Research Article
Comparison Study of Different Structures of PID Controllers

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Abstract: The paper aim to comparative study of different structures of PID controller and the values of gains of the PID controller K_p, K_i and K_d are calculated by using Levenberg-Marquardt's optimization Algorithm (LMA). The Proportional Integral Derivative (PID) structure is mainly used to achieve the desired output in case of closed loop control systems in most of the industry applications. It is difficult to obtain the proper values of the PID controller parameters K_p, K_i and K_d. The performance of this approach is evaluated by setting the objective function as the Integral of the Square Error (ISE). The proposed methods are simulated by tuning all the parameters of different structure of PID controller for PH neutralization process in MATLAB program.

Keywords: Chemical process control, cost functions, levenberg-marquardt's optimization algorithm, pH neutralization process, PID controller

INTRODUCTION

PID is a remarkable control strategy widely used in processes industry tries such as oil and gas, chemical, petrochemical, pulp and paper, food and beverage. PID controller has been proven in terms of reliability and robustness in controlling process variables ranging from temperature, level, pressure, flow, pH, etc. Other factors that attracted industry to choose PID controller could be due to low cost, easy to maintain, as well as simplicity in control structure and easy to understand. However, improper PID parameters tuning could lead to cyclic and slow recovery, poor robustness and the worst case scenario would be the collapse of the system operation. This led researchers to explore the beast method in searching optimum PID parameters (Astrom and Hagglund, 1988).

Many strategies have been proposed to determine the optimum setting of PID controllers. Ziegler and Nichols, Cohen and Coon are amongst the pioneers in PID tuning methods. They have proposed experimental PID tuning methods based on trial and error and the process reaction curve (Chopra *et al.*, 2014). However, the difficulties may arise to tune the PID controller when the system is complex such as high order, time delay, non-minimum phase and non-linear processes (Todorov *et al.*, 2013). For example, Ziegler and Nichols method may give overshoots, highly oscillatory and longer settling time for a high order system and Cohen-Coon method is only valid for systems having S-shaped step response (Ayman, 2011; Fung and Yang, 1998). To overcome these difficulties, various methods have been used to obtain optimum PID parameters ranging from conventional methods such as refined Ziegler-Nichols and pole placement and the implementation of modern heuristic optimization techniques such as genetic algorithms, simulated annealing and population based incremental learning and particle swarm optimization (Turkoglu *et al.*, 2008). Recently, Generic Algorithm (GA) has been extensively studied by many researchers in searching for optimal PID parameters due to its high potential of escaping being trapped at a local minimum (Kada and Ghazzawi, 2011).

This study is provide a better understanding of how PID controller is tuned using Levenberg-Marquardt Algorithm (LMA). The performance of LMA is searching local optimal PID parameters and its reliability to maintain the optimum value for a nonlinear system. This study also compares the transient performance of the pH neutralization system using different structures of PID controller with LMA tuning method.

METHODOLOGY

Levenberg-Marquardt Algorithm (LMA) for tuning PID controller: In mathematics and computing, the Levenberg-Marquardt Algorithm (LMA) is used to solve non-linear least squares problems. These minimization problems arise especially in least squares curve fitting. The LMA interpolates between the Gauss-Newton Algorithm (GNA) and the method of gradient descent. The LMA is more robust than the GNA, which means that in many cases it finds a solution even if it starts very far off the final minimum. The LMA is a very popular curve-fitting algorithm used in many software applications for solving generic curve-fitting

problems. However, as for many fitting algorithms, the LMA finds only a local minimum, which is not necessarily the global minimum (Astrom and Hagglund, 1988).

The primary application of the Levenberg-Marquardt algorithm is in the least squares curve fitting problem: given a set of m empirical datum pairs of independent and dependent variables, (x_i, y_i) optimize the parameters β of the model curve $f(x, \beta)$ so that the sum of the squares of the deviations becomes minimal:

$$S(\beta) = \sum_{i=1}^{m} [y_i - f(x_i, \beta)]^2$$

The Levenberg-Marquardt (LM) algorithm is an iterative technique that locates the minimum of a multivariate function that is expressed as the sum of squares of non-linear real-valued functions. In the least-squares problem a function f (x) is minimized that is a sum of squares:

$$min\ f(x) = \sum_i f_i^2(x)$$

Problems of this type occur in a large number of practical applications, especially when fitting model functions to data, i.e., nonlinear parameter estimation. They are also prevalent in control where you want the output, $y(x, t)$ to follow some continuous model trajectory, $\varphi(t)$ for vector x and scalar t. This problem can be expressed as:

$$Erorr = min \int_{t1}^{t2} (y(x, t) - \varphi(t))^2\, dt$$

where, $y(x, t)$ and $\varphi(t)$ are scalar functions.

Performance index of PID controller: Systems performance quantification is achieved through a performance index. The performance selected depends upon the process under consideration and is chosen such that emphasis is placed on specific aspects of the systems performance. Alternatively a performance index is a quantitative measure of systems and is chosen so that a set of parameters in the systems can be adjusted to meet the required specification optimally. Minimum or maximum value of this index then corresponds to the optimum set of parameter value. Different performance indices used in practice are:

- Integral of the Absolute Error (IAE) where:

$$IAE = \int_0^\infty |e(t)|\, dt$$

- Integral of the Square Error (ISE) where:

$$ISE = \int_0^\infty e^2(t)\, dt$$

- Integral of the Time-weighted Absolute Error (ITAE) where:

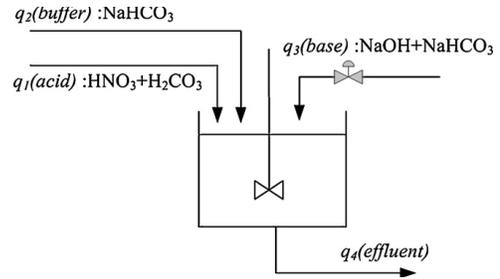$$ITAE = \int_0^\infty t|e(t)|\, dt$$

Fig. 1: pH neutralization process

- Integral of the time-weighted square error or Mean Square Error (MSE) where:
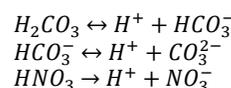
$$MSE = \int_0^\infty te^2(t)\, dt$$

In all of the above '$e(t)$' is the error at time't'. The error is taken as the input to the PID controller which can be expressed as:
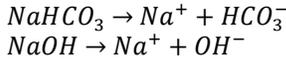
$$u(t) = K_p e(t) + K_i \int_0^t e(t)\, dt + K_d \frac{de(t)}{dt}$$

In this study, the optimum values of the controller parameters, which minimize the cost function, are accurately computed using LMA method. In the pH neutralization model, considering PID controller which is using standard ISE cost function for developing controlling parameters. Also, different types of structure of PID controller are tested for this study. Proper parameter setting is of great importance for a system to be stable. Having secured a stable system, cost function must be to adjust the controller parameters until an optimum response is achieved. Performance indices or cost function have proved to be the most meaningful measures of dynamic performance. The control parameters to be tuned through the optimization algorithm are feedback gains of PID of output of the manipulated variable in the pH neutralization system. In this study, the Integral of the Square Error (ISE) is chosen as the objective function. The goal of LMA is to seek for minimum fitness value.

Case study:
pH neutralization process: Consider a pH neutralization process as shown in Fig. 1. The flow rates of acid, buffer, base and effluent streams are denoted by q_1, q_2, q_3 and q_4, respectively. Output of the process is the pH value of the effluent stream and the flow rate of base stream, q_3 is the manipulated variable. A dynamic model is derived using the conservation laws and reactions equilibrium. The modeling assumptions include perfect mixing, constant volume of the neutralization tank (V) and complete solubility of the ions involved. The chemical reactions in the system are as follows:

$$H_2CO_3 \leftrightarrow H^+ + HCO_3^-$$
$$HCO_3^- \leftrightarrow H^+ + CO_3^{2-}$$
$$HNO_3 \rightarrow H^+ + NO_3^-$$

$$NaHCO_3 \rightarrow Na^+ + HCO_3^-$$
$$NaOH \rightarrow Na^+ + OH^-$$

The equilibrium constants for these reactions are:

$$Ka_1 = \frac{[HCO_3^-][H^+]}{[H_2CO_3]}$$

$$Ka_2 = \frac{[CO_3^{2-}][H^+]}{[HCO_3^-]}$$

$$Kw = [H^+] + [OH^-] + [CO_3^{2-}]$$

The chemical equilibrium equations are modeled using the reaction invariant concept (Gustafsson and Waller, 1983). For this system, concentrations of reaction invariants are defined as:

$$x_1 = [NO_3^-]$$

$$x_2 = [Na^+]$$

$$x_3 = [H_2CO_3] + [HCO_3^-] + [CO_3^{2-}]$$

Denoting, $y = pH$ the ions neutrality balance in the tank results the following static equation:

$$h(x, y) = -x_1 + x_2 + x_3 c_{x3} + 10^{-y} - 10^{y-PK_w} = 0 \tag{1}$$

$$c_{x3} = \frac{2 + 10^{PK_2 - y}}{1 + 10^{PK_2 - y} + 10^{PK_1 + PK_2 - 2y}}$$

$$PK_1 = -\log_{10} Ka_1$$

$$PK_2 = -\log_{10} Ka_2$$

The dynamic equations are given by:

$$\frac{d\dot{x}_1}{dt} = \frac{q_1}{V}(w_{11} - x_1) + \frac{q_2}{V}(w_{21} - x_1) + \frac{q_3}{V}(\propto_1 - x_1) \tag{2}$$

$$\frac{d\dot{x}_2}{dt} = \frac{q_1}{V}(w_{12} - x_2) + \frac{q_2}{V}(w_{22} - x_2) + \frac{q_3}{V}(\propto_2 - x_2) \tag{3}$$

$$\frac{d\dot{x}_3}{dt} = \frac{q_1}{V}(w_{13} - x_3) + \frac{q_2}{V}(w_{23} - x_3) + \frac{q_3}{V}(\propto_3 - x_3) \tag{4}$$

where,
V : Volume of the mixing tank, mL
Kw : Dissociation constant of water, 10^{-14}
Ka_i : ith dissociation constant of acid
w_i : Concentration of the ith species in the process stream, mol/L
w_{1i} : Concentration of the ith species in the acid stream, mol/L
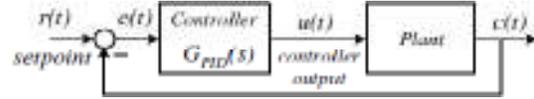w_{2i} : Concentration of the ith species in the buffer stream, mol/L

Fig. 2: Control system

q_i : Flow rate of acid, buffer and base stream in simulation, mL/sec

$\propto_i$: Concentration of the ith species in the titrating stream, mol/L
x_i : Reaction invariant of ith species, mol/L
y : Process variable, pH
u : Flow rate of the titrating stream, mL/min or mL/sec

Different PID structures: The block diagram of a simplified control system is shown in Fig. 2. Not all manufactures produce PID's that conform to the ideal textbook structure. So before commencing tuning it is important to know the configuration of the PID algorithm. The majority of text-book tuning rules are only valid for the ideal architecture. If the algorithm is different, then the controller parameters suggested by a particular tuning methodology will have to be altered.

The control signal provided by standard PID controller is dependent upon three terms and is given by:

$$u(t) = K_P e(t) + K_i \int_0^t e(t)\, dt + K_d \frac{de(t)}{dt} = K_P \left[e(t) + \frac{1}{T_i} \int_0^t e(t)\, dt + T_d \frac{de(t)}{dt} \right]$$

where $e(t)$ is the error, $u(t)$ the controller output and K_P, K_i and K_d are the proportional, Integral and derivative gains.

Ideal PID structure: The transfer function of the ideal PID controller is expressed as:

$$G_{PID}(s) = \frac{u(s)}{e(s)} = K_p\left(1 + \frac{K_i}{s} + K_d s\right)$$

One disadvantage of this ideal configuration is that a sudden change in set point and hence, the error will cause the derivative term to become very large and thus provide a derivative kick to the final control element, this is undesirable. The derivative mode acts on the measurement and not the error is an alternative implementation. After a change in set point the output will move slowly avoiding derivative kick after set point changes. This is therefore a standard feature of most commercial controllers. The SIMULINK model of ideal PID construe is shown in Fig. 3.

Series (interacting) PID: The transfer function of the series PID controller is expressed as:

$$G_{PID}(s) = \frac{u(s)}{e(s)} = \left[K_p\left(1 + \frac{K_i}{s}\right) \right](K_d s + 1)$$
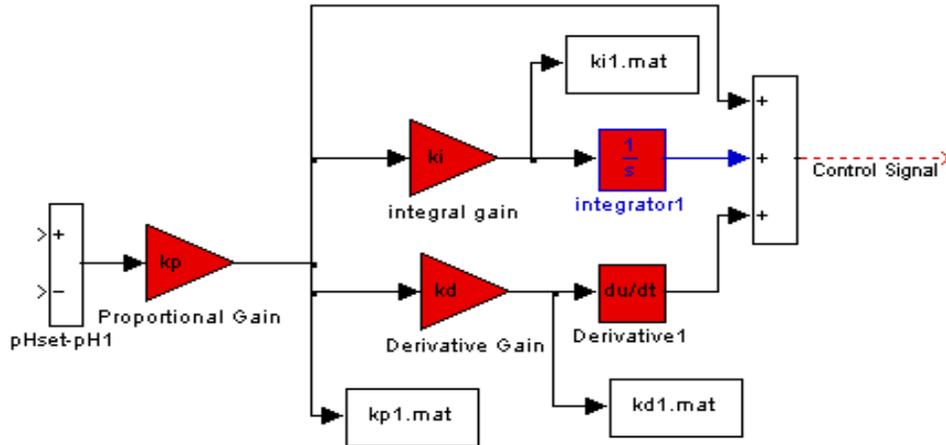
Fig. 3: SIMULINK model of ideal PID configuration
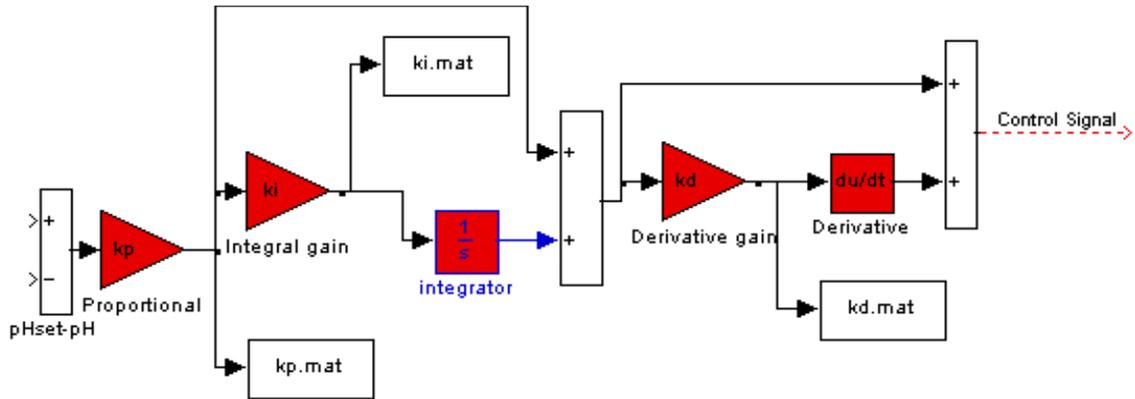

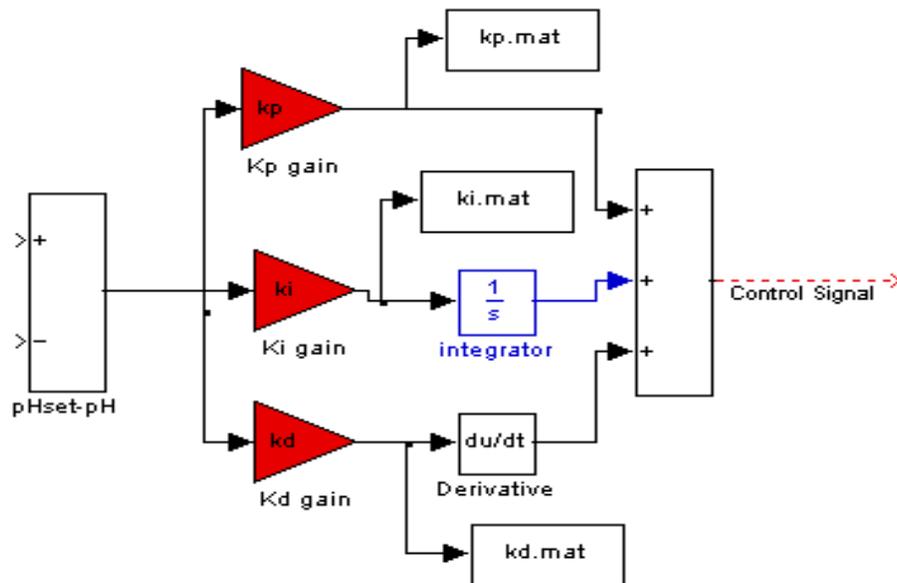Fig. 4: SIMULINK model of series PID configuration

Fig. 5: SIMULINK model of parallel PID configuration

As with the ideal implementation the series mode can include either derivative on the error or derivative on the measurement. The SIMULINK model of series PID is shown in Fig. 4.

Parallel PID: The transfer function of the parallel PID controller is expressed as:

$$G_{PID}(s) = \frac{u(s)}{e(s)} = K_p + \frac{K_i}{s} + K_d s$$

The proportional gain only acts on the error, whereas with the ideal algorithm it acts on the integral and derivative modes as well. The SIMULINK model of parallel PID construe is shown in Fig. 5.

SIMULATION RESULTS AND DISCUSSION

The pH neutralization closed loop control system was solved using Levenberg-Marquardt's optimization approach with sampling time of 0.001 sec. The simulation method combines SIMULINK module and M-file where, the main program is realized in SIMULINK and the optimized PID controller is predicted using M-file. The closed loop response of the pH neutralization transfer function subjected to a step
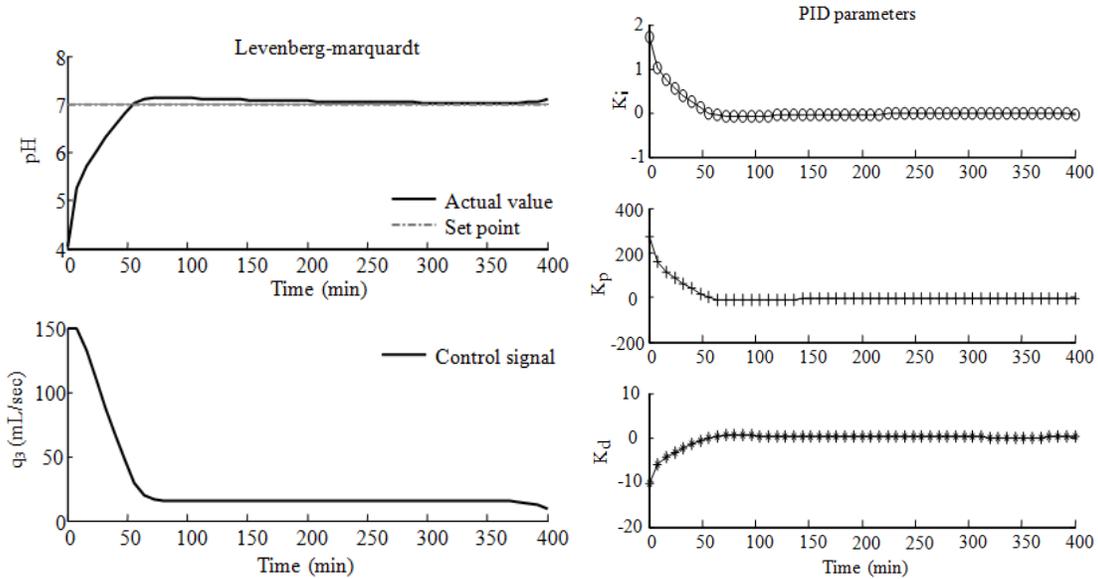

Fig. 6: Step response and PID parameters values of ideal PID configuration
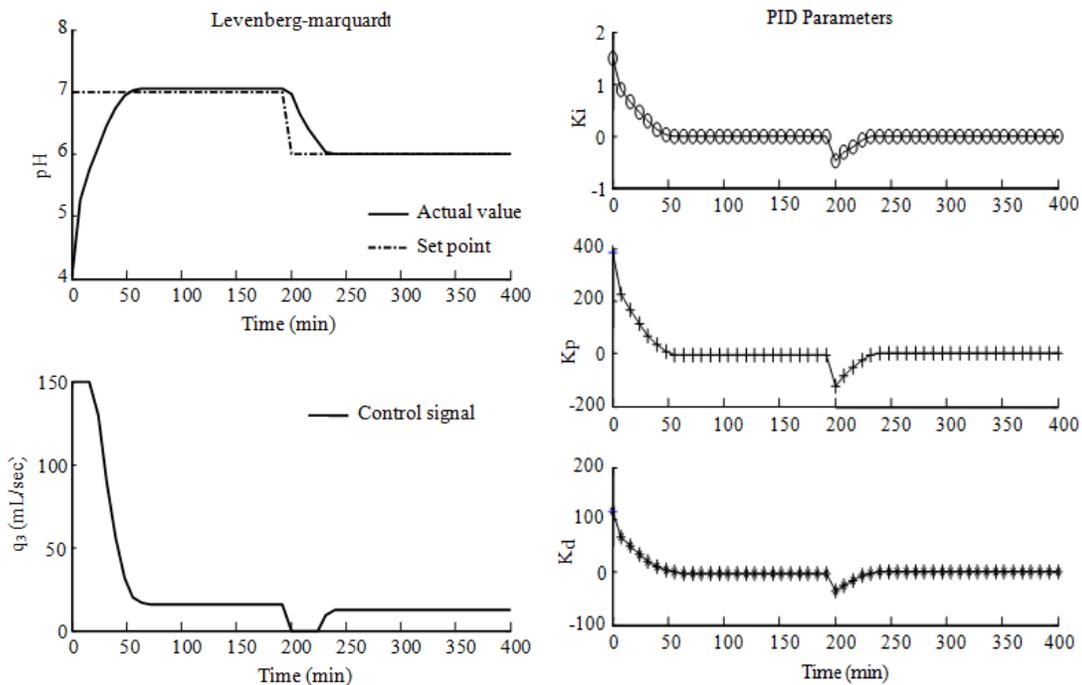

Fig. 7: Step response and PID parameters values of ideal PID configuration

input with different methods has been shown in Fig. 6 to 11 using different PID controller. The comparison among different structure of PID controller in terms of various performance specification such as rise time, settling time, overshoot and steady state error using Levenberg-Marquardt's optimization approach has been shown in Table 1. The various performance specifications have been ideal PID controller, the best
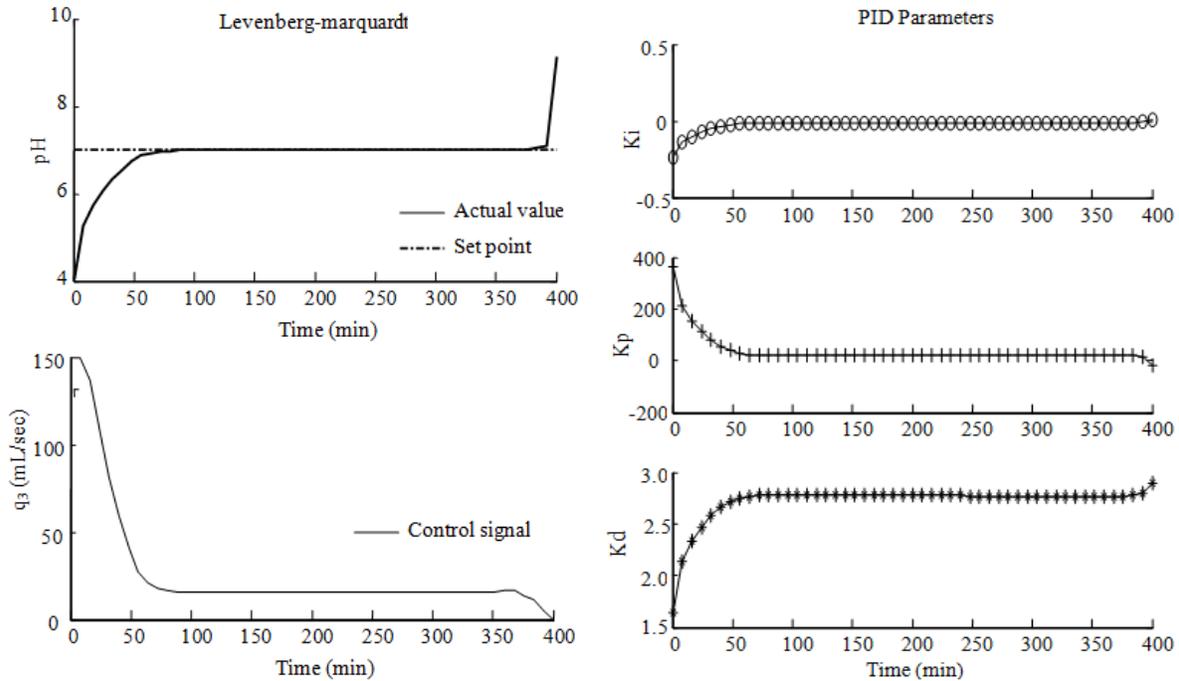

Fig. 8: Step response and parameters values of series PID configuration
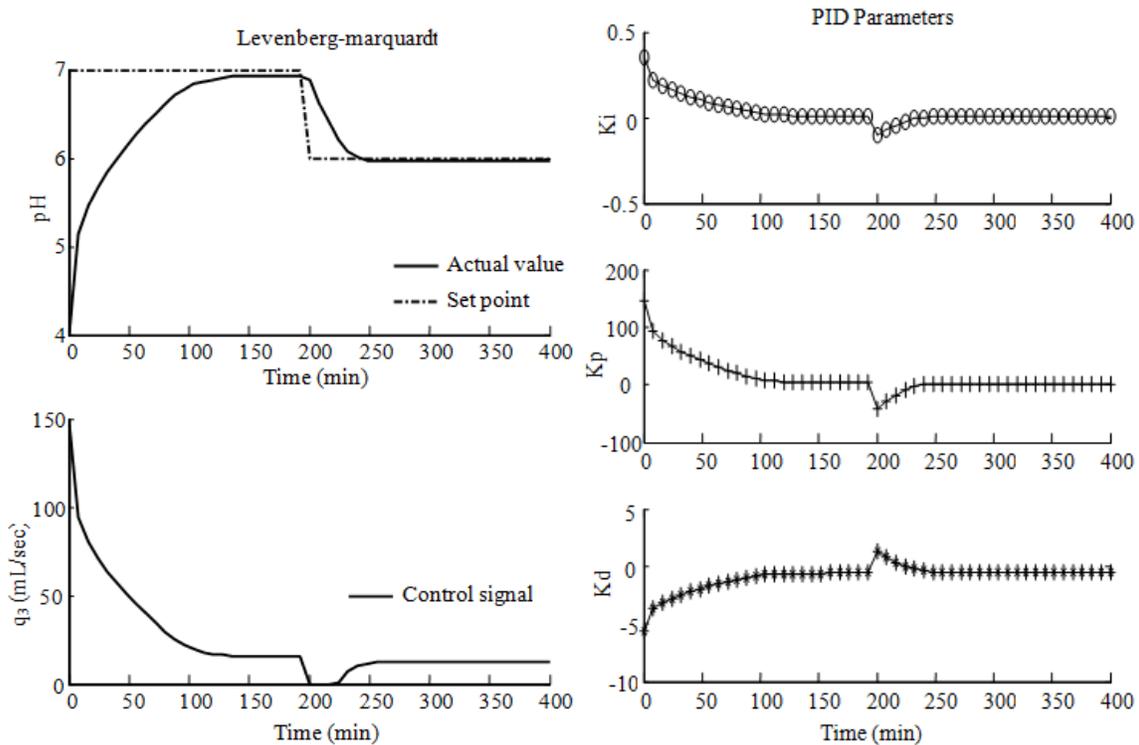

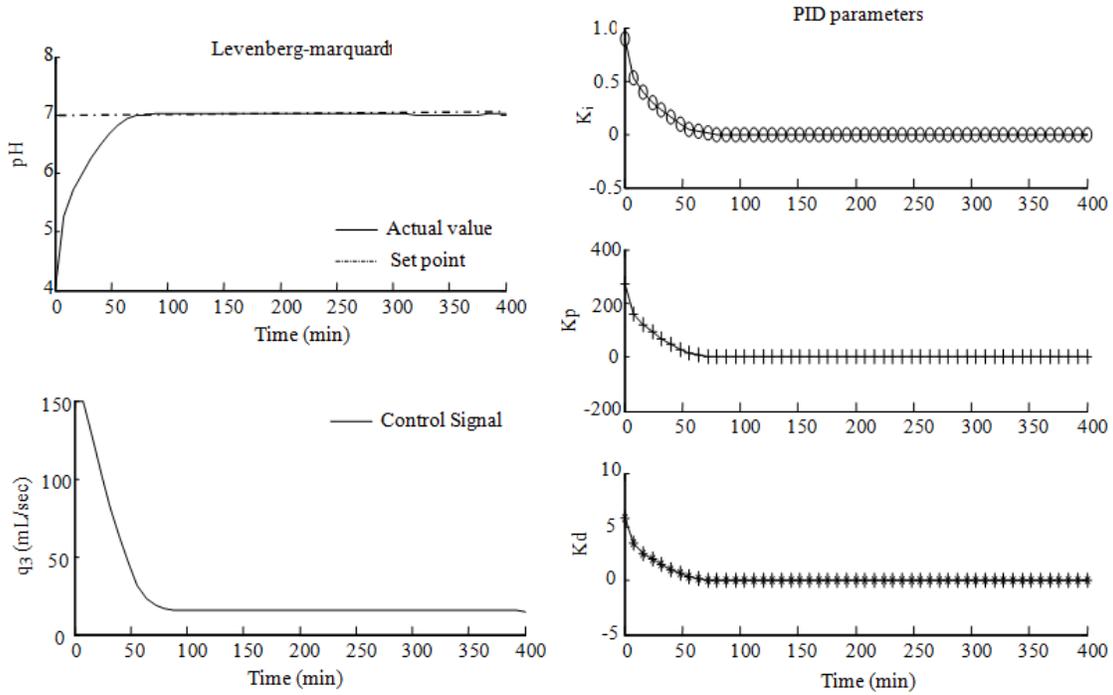Fig. 9: Step response and parameters values of series PID configuration

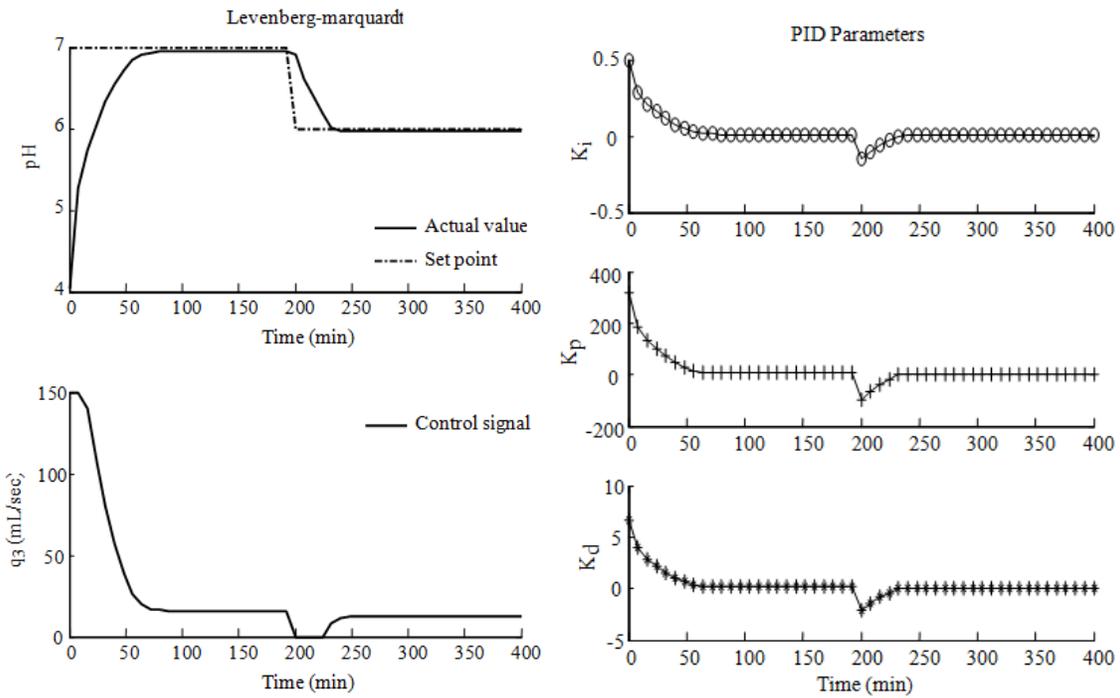Fig. 10: Step response parameters values of parallel PID configuration

Fig. 11: Step response parameters values of parallel PID configuration

Table 1: Comparison among different PID controllers

Performance specification parameters	LMA tuning method		
	Ideal PID controller	Series PID controller	Parallel PID controller
Rise time	40 min	45 min	30 min
Settling time	50 min	70 min	65 min
Overshoot	10%	0	0
Steady state error	0	0.5%	0

performance in terms of settling time and overshoot has been given by Parallel PID controller.

CONCLUSION

A comparison between the different structures of PID controller is presented. Optimization of the PID controller gains for a pH neutralization systems using LMA technique has been proposed. The LMA technique is utilized to evaluate the PID controller gains which improve the dynamic performance of the pH neutralization system to an operating condition with perturbations.

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