Research Journal of Applied Sciences, Engineering and Technology 11(8): 921-926, 2015
DOI: 10.19026/rjaset.11.2104
ISSN: 2040-7459; e-ISSN: 2040-7467

Submitted: June 12, 2015 Accepted: September 5, 2015 Published: November 15, 2015

Research Article

Theoretical Calculation of Critical Values of Technological Residual Stresses

T. Mendebayev, A. Gabdullina, Y. Shayakhmetov, R. Ibragimova and Sh. Rakisheva
Department of Standardization, Certification and Technology of Machine Tool Building, Kazakh National Technical University Named after K.I. Satpayev, Almaty, Kazakhstan

Abstract: In the present article the analysis of deformations when processing low-rigid and thin-walled products is carried out, calculation of values of critical values of residual stresses is made. The principles of creation of technological processes for such details, the called "stabilizing technologies" are offered.

Keywords: Critical levels of residual stresses, deformation of a jacket, residual stresses, stabilizing technologie

INTRODUCTION

In the construction process for the machining of thin-walled parts is extremely important picture diagrams circumferential residual stress getting such that the amount of deformation of the shell of their action does not exceed the tolerance on the diametrical size. Character diagrams district residual stresses determined by the conditions of the machining operation and is caused mainly removing irregular allowances, uneven texture, changes in the shape and size of the grains, as well as violations of the integrity of the material surface layer.

Diagrams of residual stresses are experimentally measured investigated and widely used in mechanical engineering technology the method of Davidenkova-Zaks, by electromechanical etching strained parts of the surface layer (Burtsev *et al*., 1999; Isaev and Malinovski, 2001; Mendebayev, 1995; Suslov, 2000; Sinopalnikov and Grigoryev, 2003; Timiryazev *et al*., 2011; Vasilyev *et al*., 2005).

These factors lead to the localization of the surface layer parts of significant quantities of residual stresses. For certain values of these variables, even uniformly distributed around the circumference of the stress, thin-walled shells may lose its stable circular shape and go to another intermediate equilibrium state. Since the field of residual stresses discrete, the resulting equilibrium state is maintained.

Theoretical and practical calculations of the critical values of technological residual stresses are given in the works (Burtsev *et al*., 1999; Isaev and Malinovski, 2001; Mendebayev, 1995; Suslov, 2000; Sinopalnikov and Grigoryev, 2003; Timiryazev *et al*., 2011; Vasilyev *et al*., 2005).

Vasilyev *et al*. (2005) and Burtsev *et al*. (1999) experimentally checked that residual stresses affect the stability of the form of thin-walled of cylindrical shells. They pointed out that the sign and quantities of residual stresses influence the cutting conditions and the geometric parameters of cutting tools. It is assumed that a scientifically substantiated process technology should be optimal. It must provide a predetermined quality of the machines and it should save to the long time, which in turn depends on the critical values of residual stress in the surface layer.

Therefore the aim of this study is the theoretical definition of the relationship between the residual stresses and distortions of the form of thin-walled of cylindrical shells.

MATERIALS AND METHODS

In theory, the task of establishing the relationship between the district and the residual voltage distortion of the shell performed using the energy method of resolving equations of elastic stability of cylindrical shells (Mendebayev, 1995).

For a more complete scientific study of this important theoretical concepts necessary to clarify another aspect of the problem associated with the critical value of the residual stress, which takes the form of a shell or out of balance leads to hardening and cracking of the treated surface of the part again.

These critical levels of residual stresses established theoretical calculation must undergo tested in a production environment and to find their practical confirmation. If they really are true, the concept of building manufacturing processes little hard and thin-walled parts must be completely based on scientific principles and other provisions than the existing basic

Corresponding Author: T. Mendebayev, Department of Standardization, Certification and Technology of Machine Tool Building, Kazakh National Technical University Named After K.I. Satpayev, Almaty 050013, Kazakhstan

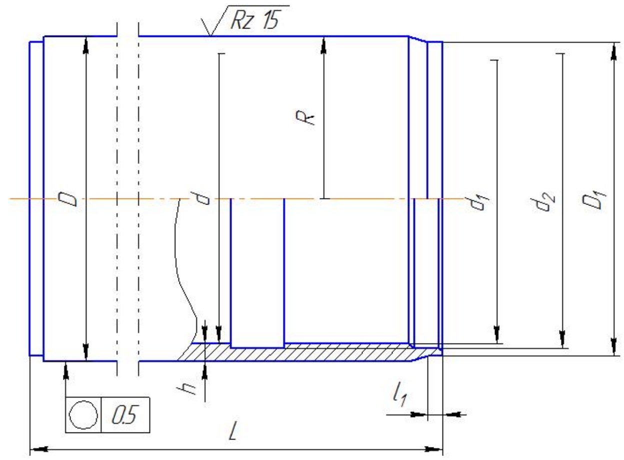

Fig. 1: Thin-walled casing of an underwater torpedo

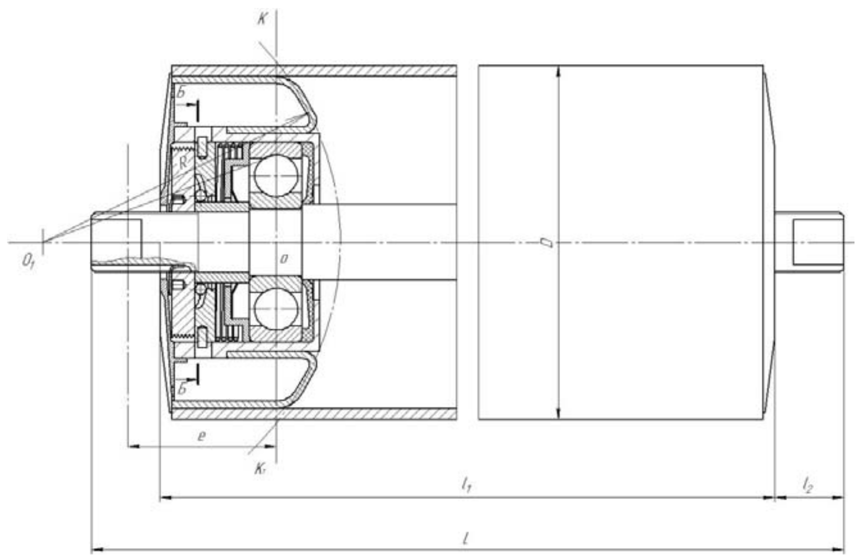

Fig. 2: Pipeline roller. 1 shell of a roller of the pipeline

principles and provisions of the construction of traditional technologies.

The similar concept claiming for the basic principles of development of stabilizing technologies is supposed in case of manufacture of the thin-walledcasing of underwater torpedoes (Fig. 1), shells of rollers of pipelines (Fig. 2) etc. Mechanical engineering details. The casing of torpedoes is made of hot-rolled sheet steel of brand 1H16N4B of a martensit class.

RESULTS AND DISCUSSION

The theory of Biot (Bozhanov, 1997) within Kirchhoff-Lyava's hypothesis is applied to establishment of value of critical value of residual stress and the task of stability of a jacket from residual stresses is considered.

After obtaining the equations of stability for any shell make the transition to a semi-infinite cylindrical shell, as a special case and the problem is solved using the Bubnov-Galerkin (Bozhanov, 1997; Vlasov, 1994).

The expression defining critical value of environing effort (tension) depending on the form of loss of stability is found.

For receiving the equation of the perturbed status we use the equilibrium equations received in operation (Bozhanov, 1997), assuming that lengthenings and shift are small in comparison with unit.

When this is not considered a deformation of the middle surface. If the subcritical state of the shell considered momentless, then the equilibrium equations are of the form:

$$\left.\begin{array}{c}\frac{\partial}{\partial\alpha_1}A_2T_{11}+\frac{\partial}{\partial\alpha_2}A_2T_{21}+T_{21}+T_{21}\frac{\partial A_1}{\partial\alpha_2}-T_{22}\frac{\partial A_2}{\partial\alpha_2}-T_{22}\frac{\partial A_2}{\partial\alpha_2}-T_{22}\frac{\partial A_2}{\partial\alpha_1}+A_1\cdot A_2\cdot X_1=0\\ \frac{\partial}{\partial\alpha_2}A_1T_{22}+\frac{\partial}{\partial\alpha_1}A_2T_{21}\frac{\partial A_2}{\partial\alpha_1}-T_{11}\frac{\partial A_1}{\partial\alpha_2}+A_1\cdot A_2\cdot X_1=0\\ -A_1\cdot A_2(K_{11}\cdot T_{11}+K_{22}\cdot T_{22}-X_3)=0\\ T_{11}=\int_{-h/2}^{h/2}S_{11}d\gamma;T_{22}=\int_{-h/2}^{h/2}S_{22}d\gamma;\ T_{12}=\int_{-h/2}^{h/2}S_{12}d\gamma\end{array}\right\}\tag{1}$$

Here,
A_1, A_2 : Parameters to Lyama
K_1, K_2 : Principal curvature

The critical state of the shell of torque and equilibrium equation have the form:

$$\left.\begin{array}{l}\frac{\partial}{\partial\alpha_1}A_2T_{11}^*+\frac{\partial}{\partial\alpha_2}A_1T_{21}^*+T_{21}^*\frac{\partial A_1}{\partial\alpha_2}-T_{22}^*\frac{\partial A_2}{\partial\alpha_1}+A_1\cdot A_2(K_1^*\cdot N_1+X_1^*)=0\\ \frac{\partial}{\partial\alpha_2}A_1T_{22}^*+\frac{\partial}{\partial\alpha_2}A_2T_{12}^*+T_{21}^*\frac{\partial A_2}{\partial\alpha_1}-T_{11}^*\frac{\partial A_2}{\partial\alpha_1}+A_1\cdot A_2(K_2^*\cdot N_2+X_2^*)=0\\ \quad\frac{\partial}{\partial\alpha_1}A_2N_1\ +\frac{\partial}{\partial\alpha_2}A_1N_2+A_1\cdot A_2(K_1^*\cdot T_{11}+K_2^*\cdot T_{22}X_3^*)=0\\ \frac{\partial}{\partial\alpha_1}A_2m_{11}+\frac{\partial}{\partial\alpha_2}A_1m_{21}+m_{12}\frac{\partial A_1}{\partial\alpha_2}-m_{22}\frac{\partial A_3}{\partial\alpha_1}-A_1\cdot A_2(N_1-L_1)=0\\ \frac{\partial}{\partial\alpha}A_1m_{22}+\frac{\partial}{\partial\alpha_1}A_1m_{12}+m_{21}\frac{\partial A_2}{\partial\alpha_1}-m_{11}\frac{\partial A_1}{\partial\alpha_2}-A_1\cdot A_2(N_2-L_2)=0\end{array}\right\}\tag{2}$$

where, X_i, L_i -проекции векторов projections of vectors of external effort and the moment to the direction of axes α_1 and α_2.
Change of curvature is taken as in operation (Amandosov and Bozhanov, 1975):

$$K_i=k_i+\frac{\partial^2 W}{\partial\alpha_i^2}\tag{3}$$

According to Bio theory, the body is in the perturbed status under the influence of initial stresses of S_{ij} and the perturbed status is characterized by a small increment of tension S_{ij} and full tension in the perturbed status is defined by the following formula:

$$\sigma_{ij}=S_{ij}+s_{ij}\tag{4}$$

Then internal forces and the moments of t_{ij}, N_{ij}, m_{ij}, which are logging in (2) and arising in connection with transition from one status to another, are defined as follows:

$$t_{ij}=\int_{-h/2}^{h/2}S_{ij}d\gamma;\ N_{ij}=\int_{-h/2}^{h/2}S_{ij}d\gamma;\ m_{ij}=\int_{-h/2}^{h/2}S_{ij}\gamma d\gamma;\tag{5}$$

These internal forces and the moments arising owing to transition from one status to another are so small that their levels above the first and works can be neglected.
Comparing system of Eq. (2) and (1), we find two groups of the equations:

$$\left.\begin{array}{l}\frac{\partial}{\partial\alpha_1}A_2m_{11}+\frac{\partial}{\partial\alpha_2}A_1m_{21}+m_{21}\frac{\partial A}{\partial\alpha_2}-m_{22}\frac{\partial A}{\partial\alpha_1}-A_1\cdot A_2(N_1-L_1)=0\\ \frac{\partial}{\partial\alpha_2}A_1m_{22}+\frac{\partial}{\partial\alpha_1}A_2m_{12}+m_{21}\frac{\partial A_2}{\partial\alpha_{12}}-m_{11}\frac{\partial A_1}{\partial\alpha_2}-A_1\cdot A_2(N_2-L_2)=0\end{array}\right\}\tag{6}$$

From which the Eq. (6) -the balance of power equation and the Eq. (6) -the equation of equilibrium points.
Solving them together, we get the condition of critical state of the shell:

$$\frac{A_2}{A_1}\cdot\frac{\partial^2 m_{11}}{\partial\alpha_1^2}+2\frac{\partial^2 m_{12}}{\partial\alpha_1\partial\alpha_2}+\frac{A_1}{A_2}\frac{\partial m_{22}}{\partial\alpha_2^2}+\frac{A_2}{A_1}\cdot\frac{\partial^2 W}{\partial\alpha_1^2}T_{11}+A_2\cdot A_1[K_{11}t_{11}+K_2t_{22}]=0\tag{7}$$

Communication "tension-deformation" According to Bio theory in relation to the two-dimensional task will register the following formula:

$$\left.\begin{aligned} S_{11} &= \lambda\varepsilon + 2\mu\varepsilon_{11} + S_{11}\varepsilon_{22} + S_{12}\varepsilon_{12} \\ S_{22} &= \lambda\varepsilon + 2\mu\varepsilon_{22} + S_{22}\varepsilon_{11} + S_{12}\varepsilon_{12} \\ S_{12} &= \mu\varepsilon_{12} - \tfrac{1}{2}S\varepsilon \end{aligned}\right\} \tag{8}$$

where,
λ, μ = Lame constant, $\varepsilon = \varepsilon_{11} + \varepsilon_{22}$.

Deformation of any layer of the characteristics of deformation of the middle surface of the shell to define (Geier, 1973):

$$\left.\begin{aligned} \varepsilon_{11,h} &= \varepsilon_{11} + \gamma\chi_{11,} \\ \varepsilon_{22,h} &= \varepsilon_{22} + \gamma\chi_{22,} \\ \varepsilon_{12,h} &= \varepsilon_{12} + \gamma\chi_{12,} \end{aligned}\right\} \tag{9}$$

In considering the problems of stability of the shell can take the assumption of "non-stretchable" middle surface (9) and assume a massive body shell in his "plane". Then, at a constant value of Lame parameters (9), we obtain:

$$\left.\begin{aligned} \varepsilon_{11} &= \frac{\gamma}{A_1^2}\cdot\frac{\partial^2 W}{\partial\alpha_1^2} + \frac{1}{A_1^2}\cdot A\left(\frac{\partial W}{\partial\alpha_1}\right)^2 + K_1 W \\ \varepsilon_{22} &= \frac{\gamma}{A_2^2}\cdot\frac{\partial^2 W}{\partial\alpha_2^2} + \frac{1}{A_2^2}\cdot A\left(\frac{\partial W}{\partial\alpha_2}\right)^2 + K_2 W \\ \varepsilon_{12} &= -\frac{2\gamma}{A_1 A_2}\cdot\frac{\gamma^2 W}{\partial\alpha_1\partial\alpha_2} + \frac{1}{A_1 A_2}\cdot\frac{\partial W}{\partial\alpha_1}\cdot\frac{\partial W}{\partial\alpha_2} \end{aligned}\right\} \tag{10}$$

In expression (10) is omitted index "n". Knowing (10) and (8) can establish a connection between the forces, moments and deflection "W".
Substituting it into (7) we can obtain the equation of the perturbed state relative deflection "W".

According to Vlasov (1994), Ymamoto *et al*. (1969) and Volmir (1967) the residual stresses are purely elastic regardless of the causes. Considering the above statement, Eq. (7) with the law (8), with the symbols (5) yields the critical value of the residual stress forces.

Consider a semi-infinite resistance cylindrical shell (Fig. 1), the radius R, the wall thickness h coordinate system z, φ, R, where the z axis is directed along the axis of the cylinder parallel to the generator. Parameters A_1, A_2 and Lame curvature in a coordinate system z, φ, R have the following meanings:

$$A_1 = 1; A_2 = R; K_1 = 0; K_2 = \frac{1}{R} \tag{11}$$

Rewrite (10) with (11):

$$\left.\begin{aligned} \varepsilon_{zz} &= -\gamma\cdot\frac{\partial^2 W}{\partial z^2} + \frac{1}{2}\left(\frac{\partial W}{\partial z}\right)^2 \\ \varepsilon_{\varphi\varphi} &= \gamma\frac{\partial^2 W}{\partial\varphi^2} + \frac{1}{2R}\left(\frac{\partial W}{\partial\varphi}\right)^2 + \frac{1}{R}W \\ \varepsilon_{z\varphi} &= \frac{1}{R}\cdot\frac{\partial W}{\partial z} + \frac{\partial W}{\partial\varphi} - 2\frac{\gamma}{R}\cdot\frac{\partial^2 W}{\partial z\cdot\partial\varphi} \end{aligned}\right\} \tag{12}$$

Equation (7) in cylindrical coordinates has the form:

$$R\frac{\partial^2 m_{11}}{\partial z^2} + 2\frac{\partial^2 m_{z\varphi}}{\partial z\partial\varphi} + \frac{1}{R}\frac{\partial^2 m_{\varphi\varphi}}{\partial\varphi^2} + R\frac{\partial^2 W}{\partial\varphi^2}T_{\varphi\varphi} + t_{\varphi\varphi} \tag{13}$$

Substituting (8) and (5) determine the forces and moments:

$$\left.\begin{aligned}
t_{zz} &= \frac{h}{2}(\lambda+2\mu)\left(\frac{\partial W}{\partial z}\right)^2 + \frac{1}{2R^2}\frac{\partial^2 m_{\varphi\varphi}}{\partial\varphi^2} + R\frac{\partial^2 W}{\partial\varphi^2}\cdot T_{\varphi\varphi} - \frac{2}{R}\frac{\partial^2 W}{\partial z\cdot\partial\varphi}\cdot M_{z\varphi} + \\
&\quad + \frac{1}{R}(T_{zz}+h\lambda)W - \frac{1}{R^2}\cdot\frac{\partial^2 W}{\partial\varphi^2}\cdot M_{zz} + \frac{1}{R}\frac{\partial W}{\partial z}\cdot\frac{\partial W}{\partial\varphi}\cdot T_{z\varphi}; \\
t_{\varphi\varphi} &= \frac{h}{2R^2}(\lambda+2\mu)\left(\frac{\partial W}{\partial\varphi}\right)^2 + \frac{1}{2}\left(T_{\varphi\varphi}+\lambda h\right)\cdot\left(\frac{\partial W}{\partial z}\right)^2 + \frac{1}{R}\frac{\partial W}{\partial z}\cdot\frac{\partial W}{\partial\varphi}\cdot T_{z\varphi} + \\
&\quad + \frac{h}{R}(\lambda+2\mu)\cdot W; \\
t_{z\varphi} &= \frac{\mu h}{R}\frac{\partial W}{\partial z}\cdot\frac{\partial W}{\partial\varphi} - \frac{1}{R}\cdot T_{z\varphi}W - \frac{1}{4}\left[\left(\frac{\partial W}{\partial z}\right)^2 + \frac{1}{R}\left(\frac{\partial W}{\partial\varphi}\right)^2\right]\cdot T_{z\varphi}
\end{aligned}\right\} \tag{14}$$

$$\left.\begin{aligned}
m_{zz} &= \frac{h^3}{12}(\lambda+2\mu)\left(\frac{\partial^2 W}{\partial z^2}\right) - \frac{\lambda}{R^2}\left(J_{zz}+\frac{\lambda h^3}{12}\right)\frac{\partial^2 W}{\partial\varphi^2} + \frac{1}{2R}\left(\frac{\partial W}{\partial\varphi}\right)^2\cdot M_{zz} + \frac{1}{R}\frac{\partial W}{\partial z}\cdot\frac{\partial W}{\partial\varphi}\cdot M_{z\varphi} - \\
&\quad - \frac{2}{R}\cdot\frac{\partial^2 W}{\partial z\partial\varphi}\cdot J_{z\varphi} + \frac{W}{R}\cdot M_{zz}; \\
m_{\varphi\varphi} &= -\left(\frac{\lambda h^3}{12}+J_{\varphi\varphi}\right)\frac{\partial^2 W}{\partial z^2} - \frac{h^3}{12R^2}(\lambda+2\mu)\left(\frac{\partial^2 W}{\partial\varphi^2}\right) + \frac{1}{2}\left(\frac{\partial W}{\partial z}\right)^2 M_{\varphi\varphi} + \frac{1}{R}\frac{\partial W}{\partial z}\cdot\frac{\partial W}{\partial\varphi}\cdot M_{z\varphi} - \\
&\quad - \frac{2}{R}\frac{\partial^2 W}{\partial z\partial\varphi}\cdot J_{z\varphi}; \\
m_{z\varphi} &= -2\mu\frac{h^3}{12R}\frac{\partial^2 W}{\partial z\partial\varphi} + \frac{1}{2}\left(\frac{\partial^2 W}{\partial z^2}+\frac{1}{R^2}\frac{\partial^2 W}{\partial\varphi^2}\right)J_{z\varphi} - \frac{1}{2R}W\cdot M_{z\varphi} - \\
&\quad - \frac{1}{L}\left[\left(\frac{\partial W}{\partial z}\right)^2 + \frac{1}{R}\left(\frac{\partial W}{\partial\varphi}\right)^2\right]\cdot M_{z\varphi}
\end{aligned}\right\} \tag{15}$$

Substituting Eq. (14), (15) in Eq. (13), we obtain the equation of state of the perturbed cylindrical shell relative deflection.
If we assume now that:

$$W^{/} = W(\varphi) \tag{16}$$

Then (14) with (15) and (16) we obtain:

$$\frac{h^{/2}}{2}\cdot\frac{\partial^4 W^{/}}{\partial\varphi^4} + T^{/}_{\varphi\varphi}\frac{\partial^2 W^{/}}{\partial\varphi^2} + \frac{1}{2}\left(\frac{\partial W^{/}}{\partial\varphi}\right)^2 + W^{/} = 0 \tag{17}$$

where, the notation:

$$W^{/} = \frac{W}{R}, h^{/} = \frac{h}{R}, f^{/} = \frac{f}{R}, T^{/}_{\varphi\varphi} = \frac{T_{\varphi\varphi}}{h(\lambda+2\mu)} \tag{18}$$

We solve the resulting equation by specifying its solution in the form:

$$W^{/} = f sinn\varphi \tag{19}$$

where, n-number of waves
Form the equation of the Bubnov-Galerkin method:

$$4n\int_0^{\pi/2n} x\cdot sinn\varphi\cdot d\varphi = 0 \tag{20}$$

где:

$$X = \frac{h^{/2}}{2}\cdot\frac{\partial^4 W^{/}}{\partial\varphi^4} + T^{/}_{\varphi\varphi}\frac{\partial^2 W^{/}}{\partial\varphi^2} + \frac{1}{2}\left(\frac{\partial W^{/}}{\partial\varphi}\right)^2 + W^{/} \tag{21}$$

Integrating (20) with (19), we find the value of the efforts of the district:

$$T^{/}_{\varphi\varphi} = \frac{1}{n^2} + \frac{2f^{/}}{3\pi} + \frac{h^{/2}}{2}\cdot n^2 \tag{22}$$

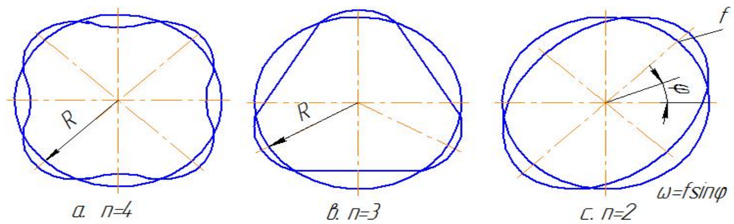

Fig. 3: Different kinds of distortions of the body from the roundness. A tetrahedron, in-cut, with oval

Substituting the value of $\sigma_{\theta kp} = 0,01(\lambda + 2\mu T\varphi\varphi; \lambda+2\mu=E1+v21-v2$, obtain:

$$\sigma_{\theta kp} = 0,01\left[\frac{1}{n^2}+\frac{1}{2}\left(\frac{h}{R}\cdot n\right)^2\right]\cdot E\,\frac{1+v^2}{1-v^2} \qquad (23)$$

Here

$\sigma_{\theta kp}$ = Critical circumferential residual stress, MPa
R, h = The radius and the wall thickness of the cylinder, mm
E = Modulus of elasticity of the first kind, MPa
v = Poisson's ratio

From the analysis of formula (23) can be seen that the critical voltage depends on the physical-mechanical properties of cladding material (E, v) and the ratio of shell wall thickness to the radius h/R. When h/R = const, the critical value of the stress is the number of waves distortion. From Eq. (23) it follows that for an ellipsoid shape distortion (n = 2) for the studied steels 1H16N4B, 10H15N27, 16H20K6N2 $\sigma_{окр}$ = 520 for a three-sided shape (n = 3) $\sigma_{окр}$ = 230 MPa for tetrahedral (n = 4) $\sigma_{окр}$ = 130 MPa (Fig. 3).

CONCLUSION

These critical levels of residual stresses Establishing theoretical calculations have been tested in a production environment and have found their practical confirmation. Therefore, the concept of building manufacturing processes little hard and thin-walled products, which are named as "stabilizing technology" should be based on scientific principles and other provisions than the existing basic principles and provisions of the construction of traditional technologies. However, authors continue to work on this subject and additions to the existing findings and, as a consequence, the development of this direction of research are possible in the future.

REFERENCES

Amandosov, A.A. and E.T. Bozhanov, 1975. Refined Equations of the Theory of Flexible Plates and their Linearization Stability Problems. Problematic Issues of Rock Mechanics, Alma-Ata.

Bozhanov, E.T., 1997. Study of the problems of stability of elastic bodies, flexible plates, membranes and their applications. M.Sc. Thesis, Almaty.

Burtsev, V.M., A.S. Vasilyev, A.M. Dalski *et al*., 1999. Machine Building Technology. In: Dalski, A.M. (Ed.), Fundamentals of Machine Building Technology: Textbook for High Schools. Publishing House of MSTU after Bauman N.E., Moscow, pp: 564.

Geier, B., 1973. Eine Diskretisierung der Gleichung fur die Berechnung der stabilitat orthotroper S and wichschalen uber rechteckidem Grundriss. Finite Elem. Static. Berlin.

Isaev, L.K. and V.D. Malinovski, 2001. Quality Assurance. Standards Publishing House, Moscow, pp: 270.

Mendebayev, T.M., 1995. Theoretical bases and methods of stabilizing the development of machining technology products such as non-rigid shells. M.Sc. Thesis, Bishkek.

Sinopalnikov, V.A. and S.N. Grigoryev, 2003. Reliability and Diagnostics of Technological Systems. RC MSTU "Stankin", Moscow, pp: 331.

Suslov, A.G., 2000. Quality of Surface Layer of Machine Parts. Machine Building, Moscow, pp: 320.

Timiryazev, V.A., A.A. Kutin and A.G. Skhirtladze, 2011. Fundamentals of Machine Building Technology. SEI MSTU "Stankin", Moscow.

Vasilyev, A.S., A.M. Dalski, Y.M. Zolotarevski and A.I. Kondakov, 2005. Directed Formation of Properties of Machine Building Products. In: Sc. Kondakov, A.I. (Ed.), Machine Building, Moscow, pp: 352.

Vlasov, V.V., 1994. Basic differential equations of the general theory of elastic shells. Prikl. Mat. Mekh., 8: 109-140. (In Russian)

Volmir, A.S., 1967. Stability of Deforming Systems. Nauka, Moscow, pp: 910.

Ymamoto, Y., K. Watanale and T. Nakamura, 1969. Buckling of a cylindrical shell reinforced by spiral stiffeners under external pressure. Bull. JSME, 12(54): 1249-1259.