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Research Article

A Postulatory Study on Portfolio Optimization Algorithms: A Survey

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Abstract: A stock represents the capital a company or corporation raises by issuing and subscribing shares. The stock market is a term used to describe the physical location where the buying and selling as well as overall market activity takes place. Companies issue stocks to acquire capital while investors buy them to own a portion of the company. Investors buy stocks with the belief that the company will grow continuously to raise the value of their shares. Every shareholder in a company will have a say on how the company runs. Making investment among various financial enterprises, industries and other categories is associated by a risk factor. Diversification is a technique that is used to mitigate the effects of such risks and creating a portfolio of stocks is the technique used in diversification. In this study, effort has been taken to describe three of the most important portfolio optimization algorithms viz. Genetic Algorithm, Particle Swarm Optimization, Simulated Annealing.

Keywords: Genetic algorithm, Markowitz model, particle swarm optimization, portfolio, portfolio management, portfolio optimization, simulated annealing

INTRODUCTION

Stocks, shares, bonds, mutual funds, cash etc. Forms a collection of investment tools and is referred to as a Portfolio. This collection relies on investor's income, budget and stipulated time frame. Since portfolios form a major strand in the business world, their proper management is a significant aspect. Hence we go for Portfolio Management. It refers to the art of decision making about how the policies are instituted in investment, by appropriately matching the investments to objectives, allocating assets to individuals and institutions and stabilizing risk and performance. This decision making is done at maximizing return and minimizing risk.

Portfolio management helps the portfolio managers to provide customized investment solutions to the clients after eliciting their requirements. A process can be executed in its best case, worst case or average case scenarios. Whenever money comes in question, only the best case scenarios will be aimed by any entity which is a part of business world. There comes the necessity of optimization of portfolio management, where we look for the best case aspect of portfolio management. This study is drafted by keeping in mind about the beginners learners difficulty in understanding the concepts of portfolio optimization. Clear vision is provided on the portfolio optimization algorithms so as to make concrete basics in these. The main objective of this study is to get a clear cut idea on portfolio optimization algorithms. Since there are not many research articles that follow such a consolidated pattern, the objective of the study is deemed to be significant.

LITERATURE REVIEW

The paper itself is aimed at providing a theoretical study on the various algorithms used for portfolio optimization and this study is not based on any references. There are no many papers available which does a similar theoretical study that we have drawn here. Citations are provided to the papers which are used as references in this study.

Context and background of the work carried out: Since portfolio optimization is a highly recognized area that has immense potential in stock market forecasting, the work is highly focused and is of high significance. Stock market, being one of the most important financial investment sectors, provides us with a major motivation to prepare such a research review article. This can also be used as a reference material for the researchers interested in developing portfolio optimization systems.

Portfolio optimization: Portfolio optimization is the process by which we choose the fraction of portfolio assets in a particular manner such that the returns are maximized whereas the risk is minimized. Markowitz model is the traditional model which is followed for portfolio optimization (Mokhtar *et al*., 2014). The fundamental principle of this model is diversification with the aim of achieving a maximum level of expected

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return with the minimization of risks involved (Steinbach, 2001). The Solution for this model is using a quadratic program, considering the ideal case of linear constraints. But in real life condition, there are a lot of non-linear constraints like-the amount of weight age that can be applied on each assets included in the portfolio, limits on the number of assets included in the portfolio etc. With the addition of these constraints, the formulation becomes a non-linear mixed integer quadratic programming problem (Cesarone *et al*., 2011). This quadratic problem becomes difficult to solve, computationally i.e., NP-Hard. For the above reasons the problem can be solved using soft computational methods or techniques (Nair *et al*., 2013). Figure 1 shows the methods discussed in this study.

Portfolio optimization is usually used to produce a well-diversified portfolio which can be achieved by diversifying the investor's investment into different assets, which helps the investor from investing in only one asset which may be worse or come out as the best. The returns gained from a diversified portfolio will always be more than the investment which may perform the worst (Sullivan and Sheffrin, 2003).

The simplest example of diversification can be seen in Software Project Management, where people are assigned with different modules of a project rather than assigning a single person to do the entire project. Here if the single person fails, that could cost the entire project being stalled, whereas when diversification is used, failure of a single person can cost a single module being stalled which does not affect the entire project.

Markowitz model: Markowitz Model helps in assisting the selection of best portfolio from the various available portfolios that do not vary in the same direction out of the given securities by analyzing them. Thus it helps in risk reduction as it is based on the expected returns and risk of the various portfolios. Arithmetic Mean is used to calculate the expected return and Standard Deviation is used for calculating risk. Hence this model is also known as Mean-Variance Model (Markowitz, 1952). Markowitz Model is a quadratic programming problem, but with the addition of nonlinear constraints like transaction costs, the number of assets selected, limits on constraints like budget allocation and capital allocation, the quadratic program becomes more complex to achieve the solutions. Soft computing methods come in handy here to obtain optimal solutions.

GENETIC ALGORITHM

A Genetic Algorithm solves problems based on a natural selection process that works like a biological evolution (Samira, 2014). It can find solutions for both constrained and unconstrained optimization problems. The algorithm works by modifying a set of individual solutions from a population. The algorithm selects the individual solutions as parents, randomly, from the present population and generates the children for next generation. Parents and children are collected together to form the mating pool, from which the next generation of solution is obtained. For every generation processes like selection, crossover and mutation is repeated which helps in improving the results for the fitness of the population over generations. The population eventually evolves to an optimal solution, from the successive generations. It stops when a predefined terminating condition is met or when the fitness function has no further improvement in the fitness of the population over many generations (Yang, 2006). Figure 2 summarizes Genetic Algorithm.

Figure 3 shows the pictorial representation of the working of Genetic Algorithm.

Process of optimization by GA:

- Start with a random solution called population comprised of chromosomes, which are the parameters solved by GA
- Solution to the problem are described by chromosomes, having genes representing the weights given to the stocks

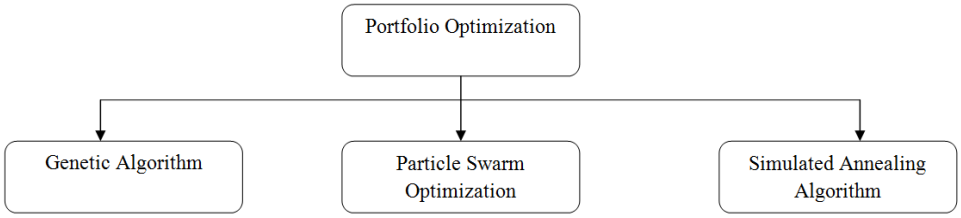

Fig. 1: Methods used for portfolio optimization

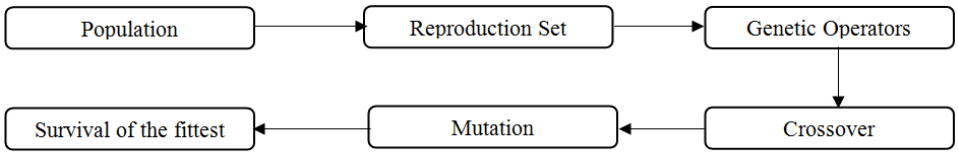

Fig. 2: Summarization of genetic algorithm

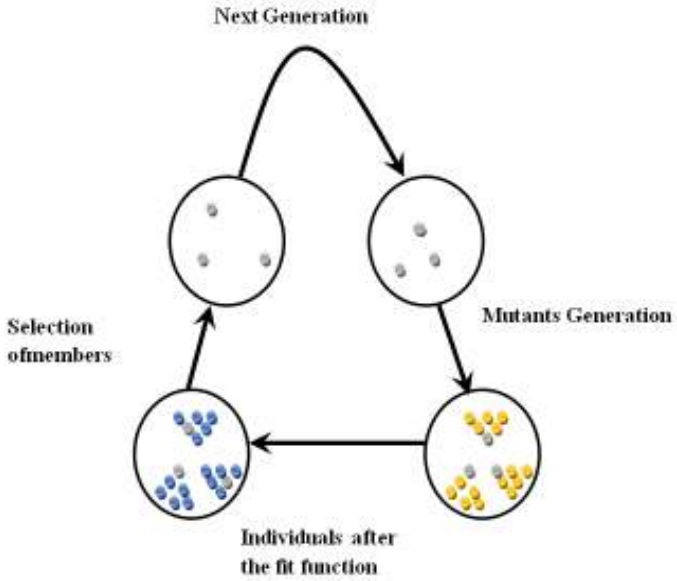

Fig. 3: Genetic Algorithm workflow; Image courtesy: http://nnpdf.hepforge.org/images/GA.png

- The iterative genetic evolution of chromosomes produces new population
- A fitness value is provided by each chromosome and this value of existing as well as evolving chromosomes are compared
- Fitness value is calculated using an objective function
- At the end of all iterations, chromosome with highest fitness value is selected
- Genetically suitable risk and return are provided by weight optimization represented by the selected chromosome (Sinha *et al*., 2013)

How GA in Markowitz: The variables in Markowitz model which can be optimized are: the upper limit and lower limits on the investment cost for each individual asset, the number of assets that can be included in the portfolio and the upper limit and lower limits of the budget that can be invested on the assets. By optimizing these variables we can expect to get a portfolio which will maximize the returns and minimize the risk.

Equation (1) gives the objective function for GA in portfolio optimization.

Maximization of:

$$f(chromosome) = \frac{Mean(returns)}{standard\ deviation(risk)} \quad (1)$$

PARTICLE SWARM OPTIMIZATION

PSO is one of the population based optimization technique and is one of the evolutionary algorithm which is based on social interaction and communication like bird flocking and fish schooling (Yourdkhani, 2014). In PSO method, we can easily find out the sub-optimal solution by global search which is the best when compared to other optimization algorithms like dynamic programming (Zhu *et al*., 2011). In Markowitz model, we use PSO algorithm for the optimization of selected good quality assets. PSO is exactly a simulation of behaviors of swarm like bird flocking. PSO is an evolution method which is a one-way sharing mechanism to find out the best solution (Jones, 2005). In this following scenario, consider a group of birds is randomly searching food in an area where only one piece of food is available and all the birds do not know where to search for the food. But, they can easily estimate how far the food is at each iteration. Thus, the problem here is to understand the best strategy to find the food. Here, the best effective solution is to follow the bird nearest to the food. PSO was invented after they learned this scenario and it became a very popular method for solving many optimization problems. PSO swarm consists of a population and each particle in particle in this population is considered as a bird and the search space is the area to explore (Cura, 2009). All the particles in PSO has a fitness values which is computed by using a fitness function and have velocities to direct or fly towards the optimum, food. Figure 4 shows the pictorial representation of PSO Algorithm (Dashti *et al*., 2007).

PSO for optimizing portfolio: The portfolio optimization method which uses the Markowitz model requires accurate forecasts of future expected returns and risk. This model which not only focus on the maximization of expected return but also on minimizing the risk. This theory becomes a reasonable

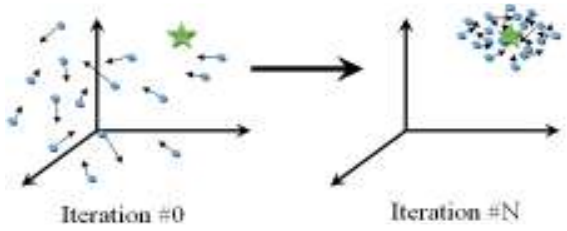

Fig. 4: PSO algorithm pictorial representation; Image courtesy: http://wirelesstech thoughts. blogspot. in/ 2013/06/ an-introduction-to-particle-swarm.html

trade-off between returns and risk involved (Kendall and Su, 2005). This optimization model can be solved by Quadratic Programming but PSO seems well-suited for this application because of its parallelism nature.

This is done in PSO by approaching to the best solution from all directions in the exploring space, so there is a great chance that the neighboring solutions will be invented by any particle (Wei and Qiqiang, 2004). In Portfolio Optimization, the objective function for PSO is the rational trade-off between maximizing return and minimizing risk. So, we maximize the Sharpe Ratio which is represented as a quantity of the amount of additional return compared with the risk involved. In this way, maximizing this objective function for each particle becomes the criteria for PSO to perform the selection. In portfolio, the total summation of weights is considered as a particle and every particle has a specific position in its exploring area. The position of each asset is determined by its own experience and its neighboring assets (Kuo and Hong, 2013).

Equation (2) shows the Objective function to maximize here is:

$$\max S_p = \frac{M(R_T)}{\sqrt{V(R_T)}} \quad (2)$$

where,

R_T : Returns

M (R_T) : Mean of Returns (Expected Returns)

V (R_T) : Variance of Returns (Risk Volatility)

Figure 5 shows the work flow of general PSO algorithm.

SIMULATED ANNEALING ALGORITHM

This is a metaheuristic technique which is based on probability and is inspired from the physical process of annealing the crystalline materials (Hwang, 1988). In this process, it involves heating a material at a high temperature and then it is slowly cooled to enhance its crystals size. After heating, the atoms' acquire lofty energy which makes them to change positions and can take large random movements in the material. Slowly, the cooling takes place which reduces the atoms' energy and their movement. This causes different transitory states which helps to attain good quality homogeneous materials. The Simulated Annealing algorithm is easily implemented in the portfolio optimization method by associating every point's random movements with a probability of a dependent variable which represents the temperature of the material (Bilel, 2013).

Equation (3) shows the form of SA:

$$Min\ F(x)\ s.t.\ x \to X; \quad (3)$$

The basic principle of SA heuristic is that it starts from a current solution x and the next solution y generates in some neighborhood of x. If this new

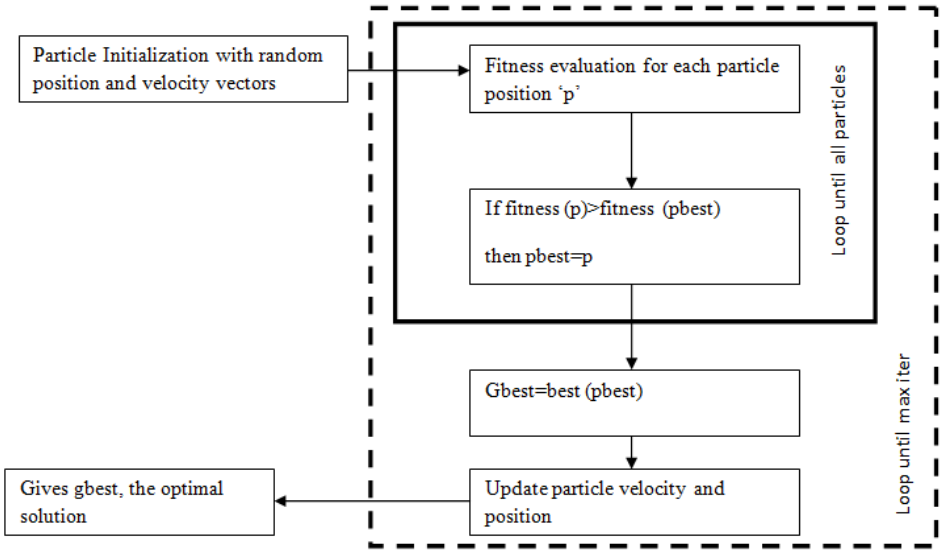

Fig. 5: General PSO algorithm: Workflow; Image courtesy: Dashti *et al*. (2007)

value y improves the objective function, then x gets replaced by y . Else, the new solution y will be accepted with a magnitude of probability which decreases its value after some iteration.

Figure 6 depicts the general structure of the simulated annealing algorithm.

Simulated annealing for optimizing portfolio: The existing Markowitz model used for portfolio optimization consists of many constraints like minimum trading quantities and maximum number of assets, this problem enters into a field of mixed integer nonlinear programming which becomes very difficult to find solutions. So, we apply SA heuristic algorithm to solve this model to find good quality solutions. Since a portfolio involves continuous and discrete variables, SA is the best method to use for portfolio selection.

There are two important approaches when a SA algorithm is applied to the portfolio selection problem. The first approach ensures that feasibility exists throughout all the iterations in SA algorithm and there will be no consideration of any solution that violates the constraints.

Thus making the neighborhood of the current solution fully with feasible solutions. The second approach is that as the violation of the constraint increases, the value of objective function also increases. So a portfolio which is not acceptable by an investor should be penalized enough to be rejected.

When penalties are considered, the value of penalty depends on the amount of violation of the constraint. Equation (4) shows the penalty expression:

$$a \times |violation|^p \quad (4)$$

where, a and p are scaling factors which has some selected values in our implementations. Here $a = {}^{v}/_{\epsilon^n}$ where v is the variance of the most risky asset. It is a measure of accuracy. As the variance of the portfolio lies in the interval range of $[0, v]$, the value of a assures that the portfolio attains a better value for the objective function which violates a constraint by ϵ or more. Here, only trading constraints are considered which shows that only few asset holdings are changed during the move (Crama and Schyns, 2003).

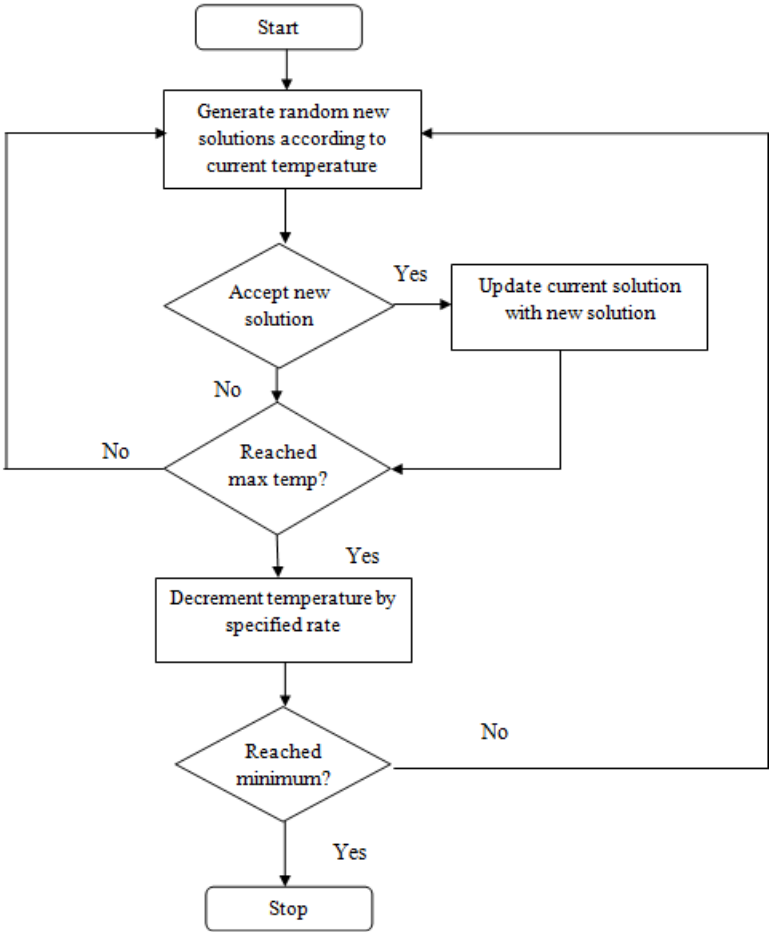

Fig. 6: The structure of simulated annealing algorithm; Image courtesy: http://www.heatonresearch.com/online/introduction-neural-networks- csedition-2/chapter-7/page2.htm

CONCLUSION

This study analyzed the concept of portfolio optimization and the major algorithms used in it. Genetic Algorithm optimizes five different variables so as to perform portfolio optimization. Particle Swarm Optimization implements parallelism as a strategy in which approach is made from all directions in to the exploration space so as to find optimized solutions. The presence of discrete and continuous variables make Simulated Annealing, a potential portfolio optimization algorithm. This algorithm implements optimization by ensuring feasibility and by providing a direct proportional relation between objective function and constraint violations. The study can be further enhanced as a book which pays detailed attention to the implementation aspects of these algorithms.

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