

Research Article

Concrete Depth on Moment Capacity of Reinforced Concrete Slab: Safety Performance

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Abstract: Aimed to determine whether structural safety response of reinforced concrete slab will fall short of the code target safety value for flexural system and also determine its relation to moment capacity and minimum flexural reinforcement area requirement in consideration of the influence of un-certainties associated with the design parameters. The randomness of the design parameters makes the present deterministic design overly conservative, as such; it is becoming increasingly difficult to ignore the importance of using reliability approach in structural design application, because of its known advantage in accounting for the inherent variability in both loads and resistance variables. The study examined moment capacity violation and parametric sensitivity analysis with different concrete strengths class and span lengths, subjecting the deterministically designed required slab depth to further decrease and obtained the safety index value using First Order Reliability Method. The results indicate that concrete strength has little influence on minima reinforcement requirement with a value of about 3% surge for 500 mm change in span, which also results in increased ultimate moment of resistance of the concrete section with marginal decline in safety value. Interestingly, the safety value from the reliability analysis with optimized slab depth is well above the target safety limit specified for RC slab.

Keywords: Concrete strength, depth, FORM, moment, reinforced concrete slab

INTRODUCTION

The fundamental reason to design is to have a unit or components which will not only be economical to construct but have some level of tolerance in terms of performance (Enevoldsen and Sørensen, 1994; Gouverneur *et al.*, 2013). In other words, it is becoming increasingly difficult to ignore the desire to have low failure probability in the design of Reinforced Concrete (RC) structures, i.e., low probability of getting action value higher than the resistance (Larsen, 1995). Historically, structural safety is brought into lime light through various works by Freudenthal *et al.* (1964) and Pugsley (1955). The authors' works aid in harnessing and easy implementing of structural codes within a short period around the world.

In recent years, RC structural design is moving towards more rational and probability based design (Dolinski, 1982), where it takes into account more information than the deterministic method that uses safety factor that have the potential to make our design overly expensive and result in low probability of failure (Neves *et al.*, 2006). In this method, uncertainties associated with material strength, in loads and their combination, as well as modelling errors is included. These uncertainties which are inherent in our design problems, which could be as a result of variability to

materials properties, loads and dimensions or those related due to lack of knowledge about it that are usually associated to errors in modelling, sampling and measurement (Diniz, 2008; Dolinski, 1982).

Many authors' (Ellingwood and Galambos, 1982; Rosowsky and Stewart, 2001; Tu *et al.*, 1999) have demonstrated the use of reliability-based approach as it offers great advantages in addressing the issue of variability in our strength and load models.

Tu *et al.* (1999) show the needs for reliability based design models as a result of inherent uncertainties in our design variables, this will result in powerful and less conservative design output. So far, there has been little discussion on how to use reliability method as a weapon in presenting optimized the design of RC Slab. From literature consulted most research to date focuses on establishing target safety indices for either wide range of loadings or for various types of slab rather. And that target safety will be use as datum in this study.

It is known that several parametric studies including concrete strength and reinforcement yield are conducted on safety performance of RC slab; for example the work by the Low and Hao (2001), subjected to explosion and similar geometric characteristics influence on both moment capacity and safety considerations of RC slab with concentrated load (Jinxin *et al.*, 2011). However, slab depth optimization

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in relation to safety performance of RC slab has not been determined. The idea of decreasing slab depth designed deterministically is borne out of assumption that may not only enhanced the volumetric concrete requirement, it is arbitrary thought to interchange the excess of safety factors that contribute to high level of conservatism in our design using deterministic approaches which are as a result of uncertainties associated with load and resistance models. Therefore in this study we asked whether the structural safety response will fall short of the target safety index specified for RC slab deck by optimizing slab depth and equally how does the sensitivity relates to moment capacity and minima flexural reinforcement requirement. The case of simply supported condition with different span lengths is studied.

REVIEW ON RELIABILITY APPLICATION ON RC SLAB

RC slab are flexural member which are commonly used in residential building (both public and private) and in some cases are relatively used to resist seismic forces more specifically in seismic prone area like the Mediterranean (Benavent-Climent *et al.*, 2012). In most cases it is supported directly by column (commonly known as flat slab), which are very much easy to build (Sahab *et al.*, 2005). A considerable amount of literatures is available in the areas related to Reliability studies on RC slab or decks (Chul-Woo *et al.*, 2007; Jinxin *et al.*, 2011; Low and Hao, 2001; Marsh and Frangopol, 2008).

Low and Hao (2001), investigates the reliability of reinforced concrete slab designed according to BS8110, subjected to sudden level of explosion. The result shows for the three material strengths investigated variation of young modulus of elasticity of concrete show high influence of the failure probability, while crushing strength of concrete has the least effect compared to the yield of the reinforcement. Similarly, Chul-Woo *et al.* (2007), carried out studies to outline the effect of impact coefficient on reinforced concrete slab on a steel girder bridge on probabilistic approach. The findings reveals, codes specified impact coefficient may lead to conservative fatigue design, while higher value of impact coefficient resulting from worst loading surface may cause fatigue failure as well. It also adds the importance of impact coefficient through analytical approach to create rational criterion for slab performance level and decision making in relation to bridge management. Similarly, it was established that ultimate moment capacity of RC slab largely depends on slab depth as well as the reinforcement (Jinxin *et al.*, 2011).

RELIABILITY METHOD

First Order Reliability Method (FORM) a very powerful, robust and accurate for computing of failure probabilities is used in the evaluation of RC structures

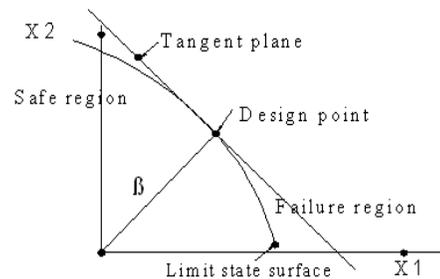


Fig. 1: Typical FORM approximation surface

in the context of non-linear analysis, which yields accurate prediction of structural behavior in the context of probability analysis (Hyo-Nam *et al.*, 2004; Val *et al.*, 1997).

The probability of failure, f_p is use to express the structural safety are typically represented by reliability index, β . This point is the shortest distance from the origin of the Cartesians point to the approximated failure surface as shown in Fig. 1. The failure surface is known as the limit state surface, $g(x)$ evaluated at the undesired level. In this study, this point β is, if the design moment, M_d exceeds the Resistance moment of the section, M_r , as in (1):

$$g(x) = g(x_1, x_2, \dots, x_n) = M_r - M_d \quad (1)$$

where, X_i represent the basic random variables in the load and strength domains. The design point shown in Fig. 1, is generally known as the checking point. At this point further iteration of the performance function in (1) will return the same value for the safety index. The major computation work in FORM is the determination of the design point and the subsequent β value from the point to the origin. This value is related to the failure probability, f_p through the use of (2), (Rackwitz, 2001). For Gaussian variables, this relation between f_p and β is shown in Table 1 for flexural members. Higher β value result in decreasing f_p chance and this translate to low cost of measure in terms of failure related consequences (Table 2):

$$f_p = \phi(-\beta) = 1 - \phi(\beta) \quad (2)$$

The expression in (2), ϕ represents the standard normal distribution function (Diniz, 2008). The general details for discrete variables standardization and genetic algorithm for β value determination are well explained in literatures; for example Ang and Tang (1990), Melchers (1999) and Ditlevsen and Madsen (2005).

Slab parameters and failure mode: Codes provision governs the design of RC slab. The formulation of limit state function of moment capacity in this study is according to euro-code 2 provisions for the design

Table 1: Relationship between β and f_p

f_p	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-6}
β	1.3	2.3	3.1	3.7	4.2	4.7	5.2

After JCSS (2000)

Table 2: Safety value in relation to relative cost of measure

Relative cost of measure	Target index
High	$\beta = 1.3$
Normal	$\beta = 1.7$
Low	$\beta = 2.3$

Table 3: Basic variables statistical characteristics

x	Distribution type	Nominal value	COV	Reference
f_{yk} x (1)	Normal	460 MPa	0.03	Low and Hao (2002)
h x (2)	Normal	210 mm	0.05	-
f_{cu} x (3)	Log-normal	Strength class type	0.17	Unanwa and Mahan (2012)
Q_R x (4)	Log-normal	1.1	0.07	JCSS (2000)
γ_{con} x (5)	Normal	25 kN/m ³	0.03	JCSS (2000)
q_1 x (6)	Gumbel	3 kN/m ²	0.25	-
Q_s x (7)	Log-normal	1.0	0.20	JCSS (2000)

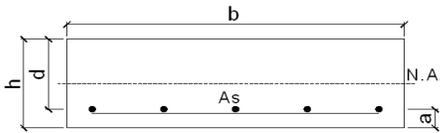


Fig. 2: Typical RC slab section

of RC slab. A variable load of 3 kN/m² is assumed for this study, with statistical distribution parameters of Gumbel type and Coefficient of Variation (COV) of 0.25. As a result, the preliminary deterministic design of the RC slab, a slab depth, h thickness of 210 mm satisfies standard resistance against insulation, REI 120 requirement as obtained in Table 3 (EN, 1992). The span lengths varied from 3.5 to 5.0 m and with mean values of four concrete strengths, f_{cu} class were used for the parametric study in this study. Similarly, 25 kN/m³ is the Concrete density, γ_{con} and its statistical parameters are normally distributed and cov value of 0.03. In the same vain, the mean strength is assumed to have the same value with the nominal design strength for the parameters involve (bias factor = 1.0). The general statistical characteristics of these basic variables are shown in Table 3.

Failure mode: The study failure mode in flexure is the moment capacity requirement for a typical RC slab section as shown in Fig. 2. The limit state function is given in (3) in accordance with Benavent-Climent *et al.* (2012):

$$g(x) = Q_R A_s f_y \left(h - a - \frac{0.5 A_s f_y}{f_{cu} b} \right) - Q_S \left[\frac{(g_k + q_1) span^2}{8} \right] \quad (3)$$

where, $g_k = h \gamma_{con}$, A_s and q_1 are the steel area and liveload value Q_R , Q_S are model R and S uncertainty factor, $a = c - \phi_{bar}/2$ with concrete cover of 30 mm and steel diameter of 10 mm

The parameters in (3) expressed with the study defined basic variables (Table 3) is shown in (4):

Table 4: Tensile strength factor

f_{ek} (MPa)	30	35	40	45
f_{ctm}	2.9	3.2	3.5	3.8

$$g(x) = X(4) A_s X(1) \left(X(2) - 25 - \frac{0.5 A_s X(1)}{X(3) b} \right) - X(7) \left[\frac{X(2) X(5) + X(6)}{8} \right] l^2 \quad (4)$$

where, l and A_s are the span length and reinforcement area per unit area defined in (5). However, in this study, where the A_s value is less than the limiting value as in (6), the A_s requirement is kept to the value obtain in (6):

$$A_s = \frac{(h \gamma_{con} + q_1) l^2}{6.96 f_y Z} \quad \text{where } Z = d \left[0.5 + 0.5(1 - 3.53k)^{0.5} \right] \leq 0.95d$$

$$k = \frac{(h \gamma_{con} + q_1) l^2}{8 b d^2 f_{cu}} < 0.598\delta - 0.18\delta - 0.21 \quad \text{with } \delta = 1.0 \text{ in this study} \quad (5)$$

For a typical RC slab, that is singly reinforced section, the minimum A_s requirement for the section is a function of concrete tensile, f_{ctm} strength and Table 4 shows its value for different concrete strengths class:

$$A_{s, \min} = \frac{0.26 f_{ctm}}{f_{yk}} b d \leq 0.04 b h$$

where $f_{yk} = 500$ MPa

RESULTS AND DISCUSSION

Four different spans of simply supported RC Slab subjected to variable load of 3 kN/m² and a dead load due to self-load only were studied to show its characteristics behavior in relation to its moment capacity violation. The slabs are singly reinforced with only bottom reinforcement at the tension face, because the parametric values under considerations as regards to the slab design does not requires the compression reinforcement at the top face. The relation also

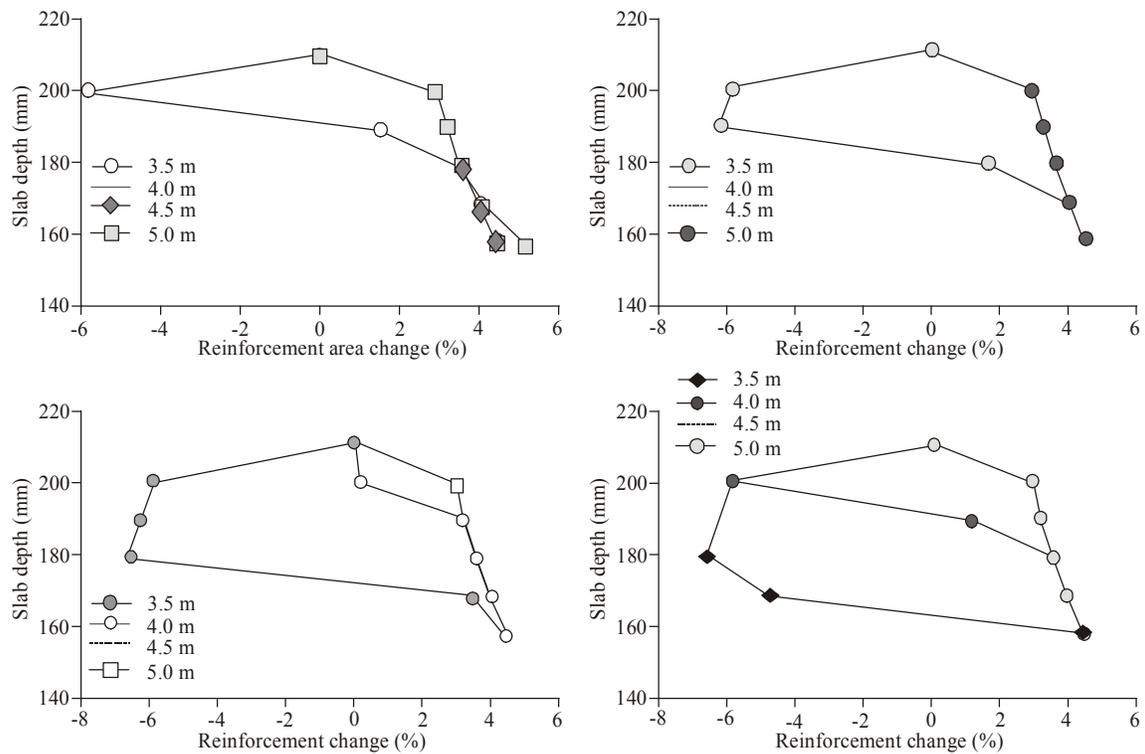


Fig. 3: Reinforcement requirements with depth change

considers the effect of (4) different concrete strength classes as their mean values were used in the reliability analysis of the slab that subsequently yields the safety index values (β) under various geometric conditions.

The question addressed by this study is to know the structural safety response of geometrically optimized section, whether or not it will fall short of the target safety index specified for RC slab deck. Similarly, with parametric evaluation, the study also presents, safety value sensitivity relating to moment capacity and minima flexural reinforcement requirement of simply supported RC slab of various span lengths through reliability analysis tool.

Reinforcement requirements: For the flexural section to be in equilibrium at minimum, the reinforcement steel augments the depth loss. Figure 3, shows the percentage variations in minimal reinforcement requirement for flexural capacity of RC slab under different geometric and concrete strengths class. In this Fig. 3, the levels A, B, C and D corresponds to 30, 35, 40 and 45 MPa concrete strengths class respectively. Averages of 4% change in minimum reinforcement required observed (spans 4-5.0 m) as the slab depth reduces progressively from 210-157.5 mm (signifying up to 25% reduction in depth). The optimum value of 6% reduction in reinforcement with initial 5% depth change recorded for 3.5 m span with any of the four (4) concrete strengths class used. This suggest the concrete characteristic strength has little or no effect on the reinforcement required as the depth reduces through 0-

25% of the initial slab depth of 210 mm, which was in agreement with findings that shows crushing strength of concrete has least effect on failure problem compared with reinforcement yield (Low and Hao, 2001). In a general, the result, suggest an average of 3% surge in minimal reinforcement required for a 500 mm change in span from 3.5-5.0 m length which is independent of the concrete strength class, This could be said to be true to augment the concrete resistance as the depth reduces on average of 5% of 210 mm.

Moments of resistance: The ultimate moment of resistance of the section will make the section to be able counter the effect of the negative moment because of the loadings. For the case under consideration, the worst effect occurs at the mid-span and supposedly zero at the ends, because it is a simply supported system as assumed. To assess the reliability of R. C. Slab, for the section to be stable and safe from collapse, it is of course logical for the sectional moment to be greater than the applied moment. Thus, Fig. 4 shows changes in moment of resistance and beta values from the optimized depth at different span lengths and strengths class. In a similar analysis presented in the previous section, the levels A, B, C and D in Fig. 4 shows the four strengths class as defined before. Similarly, in the same figure, the notations β_i and M_i represents the safety index and resistance moments for i^{th} span lengths that range from 3.5-5.0 m.

It is apparent from the results in Fig. 4, the moment of resistance increases as the span length increases from

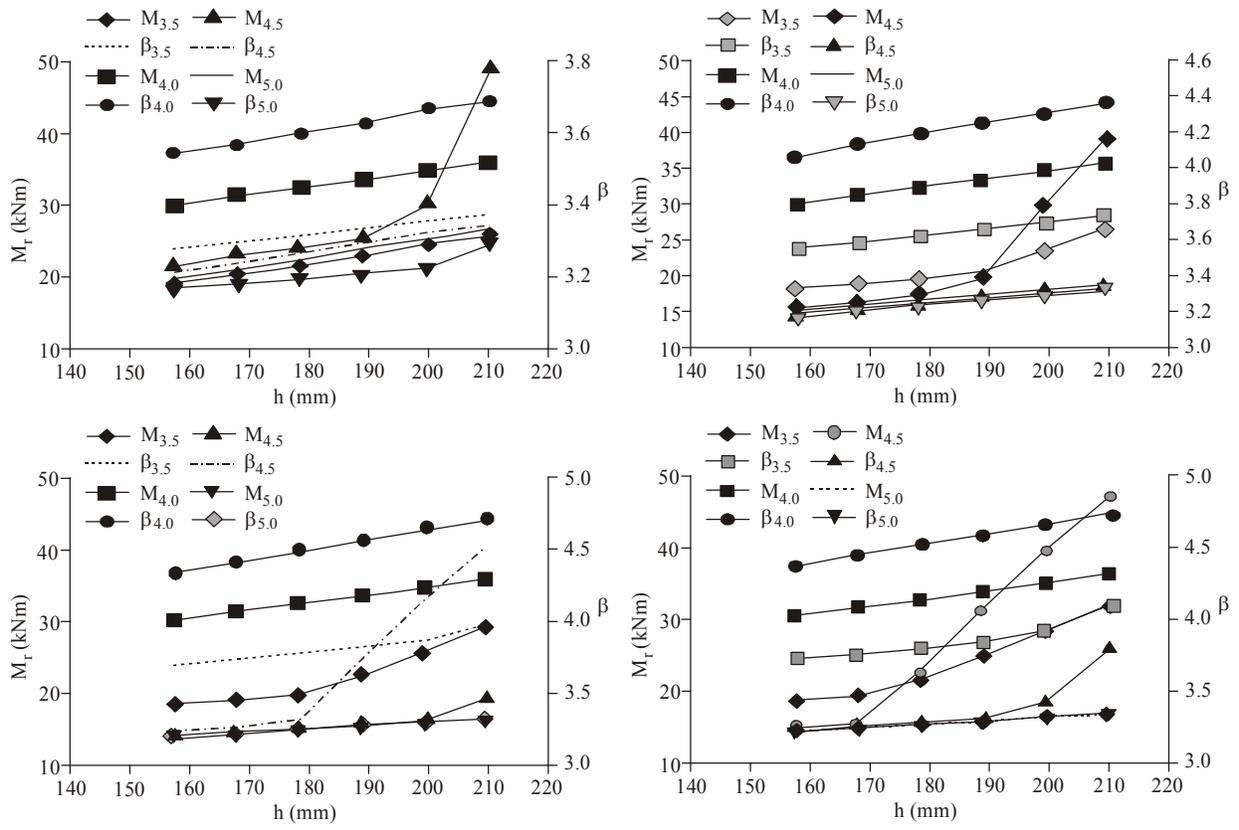


Fig. 4: Resistance moments and safety indices values

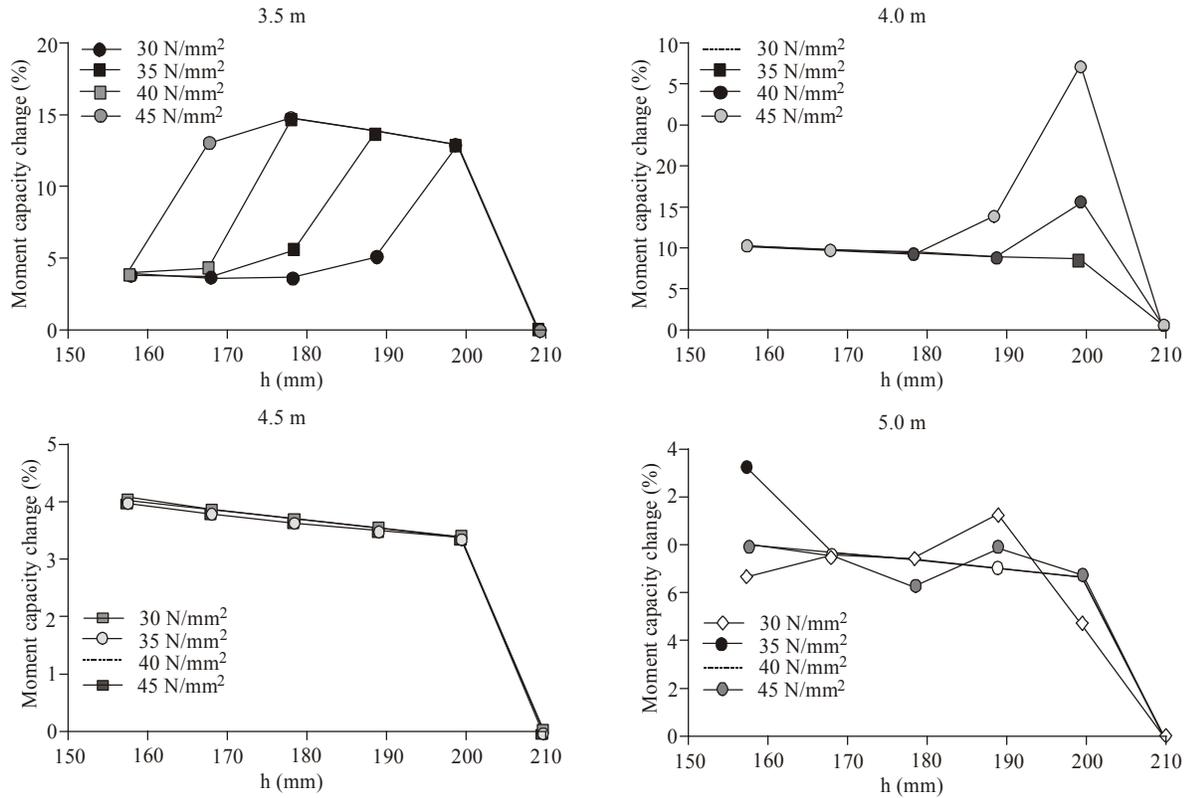


Fig. 5: Moment capacity variations

3.5-5.0 m, which equally stands while using any concrete strengths class. Similarly, the safety values obtained from the reliability analysis of simply supported RC slab for optimized slab depth up to 25% reduction in initial depth of 210 mm is well above the target safety limit specified for RC slab (Fig. 4). Previously, the study results reveals surge in reinforcement area required, because of absolute decrease in moment capacity. The decrease in moment capacity insignificantly influences the safety levels. In other words, the safety indices reduce marginally because of slab depth reduction. The result suggest linear relations between safety value and the corresponding slab depth changes as shown in Fig. 4. The result similarly suggests that the safety value of simply supported RC Slab is inclined to the reinforcement steel than the concrete strength. This was in agreement with the findings of Low and Hao (2001) which reveals that the crushing strength of concrete has least effect compared to yield of reinforcement on the reliability. Moreover, this change marginally affects the performance index of RC slab at all slab depth considered. Consequently concrete strengths shows positive impact on performance index of RC slab as Fig. 4 shows, a considerable gain in β value as associated with concrete strength class change from C30-C45. However, these value decreases with decreasing slab thickness and moment capacity, and this suggest moment capacity of RC depends heavily on the slab depth in addition to the reinforcement required (Jinxin *et al.*, 2011). Moreover, the moment capacity increases with span increase, this phenomenon may be unconnected with concrete volume increase because of span length increase.

To further understand the change in moment capacity, Fig. 5 presents the depth influences between the resistance and the design moment for the respective span lengths and strengths class considered in this study. In Fig. 5, the results show an average of 9% change in moment capacity between 5-25% decreased in slab depth of 210 mm for the study shortest span length. However, for span lengths between 4-5 m, average variations of about 4% in moment capacity variation are observed. This equally implied that it is possible to decreased design moment by 4% and achieved optimal and safer RC slab deck.

CONCLUSION

In summary, the study presented the safety indices values from the use of reliability tool in the analysis of slab depth optimized for simply supported RC slab. The study has explained the central importance for the application of reliability measures on the design of RC slab, as it was acknowledge that it will carter for the uncertainties in our design models. It is known that this un-certainties makes design overly conservative and the study have equally demonstrated the possibility of

achieving economical and safe design within code specified bounds of safety value. The results from the study suggest that design slab depth from deterministic approach will be enhance through slab depth optimizing and achieved safe state of structural health using reliability analysis method.

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