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Research Article

Rainfall Storm Modeling of Neyman-Scott Rectangular Pulse (NSRP) using Rainfall Cell Intensity Distributions

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Abstract: In this research a rainfall storm modeling using stochastic model of Nyeman-Scott Rectangular Pulse (NSRP) will be applied using different distributions on the rainfall cell intensity such as exponent, mix exponent, gamma and weibull distributions. The modeling conducted on the rain station of Alosetar uses data taken every hour during a period of 38 years (1971-2008) and it shows that most of the months the rain cells on NSRP model have been distributed to exponential and gamma. Based on parsimoni principles, it can be concluded that NSRP modeling on the rain station of Alorsetar has the intensity of rainfall cells exponentially distributed. Furthermore, in this research also yields a fact that the mix exponent distribution is unfitted for the rainfall cells using NSRP modeling on the rain station of Alorsetar.

Keywords: Gamma, mix exponent, NSRP, rainfall storm, weibull

INTRODUCTION

Stochastic modeling on hourly rainfall data is a prominent research on hydrology area, for example, it can be used on how to estimate early flooding water and to predict water resources availability. On this research the study will focus on rainfall storm using Neyman-Scott Rectangular Pulse (NSRP) modeling. Rodriguez-Iturbe *et al*. (1987b) had examined the drawback of the use of NSRP model and Bartlett-Lewis, Entekhabi *et al*. (1989) modeling had used NSRP on the intensity of rainfall cells and it exponentionally distributed and came with a good outcome for the summer's rainfall data in Denver. Cowpertwait (1991) also used dry (not rain) probability on the NSRP modeling for hourly rainfall data over 10 years in the UK. Cowpertwait (1994) used NSRP modeling to examine extreme rainfall data. Researches related to stochastical rainfall storm modeling using NSRP process are often conducted on different distributions of rain cell intensity, for example, exponential, exponential-mix, gamma an weibull are frequently used. Most researchers used exponential distribution on the rainfall cell intensity to model the rain and applied NSRP process, (Rodriguez-Iturbe *et al*., 1987a, 1989; Cowpertwait *et al*., 1996a, 1996b), for instance. Cowpertwait *et al*. (1996a, 1996b) had conducted this modeling by applying Weibull distribution of rainfall cell intensity while investigating extreme rainfall properties, furthermore, Cowpertwait (1998) used exponential and gamma distributions of the rainfall cell intesity to define the extreme rainfall model, while Mondonedo *et al*. (2010) used exponential, gamma and weibull distribution in examining the properties of extreme rainfall experienced in Kyushe and Okinawa. NSRP 's parameters estimation has been performed by some researchers, such as some famous researchers; Rodriguez-Iturbe *et al*. (1987a, 1987b), Cowpertwait *et al*. (1996a, 1996b), Calenda and Napolatino (1999) and Favre *et al*. (2004). On this research the rainfall storm modeling will use NSRP process over 4 distinct distributions on rainfall cell intensity, namely, exponential distribution, exponential-mix, gamma and weibull. The research applies hourly rainfall data on rain station of Alorsetar over 38 years (1971 – 2008). Some missing data on this station will be substituted by generating data through simulation using Marcov chain, as it was conducted by Wilks (1999) and Lennartsson *et al*. (2008).

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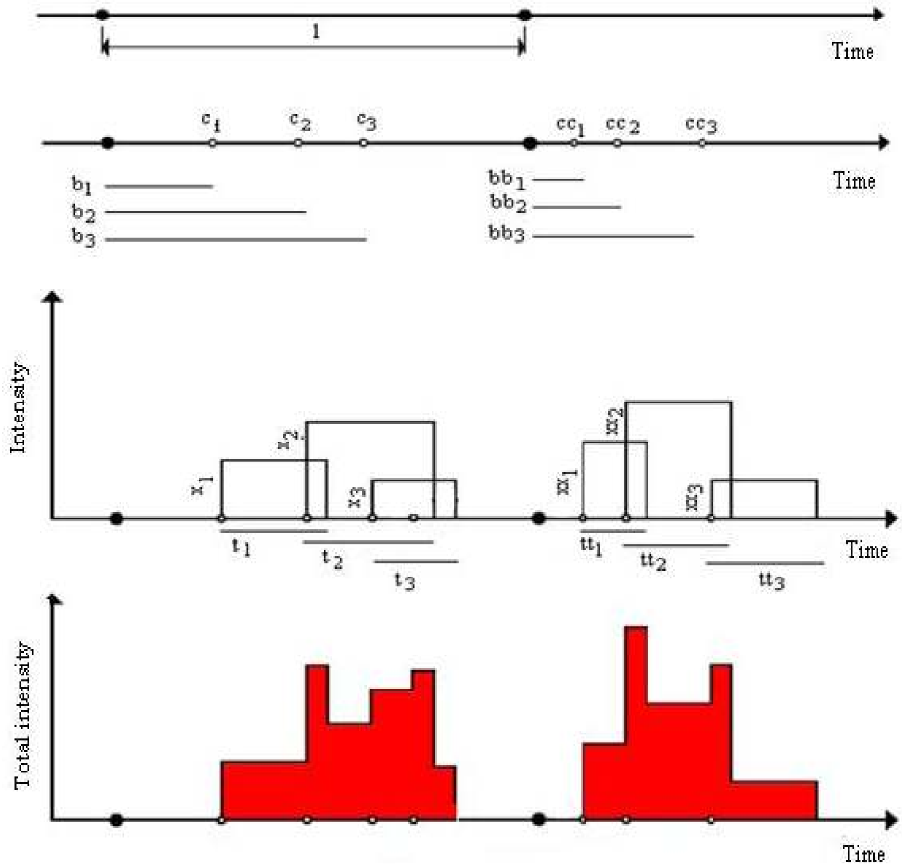

Fig. 1: NSRP modeling; (I) arrival time between two storms, (c) rainfall cell, (b) exact time of each rainfall cell, (t) the duration of rainfall cell and (x) rainfall cell intensity

NSRP modeling: NSRP modeling depends on five random processes describing point-process of rainfall data. Rodriguez-Iturbe *et al*. (1987a, 1987b) and Cowpertwait (1998) had delivered the research that the NSRP modeling was defined by five parameters; (λ, E(C), E(X), β and η). Every arrival of storm-i through poisson process with inter-arrival between the storm has exponential distribution and an average λ, while every storm has the number of random rainfall of cell c which is transformed into several clusters, where the number of random rainfall is distributed as poisson and geometry with an average E(C). Furthermore, each time of rainfall cell b is measured from arrivals distributed exponentially with average β. Every cluster of rainfall cells would have rainfall cell duration t which is exponentially distributed with an average η and rainfall cell intensity x which is also exponentially distributed with an average E(X), as it can be seen on Fig. 1.

On Fig. 1, it can be explained that in between two storms there are three rainfall cells of the first storm and three rainfall cells of the second storm. On the real number of rainfall cells is actually random, the appearing signs which comprise two symbols such as cc, bb, tt and xx are parameter for the second storm. NSRP modeling is started by generating 5 rainfall parameters, which is obtained firstly from estimating the second moment statistics and rainfall probability generated from collected data, thus a relation can be inferred by using statistics value of the second moment and the rainfall probability and from Eq. (1) through (4) would gain four non-linier equations.

$$E\left(Y_i^{(\tau)}\right)=\frac{\lambda}{\eta}E(C)E(X)\tau \qquad (1)$$

$$V\left(Y_i^{(\tau)}\right)=\Omega_1(\lambda,E(C),E(X))\,\Psi_1(\beta,\eta)+\Omega_2(\lambda,E(C),E(X))\,\Psi_2(\beta,\eta) \qquad (2)$$

$$Cov\left(Y_i^{(\tau)},Y_{i+k}^{(\tau)}\right)=\Omega_1(\lambda,E(C),E(X))\,\Psi_3(\beta,\eta)+\Omega_2(\lambda,E(C),E(X))\,\Psi_4(\beta,\eta) \qquad (3)$$

where,

$$\Omega_1(\lambda,E(C),E(X))=2\lambda(C)E\left(X^2\right)$$

$$\Omega_2(\lambda,E(C),E(X))=\lambda E\left(C^2-C\right)E^2(X)$$

$$\Psi_1(\beta,\eta)=\frac{1}{\eta^3}\left(\eta\tau-1+exp(-\eta\tau)\right)$$

$$\Psi_2(\beta,\eta)=\Psi_1(\beta,\eta)\frac{\beta^2}{\beta^2-\eta^2}-\frac{\beta\tau-1+exp(-\beta\tau)}{\beta(\beta^2-\eta^2)}$$

$$\Psi_3(\beta,\eta)=\frac{1}{2\eta^3}(1-exp(-\eta\tau))^2 exp(-\eta(k-1)\tau)$$

$$\Psi_4(\beta,\eta)=\Psi_3(\beta,\eta)\frac{\beta^2}{\beta^2-\eta^2}-\frac{(1-\exp(-\beta\tau)^2\exp(-\beta(k-1))}{2\beta(\beta^2-\eta^2)}$$

$$PW(\tau)=1-\exp\left\{\begin{matrix}-\lambda\tau+\frac{\lambda}{\beta\mu}\left[1-\exp\left(-\mu+\mu\,e^{-\beta\tau}\right)\right]\\ -\lambda\int_0^{\infty}\left[1-p_t(\tau)\right]\,dt\end{matrix}\right\} \quad (4)$$

While

$$p_t(\tau)=(1-\exp(-\beta t)+\exp(-\beta(t+\tau))) \\ (1-\beta\,(\exp(-\beta t)-\exp(-\eta t))\ /(\eta-\beta)\ ) \\ \exp\{-\mu\beta(\exp(-\beta t)-\exp(-\eta t)) \\ /(\eta-\beta)-\mu\exp(-\beta t)+\mu\,\exp(-\beta(t+\tau)\}$$

NSRP'S PARAMETER ESTIMATION AND GOOD-FIT TEST ON RAINFALL CELL INTENSITY

Rodriguez-Iturbe *et al*. (1987a, 1987b) and Cowpertwait (1991) has used a moment method to estimate NSRP's parameter. Other methods estimating the same parameters are also conducted by other researchers, who applied the method of Log-likelihood maximum probability, such as (Smith and Karr, 1985a, 1985b; Obeysekera *et al*., 1987). Some researchers, who have provided usual procedure, which is needed to convert hourly rainfall data into aggregate rainfall data, in estimating NSRP's parameter, are Entekhabi *et al*. (1989), Cowpertwait (1991) and Velghe *et al*. (1994), Scaling is applied to obtain rainfall data of various scales, such as one hour rainfall scale, six hour rainfall scale and 24-hour rainfall scale, which use Eq. (1)-(4), thus some non-linear equations are produced. Furthermore, by optimizing Eq. (5) numerically then it comes with 5 expected NSRP's parameters:

$$Z(X)=\sum_{k,\tau}\left[1-\frac{\Theta_k(X,\tau)}{\Theta_k^*(\tau)}\right]^2 \quad (5)$$

$\Theta^*_k(\tau)$ is the second moment statistics and rainfall probability from scaled data, or generally called as observation statistics and $\Theta_k(x, \tau)$ is the second moment statistics and rainfall probability stated on Eq. (1), (4), or generally called as theoretical statistics.

The equation solution numerically needs an accurate initial value. Researches on non linier numeric model often require it in order to enable them to estimate some required parameters. Cowpertwait (1998) and Calenda and Napolatino (1999) have shown some initial values to estimate NSRP's parameters accurately. In fact, it is not easy to perform numerical solution since it needs many initial values to be tested, so that z value on Eq. (5) will be optimum. Favre *et al*. (2004) have tried to give the best method on estimating NS parameters easier, the research is conducted by dividing parameters into two sets of parameters, which comprise $\{\beta, \eta\}$ and $\{E(C), \lambda, E(X)\}$. Providing an initial value for parameter $\{\beta, \eta\}$, then numerical solution of estimating other parameters will be simpler and easier to be handled. Calenda and Napolatino (1999) also gave their contribution related to other methods in simplifying numerical solution in estimating NSRP's parameters. They conferred fluctuation scale values linking one parameter to other four parameters, so that z value can be optimum based on the four chooses parameters. In this research the number of rainfall cell of each storm will be distributed under poisson condition, thus $E(C^2\text{-}C) = \mu^2_c$ -1, this result has been well investigated (Velghe *et al*., 1994). In this research also the exponential distribution of rainfall cell intensity has a probability density function with scale parameter γ and θ:

$$f(x)=\frac{p}{\gamma}\exp\left(-\frac{x}{\gamma}\right)+\frac{1-p}{\theta}\exp\left(-\frac{x}{\theta}\right),\ 0\le p\le 1,\ \gamma>0,\ \theta>0$$

thus, some values, such as $E(X) = p\gamma + (1\text{-}p)\theta$ and $E(X^2) = 2p\gamma^2 + 2(1\text{-}p)\theta^2$, are needed to define NSRP's parameters. Furthermore, gamma distribution with scale parameters θ and shape α has probability density function as the following equation:

$$f(x)=\frac{x^{\alpha-1}\exp(-y/\theta)}{\theta^{\alpha}\,\Gamma(\alpha)},\ 0\le x<\infty,\ \alpha>0\ ,\ \theta>0$$

similar to exponential-mix distribution, gamma distribution with given value $E(X) = \alpha\theta$ and $E(X^2) = \alpha(1\text{-}\alpha)\theta^2$, is used to estimate NSRP's parameter. Then weibull distribution with probability density function:

$$f(x)=\frac{\alpha\,x^{\alpha-1}}{\theta^{\alpha}}\exp\left[-\left(\frac{x}{\theta}\right)^{\alpha}\right],\ 0\le x<\infty,\ \alpha>0,\ \theta>0$$

With scale parameter θ and shape α and it has value:

$$E(X)=\theta\,\Gamma\left(\frac{1+\alpha}{\alpha}\right) \text{ and } E(X^2)=\theta^2\,\Gamma\left(\frac{2+\alpha}{\alpha}\right)$$

Good of fit test, which is used to define the best fit distribution in rainfall cell intensity of four given distribution in this research, will be applied by sorting residual value gained from a value of the second moment statistics and observed rainfall probability and from a value of the second moment and theoretical rainfall probability. Velghe *et al*. (1994) used residual value as Eq. (6):

$$S = \frac{1}{n}\left[\sum_{n}\left|1-\frac{X_n}{X_{his,n}}\right|\right]\times 100\,\% \quad (6)$$

Let the second moment statistics and theoretical rainfall probability as X_n and the second moment statistics and the observed moment statistics as $X_{his,\ n}$ and n as the number of the second moment statistics and rainfall probability for used rainfall data scales. In this research, n is 8 showing the average of an hour rainfall, the variance used for the rainfall includes periods of 1, 6 and 24 h, auto correlation lag 1 for 1 h rainfall, auto correlation lag 1 for a 24 h rainfall scale, a probability of 1 h rainfall and probability of 24 h rainfall scale are written as $\hat{v}(1)$, $\hat{\sigma}(1)$, $\hat{\sigma}(6)$, $\hat{\sigma}(24)$, $\hat{\rho}(1,1)$, $\hat{\rho}(24,1)$, $\hat{\phi}(1)$ and $\hat{\phi}(24)$ consecutively. Calenda and Napolatino (1999) used Eq. (7) to do good-fit test which is a combination of rainfall scaling for determining the best NSRP model. X_k is the second moment statistics and theoretical rainfall probability and $X_{s,\ k}$ is the second moment statistics and probability of the observed rainfall.

$$S = \frac{1}{n}\left[\sum_{k=1}^{n}\left(1-\frac{X_k}{X_{s,k}}\right)^2\right]\times 100 \quad (7)$$

RESEARCH OUTCOMES

This research used hourly rainfall data collected from the rain station of Alorsetar for 38 years (1971-2008). NSRP's parameters of various distributions for rainfall cell intensity from the fifth to seventh month has been yielded as it is shown on Table 1 to 3. From those tables, there are 4 NSRP parameters, namely, λ, β, μ and η, while in the sixth and seventh month they have almost similar values of four different distributions used onto this research. This initial gain may be interpreted that those four distributions have given equally fit on NSRP modeling, thus in the fifth month was gained that there was different values for those four distribution. This fact led to an early conclusion that distribution fitness for rainfall cell intensity is needed to be investigated further in order to obtain the best modeling. Due to that conditions, in this research the good-fit test for the four distributions of rainfall cell intensity will be conducted as it is yielded on Table 4 to 7 and discussed on the last section.

The outcome gained on the Table 1 to 3 will be used to achieve values of the second moment statistics and theoretical rainfall probability for the four

Table 1: NSRP's parameters for hourly rainfall in May

	λ	β	μ_c	η	ξ	α(shape)	θ(scale)	γ(scale)	p
Exponent	0.0141	0.107	3.875	1.974	8.918				
Mix exponent	0.0204	0.132	3.063	2.055			11.111	0.048	0.240
Gamma	0.0205	0.110	2.974	2.056		0.571	14.811		
Weibull	0.0204	0.106	2.952	1.757		0.758	7.081		

Table 2: NSRP's parameters for hourly rainfall in June

	λ	β	μ_c	η	ξ	α(shape)	θ(scale)	γ(scale)	p
Exponent	0.0141	0.107	3.875	1.974	8.918				
Mix exponent	0.0141	0.107	3.930	2.044			11.273	2.122	0.228
Gamma	0.0141	0.107	3.875	2.023		0.711	12.539		
Weibull	0.0140	0.103	3.934	2.042		0.855	8.357		

Table 3: NSRP's parameters for hourly rainfall in July

	λ	β	μ_c	η	ξ	α(shape)	θ(scale)	γ(scale)	p
Exponent	0.0162	0.091	3.884	1.659	7.213				
Mix exponent	0.0156	0.085	3.976	1.625			9.148	5.95×10^{-5}	0.777
Gamma	0.0163	0.089	3.860	1.671		0.636	11.384		
Weibull	0.0159	0.093	3.943	1.453		0.803	6.318		

Table 4: The observed vs estimated statistics for some distributions rainfall intensity cell on May

	$\hat{v}(1)$	$\hat{\sigma}(1)$	$\hat{\sigma}(6)$	$\hat{\sigma}(24)$	$\hat{\rho}(1,1)$	$\hat{\rho}(24,1)$	$\hat{\phi}(1)$	$\hat{\phi}(24)$
Observation	0.261	3.632	38.540	163.000	0.358	0.069	0.080	0.502
Exponent	0.254	3.486	36.652	180.886	0.350	0.068	0.079	0.521
Mix exponent	0.257	3.375	35.960	179.041	0.353	0.063	0.079	0.504
Gamma	0.254	3.485	36.652	180.887	0.351	0.068	0.079	0.521
Weibull	0.252	3.464	36.482	179.670	0.353	0.069	0.079	0.522

Table 5: The observed vs estimated statistics for some distributions rainfall intensity cell on June

	$\hat{v}(1)$	$\hat{\sigma}(1)$	$\hat{\sigma}(6)$	$\hat{\sigma}(24)$	$\hat{\rho}(1,1)$	$\hat{\rho}(24,1)$	$\hat{\phi}(1)$	$\hat{\phi}(24)$
Observation	0.247	3.073	39.183	182.505	0.362	0.093	0.069	0.426
Exponent	0.247	3.240	35.926	188.780	0.372	0.093	0.069	0.490
Mix exponent	0.250	3.237	35.535	187.461	0.363	0.094	0.070	0.427
Gamma	0.247	3.240	35.926	188.780	0.371	0.092	0.069	0.426
Weibull	0.250	3.270	36.118	190.489	0.368	0.096	0.070	0.427

Table 6: The observed vs estimated statistics for some distributions rainfall intensity cell on July

	$\hat{v}(1)$	$\hat{\sigma}(1)$	$\hat{\sigma}(6)$	$\hat{\sigma}(24)$	$\hat{\rho}(1,1)$	$\hat{\rho}(24,1)$	$\hat{\phi}(1)$	$\hat{\phi}(24)$
Observation	0.272	3.233	39.241	206.071	0.413	0.1004	0.085	0.493
Exponent	0.273	3.300	38.751	203.912	0.418	0.1005	0.085	0.426
Mix exponent	0.272	3.279	38.754	204.511	0.423	0.1056	0.085	0.488
Gamma	0.273	3.310	38.643	202.742	0.415	0.1007	0.085	0.493
Weibull	0.272	3.291	38.843	204.848	0.420	0.0999	0.085	0.484

Table 7: Good of fit test for various distributions on rainfall intensity cell

	Exponent	Mix exponent	Gamma	Weibull
January	0.1050	0.1470	0.1050	0.0480
February	0.0210	0.0350	0.0210	0.1910
March	0.0006	1.2480	0.0006	0.0280
April	0.0120	0.0260	0.0120	0.0240
May	0.0160	0.0170	0.0160	0.0170
June	0.0120	0.0140	0.0120	0.0140
July	0.0009	0.0030	0.0010	0.0010
August	0.0080	0.0080	0.0080	0.0080
September	0.0002	0.0006	0.0002	0.0019
October	0.0071	5.3697	0.0071	0.0072
November	0.703	0.010	0.005	0.005
December	0.001	0.012	0.009	0.004

distributions used in this research. The values of the second moment statistics and rainfall probabilities for various scaling on rainfall data will be marked as on section 3, observed values and the theoretical second moment statistics and rainfall probability for the four distributions on the rainfall cell intensity has been given on Table 4 to 6. From the table, it can be seen that for the fifth – sixth month, they have almost similar values. From the table also gained that the rainfall cell intensity which exponentially and gamma distributed have equality properties to produce statistics $\hat{v}(1)$, $\hat{\sigma}(1)$, $\hat{\sigma}(6)$, $\hat{\sigma}(24)$ and $\hat{\phi}(1)$.

To define properly the rainfall cell intensity for every month, the good-fit test has been performed by using a method shown on Eq. (7). From Table 7 it can bee seen that various distribution on rainfall cell intensity is real on this research, so that particular research in monthly good-fit distribution test is necessary to be done properly. From Table 7 it is obvious that the best distribution is marked by thickening the value of good-fit test. Exponential and gamma distribution on thoroughly rainfall cell intensity is the best distributions for rainfall modeling based on NSRP of Alorsetar's rain station. Weibull distribution is the best distribution for NSRP model on the first month. In this research also found that exponential-mix distribution does not give good outcome for NSRP modeling through all months on the Alorsetar's rain station, except for month eight. From the Table 7, it can be concluded that for monthly data with the best fitted distribution of the rainfall intensity will produce more than one distribution, for example, as it can be seen from 2nd to 10th month which can apply only one exponential distribution. This distribution is the simplest model and properly fit to Parsimoni principles that a simplest model is the best model. On the other hand, there were two alternatives of good distributions of the rainfall intensity; gamma and weibull distributions and in the following month, month 12, an exponential distribution in the eleventh month was the best fitted distribution for NSRP modeling among other distributions.

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CONCLUSION

Various distributions on rainfall cell intensity are necessarily to be investigated thoroughly in mimicking the rainfall storm using NSRP process for each month because they will provide more accurate information in estimating statistical moment value and rainfall probability on some areas which are located near the observed rain station.

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