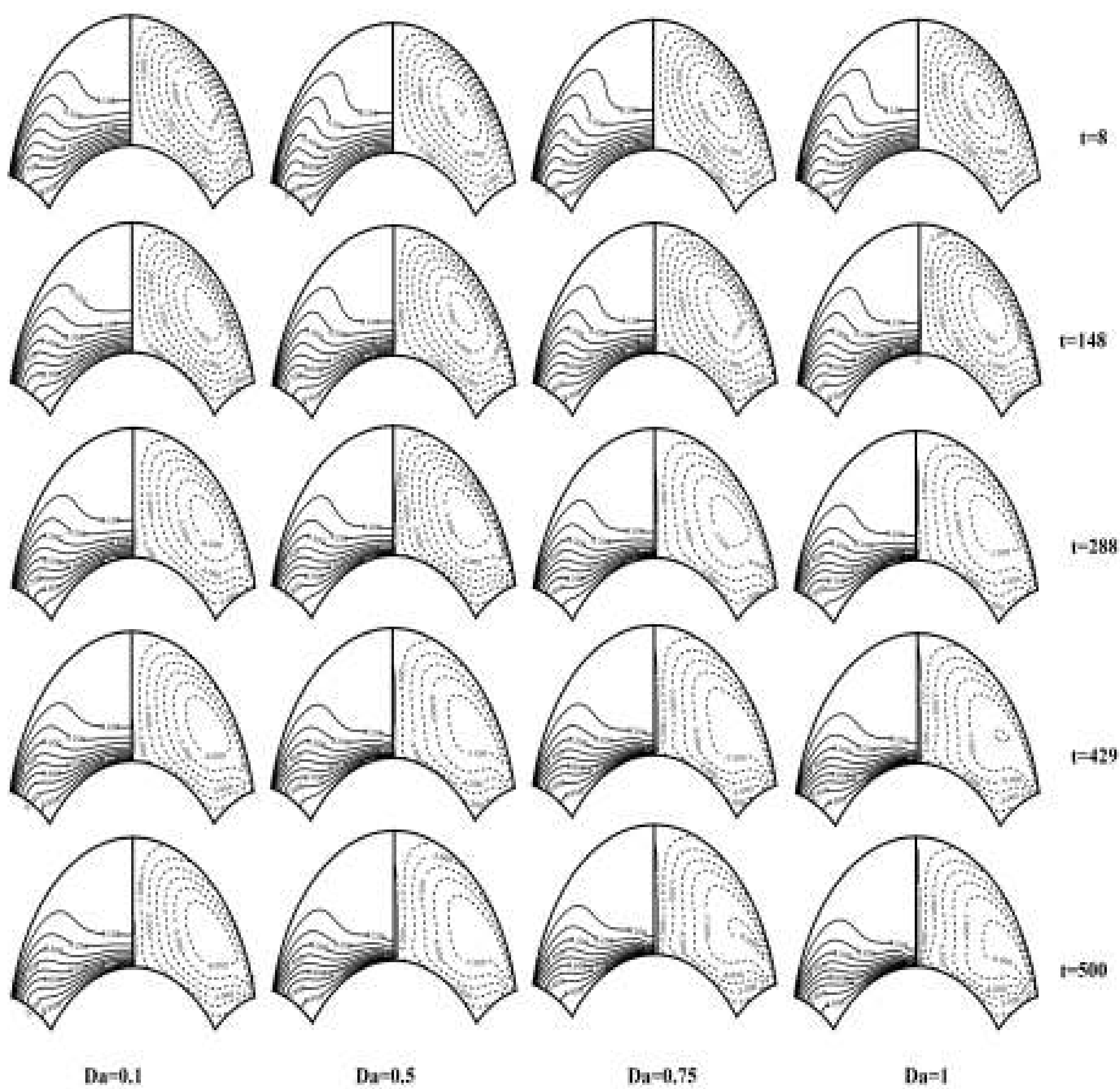

Fig. 4: Evolution of isotherms and streamlines for the different Darcy numbers at $Ra = 10^7$

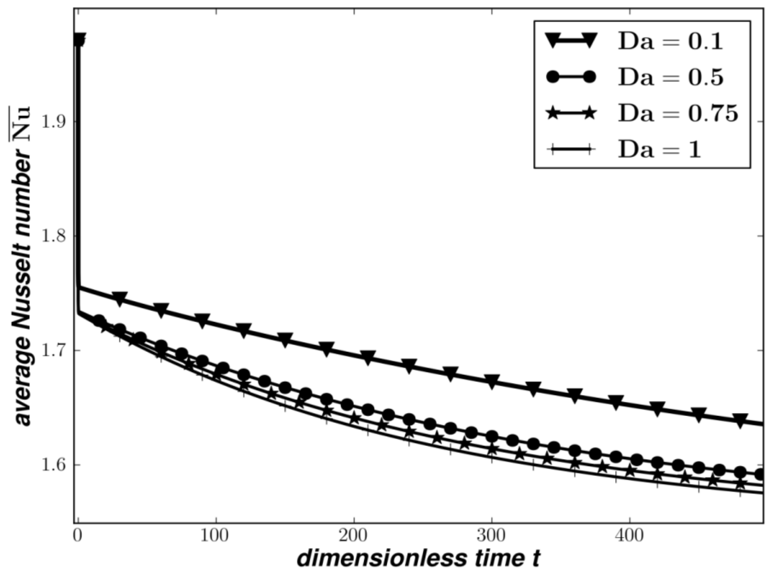

Fig. 5: Variation of the average Nusselt number against dimensionless time for various values of *Da*

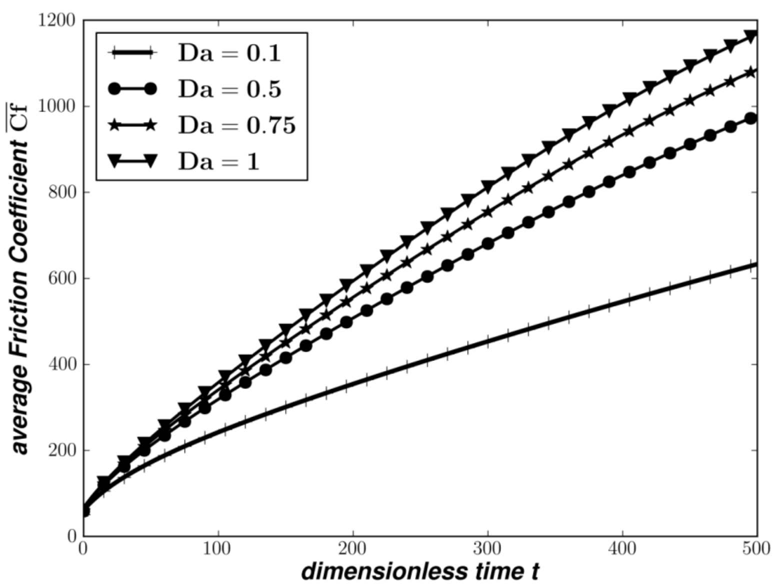

Fig. 6: Variation of the average friction coefficient against dimensionless time for various values of *Da*

When $Da = 1$, the penetration of the flow is almost total. The flow in the porous layer and the fluid merges. I tend towards the case of a flow in an entirely fluid cavity. With the values pupils of the permeability, i can expect that the porous medium does not have an influence and that i have a behavior of fluid medium some is the depth of the porous medium.

The intensity of the flow increases with the Darcy number and this growth is all the more high since the Rayleigh number is high. For the values of the Darcy number of the lowest, the porous medium behaves like an impermeable wall.

Figure 5 shows us the variation of the average Nusselt number $\overline{Nu}$ of the active wall according to time for different Darcy numbers. We can note that when we increase the Darcy number, $\overline{Nu}$ decrease. This is explained by the fact that the resistance of the flow becomes less and less significant when we increase the Darcy number. That's why the frictions too on the wall of coordinate $\theta = \theta_1$ are more important when *Da* increase (Fig. 6).

CONCLUSION

The objective of this study is to numerically study the dynamic and thermal phenomena in an enclosure delimited by portions of cylinders filled with a porous medium.

The behavior of the isotherms and the streamlines shows us that the Darcy number doesn't change significantly the flow pattern.

For the values of the Darcy number of the lowest, the porous medium behaves like an impermeable wall and a Darcy number more raises that it has a behavior of the fluid medium.

The increase in the Darcy number favors the flow in the porous environment. This medium supports the increase in the effect of the thermal forces and consequently the phenomenon becomes again pseudo convective and results in an increase in the Nusselt number and the decreasing one of the friction coefficient.

NOMENCLATURE

Latin letter:

b	:	Parameter of torus pole
C	:	Coefficient de Forchheimer
Cf	:	Friction coefficient
$\overline{Cf}$	:	Average friction coefficient
Da	:	Darcy number
F	:	Symbolic function representing the vorticity or the temperature
G_1 and G_2	:	Coefficients $G_1 = G_1(\eta,\theta) = \dfrac{1-\cos\theta\cosh\eta}{\cosh\eta-\cos\theta}$ and $G_2 = G_2(\eta,\theta) = \dfrac{\sin\theta\sinh\eta}{\cosh\eta-\cos\theta}$
H	:	Metric coefficient $H = \dfrac{1}{\cosh\eta-\cos\theta}$
Nu	:	Nusselt number
$\overline{Nu}$	:	Average Nusselt number
Pr	:	Prandtl number $\Pr = \nu/\alpha$
q	:	Heat flux density
Ra	:	Rayleigh number $Ra = \dfrac{g\beta b^4 q}{\nu\alpha\lambda}$
S	:	Surface area of the enclosure
t	:	Dimensionless time $[S]$
T	:	Dimensionless temperature $[K]$
U, V	:	Dimensionless velocity components in the transformed plane

Greek symbols:

α	:	Thermal diffusivity $\left[m^2 s^{-1}\right]$
β	:	Thermal expansion coefficient $\left[K^{-1}\right]$
η,θ,z	:	Bicylindrical coordinates
ΔT	:	Difference of temperature of the two cylinders
Δt	:	Time step $[s]$
λ	:	Thermal conductivity $\left[Wm^{-1}K^{-1}\right]$
ν	:	Kinematical viscosity $\left[m^2 s^{-1}\right]$

Centre:

Ψ	:	Dimensionless stream function
Ω	:	Dimensionless vorticity

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