

Research Article

Kinematic and Dexterity Analysis of a 3-DOF Parallel Manipulator

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Abstract: A new three Degree Of Freedom (3-DOF) parallel manipulator has been proposed in this study. Because the parallel manipulator has three Degree Of Freedom (DOF), one translation degree of freedom and two rotational degrees of freedom, it has received considerable attention from both researchers and manufacturers over the past years. The inverse kinematic and Jacobian matrix were derived. The dexterity of the parallel manipulator is presented. The key issue of how the kinematic performance in term of dexterity varies with differences in the structural parameters of the parallel manipulator is investigated. The simulation results, using MATLAB, testify the validity of the analytic model and illustrate the structural parameters have direct effect upon dexterity characteristic of the 3-DOF parallel manipulator.

Keywords: Dexterity analysis, jacobian matrix, kinematic analysis, parallel manipulator

INTRODUCTION

Parallel manipulators have become popular in recent years due to their properties in terms of a high stiffness, high accuracy and high carrying payload over their serial counterparts. Parallel manipulators typically consist of a moving platform that is connected to a fixed base by several limbs in parallel (Liu and Kim, 2003).

Most of parallel manipulators are six Degrees Of Freedom (6-DOF) based on the Gough-Stewart platform manipulator due to the advantages mentioned above. However, 6-DOF parallel manipulator suffers from the complex structure, limited workspace and coupling problem of the position and orientation movement. To overcome these drawbacks, lower mobility parallel manipulator, whose degrees of freedom are less than 6-DOF, have been recently investigated. The reduction of the degrees of the freedom of the parallel manipulator from 6-DOF to 3-DOF have attracted much attention because of simple mechanical structure, large workspace, easies in control and low manufacturing cost. The 3-DOF parallel manipulators have been used as drilling, painting, pick-and-place and assembly. Many 3-DOF parallel manipulators have been designed for specific application such as the famous manipulator with three translations is the DELTA (Clavel, 1988; Liu and Kim,

2003; Takamori and Tsuchiya, 2012), the CaPaMan (Thomas *et al.*, 2005) and HANA parallel manipulators with three spatial DOF (Liu *et al.*, 2001). Liu *et al.* (2005) presented a 3-DOF parallel manipulator with high rotational capability thanks to all single-DOF joints that are involved in the rotational DOF. Liu and Cheng (2004), Dasgupta and Mruthyunjaya (2000), Davliakos and Papadopoulos (2008), Guo *et al.* (2008) and Merlet (2006b) proposed a methodology to analyze singularity positions of a 3-RPS parallel manipulator.

The condition number of the Jacobian matrix has been proposed as a measure of dexterity for parallel manipulators. Dexterity is defined as the capability of a manipulator to move as much as possible at a particular position and orientation as well as to measure kinematic accuracy (Xu *et al.*, 1994). Positioning accuracy should be consistent with the location that exhibits condition number indices. In Merlet (2006b), the consistency of different types of condition numbers was compared with the measured positioning accuracy of the manipulator through experiments. Dexterity is sensitive to the structural parameters of the parallel manipulator because this characteristic depends on the Jacobian matrix. The deviation in the center position of the manipulator joints is called kinematics parameters in parallel manipulator calibration and error-estimation modeling (Song *et al.*, 1999). The variation in kinematics parameters affects tracking accuracy. In

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Masory *et al.* (1993), the dexterity of the Stewart platform was determined by considering the effects of kinematics parameters, such as link lengths, joint locations and design dimensions, on dexterity. In Li and Xu (2006), the dexterity index with a space utility ratio for composing a mixed performance index and optimizing translational parallel manipulators was presented. The dexterity of a 6-DOF spatial parallel manipulator was compared quantitatively in Pond and Carretero (2007). In Merlet (2006b), manipulability, condition number and dexterity index, which were reviewed for application in parallel manipulators, were defined. In Wang *et al.* (2009, 2010), different approaches for computing the dexterity of manipulators were proposed. Gosselin and Angeles (1991) proposed the Global Condition Index based on the condition number to evaluate the dexterity of a manipulator at all points in the workspace. In this study, the kinematics analysis and effect of structural parameters on the performance kinematic of a 3-DOF parallel manipulator are studied.

MATERIALS AND METHODS

Mechanism description: A 3-DOF parallel manipulator is composed of a moving platform B, a fixed base A and three limbs connecting the moving platform and base. Two of limbs are RPU-type active limbs r1 and r3 with a linear actuator. Each of the RPU limb connects the moving platform B by universal joint U at b_i ($i = 1,3$) and revolute joint R at A_i ($i = 1,3$) are located in the base A. the third limb RPS connects the moving platform B by universal joint S at b_2 , a prismatic joint P along r_2 and revolute joint R at A_2 is located in the base A. Frame $p - uvw$ is located at the center of the moving platform at p and frame $O - XYZ$ is located at the center of the base. The Z and w axes are perpendicular to the platforms and the X and Y axes are parallel to the u and v axes respectively. For the base, the Y-axis is along OA2 and the radius of the base is R. θ_1 and θ_2 be angle between OA1 and X-axis, OA3 and X-axis, respectively. A CAD model of the 3-DOF parallel kinematic machine is shown in Fig. 1.

The mobility of the lower mobility parallel manipulator can be examined by using the modified Grubler-Kutzbach criterion (Guo *et al.*, 2008). The modified Grubler- Kutzbach criterion is given by:

$$M = d(n - g - 1) + \sum_{i=1}^g f_i + v - \varepsilon \quad (1)$$

In the 2RPU+RPS parallel manipulator, the number of links is $n = 8$, the number of joints $g = 9$, the sum of degrees of freedom of the joints are $f = 13$, a public constraint is 0 and the over constrained number is $v = 2$. The degrees of freedom of 2RPU+RPS parallel manipulator are $M = 3$.

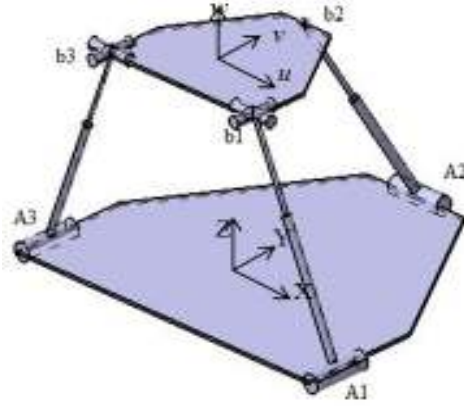


Fig. 1: CAD model of a 2RPU+RPS parallel manipulator

Position analysis of a 3-DOF parallel manipulator:

The unit vector representation of the orientation uses nine parameters which are dependent of each other. The rotational transformation matrix R_p^o is formed by rotation about YXZ axis, namely, rotation of γ about Z axis, followed by a rotation of β about Y-axis and then a rotation of α about X-axis. The unit vector P describes the position of the origin of the moving platform $\{B\}$ to the base $\{A\}$:

$$R_p^o = \begin{bmatrix} c_{\alpha i} c_{\gamma i} & s_{\alpha i} s_{\beta i} - c_{\alpha i} s_{\gamma i} c_{\beta i} & c_{\alpha i} s_{\gamma i} c_{\beta i} + s_{\alpha i} s_{\beta i} \\ s_{\gamma i} & c_{\gamma i} c_{\beta i} & -c_{\gamma i} s_{\beta i} \\ -s_{\alpha i} c_{\gamma i} & s_{\alpha i} s_{\gamma i} c_{\beta i} + c_{\alpha i} s_{\beta i} & -s_{\alpha i} s_{\gamma i} s_{\beta i} + c_{\alpha i} c_{\beta i} \end{bmatrix} = \begin{bmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_z & v_z & w_z \end{bmatrix}; P = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} \quad (2)$$

where, 'C' and 'S' are cosine and sine, respectively. The coordinate vector of joint A_i in moving platform $\{A\}$ with respect to centroid of the base platform O-xyz and the coordinate vector of joint b_i in the base $\{B\}$ with respect to centroid of the moving platform p-uvw, are expressed as:

$$A_1 = \begin{bmatrix} R c_{\theta_1} \\ R s_{\theta_1} \\ 0 \end{bmatrix}; A_2 = \begin{bmatrix} 0 \\ R \\ 0 \end{bmatrix}; A_3 = \begin{bmatrix} R c_{\theta_2} \\ R s_{\theta_2} \\ 0 \end{bmatrix} \\ b_3^B = \begin{bmatrix} r c_{\theta_1} \\ r s_{\theta_1} \\ 0 \end{bmatrix}; b_2^B = \begin{bmatrix} 0 \\ r \\ 0 \end{bmatrix}; b_3^B = \begin{bmatrix} r c_{\theta_2} \\ r s_{\theta_2} \\ 0 \end{bmatrix} \quad (3)$$

The position vector of the moving platform vertices b_i ($i = 1, 2, 3$) respect to centroid of the base platform can be expressed by a first closed loop vector equation encompasses both the moving platform and base of the manipulator:

$$\bar{q}_i = R_p^{oA} b_i^p + \bar{P} \quad (4)$$

Substituting Eq. (2) and (3) into Eq. (4), then the position kinematic equations of the moving platform vertices with respect to base are derived as:

$$\begin{aligned} \bar{q}_1 &= \begin{bmatrix} P_x + rc_{\theta_1}u_x + rs_{\theta_1}v_x \\ P_y + rc_{\theta_1}u_y + rs_{\theta_1}v_y \\ P_z + rc_{\theta_1}u_z + rs_{\theta_1}v_z \end{bmatrix} \bar{q}_2 = \begin{bmatrix} P_x + rv_x \\ P_y + rv_y \\ P_z + rv_z \end{bmatrix} \\ \bar{q}_3 &= \begin{bmatrix} P_x + rc_{\theta_2}u_x + rs_{\theta_2}v_x \\ P_y + rc_{\theta_2}u_y + rs_{\theta_2}v_y \\ P_z + rc_{\theta_2}u_z + rs_{\theta_2}v_z \end{bmatrix} \end{aligned} \quad (5)$$

The inverse kinematics of the manipulator can be solved by writing the following constraint equation:

$$\bar{r}_i = \bar{q}_i - \bar{OA}_i \quad (6)$$

where, \bar{r}_i , $i = (1, 2, 3)$ is a vector of the leg. Hence:

$$\begin{aligned} \bar{r}_1 &= \begin{bmatrix} P_x + rc_{\theta_1}u_x + rs_{\theta_1}v_x - Rc_{\theta_1} \\ P_y + rc_{\theta_1}u_y + rs_{\theta_1}v_y - Rs_{\theta_1} \\ P_z + rc_{\theta_1}u_z + rs_{\theta_1}v_z \end{bmatrix} \\ \bar{r}_2 &= \begin{bmatrix} P_x + rv_x \\ P_y + rv_y - R \\ P_z + rv_z \end{bmatrix} \\ \bar{r}_3 &= \begin{bmatrix} P_x + rc_{\theta_2}u_x + rs_{\theta_2}v_x - Rc_{\theta_2} \\ P_y + rc_{\theta_2}u_y + rs_{\theta_2}v_y - Rs_{\theta_2} \\ P_z + rc_{\theta_2}u_z + rs_{\theta_2}v_z \end{bmatrix} \end{aligned} \quad (7)$$

Based on the geometric constrain, the constraints equations are derived:

$$R_{12} \cdot R_{13} = 0, R_{32} \cdot R_{33} = 0, R_{11} \cdot r_1 = 0, R_{13} = R_{33} = u = R_{21} \cdot r_2 = 0 \text{ and } R_{31} \cdot r_3 = 0 \quad (8)$$

Lead to:

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} = 0 \quad (9)$$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} P_x + rv_x \\ P_y + rv_y - R \\ P_z + rv_z \end{bmatrix} = 0 \quad (10)$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} P_x + rc_{\theta_1}u_x + rs_{\theta_1}v_x - Rc_{\theta_1} \\ P_y + rc_{\theta_1}u_y + rs_{\theta_1}v_y - Rs_{\theta_1} \\ P_z + rc_{\theta_1}u_z + rs_{\theta_1}v_z \end{bmatrix} = 0 \quad (11)$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} P_x + rc_{\theta_2}u_x + rs_{\theta_2}v_x - Rc_{\theta_2} \\ P_y + rc_{\theta_2}u_y + rs_{\theta_2}v_y - Rs_{\theta_2} \\ P_z + rc_{\theta_2}u_z + rs_{\theta_2}v_z \end{bmatrix} = 0 \quad (12)$$

That means:

$$u_y = 0 \quad (13)$$

$$P_x + rv_x = 0 \quad (14)$$

$$(P_y + rc_{\theta_1}u_y + rs_{\theta_1}v_y - Rs_{\theta_1}) = 0 \quad (15)$$

$$(P_y + rc_{\theta_2}u_y + rs_{\theta_2}v_y - Rs_{\theta_2}) = 0 \quad (16)$$

Replacing the elements of the rotation matrix with corresponding expressions in Eq. (13), (14), (15) and (16), lead to:

$$\gamma = 0 \quad (17)$$

$$P_x = -rs_{\alpha i}s_{\beta i} \quad (18)$$

$$P_y = -\frac{1}{2}(rc_{\beta i} - R)(s_{\theta_1} + s_{\theta_2}) \quad (19)$$

Note the motion along X and Y axes are limited motion and expressed as the function of variables ($r, R, \theta_1, \theta_2, \alpha$ and β). These motions called parasitic motion and occur as a result of rotation the moving platform around u and v , i.e., α and β , respectively. Hence, the parallel manipulator has 3-DOF as 2-DOF rotations α and β around u and v axes, respectively and 1-DOF translation on Z axis.

For inverse kinematics solution, the three independent variables $X = [\alpha\beta]^T$ are given to find the actuated leg lengths, $l = [l_1l_2l_3]^T$. The leg lengths can be determined by dot multiplying l_i Eq. (6) with itself to yield:

$$l_i = \|r_i\| \quad (i = 1, 2, 3) \quad (20)$$

From Eq. (20), the leg lengths l_i ($i = 1, 2, 3$) are derived as:

$$\begin{aligned} l_1^2 &= (P_x + rc_{\theta_1}(c_{\alpha i}c_{\gamma i} + s_{\beta i}s_{\alpha i}s_{\gamma i}) + rs_{\theta_1} \\ &(-c_{\alpha i}s_{\gamma i} + s_{\beta i}s_{\alpha i}c_{\gamma i}) + c_{\theta_2}R + P_y + \\ &rc_{\theta_1}c_{\beta i}s_{\gamma i} + rs_{\theta_1}c_{\beta i}c_{\gamma i} + c_{\theta_2}R + P_z + \\ &rc_{\theta_1}(-s_{\alpha i}c_{\gamma i} + c_{\beta i}c_{\alpha i}s_{\gamma i}) + rs_{\theta_1}(s_{\alpha i}s_{\gamma i} + \\ &s_{\beta i}c_{\alpha i}c_{\gamma i})) \cdot (P_x + rc_{\theta_1}(c_{\alpha i}c_{\gamma i} + s_{\beta i}s_{\alpha i}s_{\gamma i}) + \\ &rs_{\theta_1}(-c_{\alpha i}s_{\gamma i} + s_{\beta i}s_{\alpha i}c_{\gamma i}) + s_{\theta_2}R + P_y + \\ &rc_{\theta_1}c_{\beta i}s_{\gamma i} + rs_{\theta_1}c_{\beta i}c_{\gamma i} + s_{\theta_2}R + P_z + \\ &rc_{\theta_1}(-s_{\alpha i}c_{\gamma i} + c_{\beta i}c_{\alpha i}s_{\gamma i}) + rs_{\theta_1}(s_{\alpha i}s_{\gamma i} + \\ &s_{\beta i}c_{\alpha i}c_{\gamma i})) \end{aligned} \quad (21)$$

$$\begin{aligned} l_2^2 &= P_x^2 + P_y^2 + P_z^2 + 2rP_x(c_{\alpha i}c_{\gamma i} + s_{\beta i}s_{\alpha i}s_{\gamma i}) + \\ &2rP_yc_{\beta i}s_{\gamma i} + 2rP_z(-s_{\alpha i}c_{\gamma i} + c_{\beta i}c_{\alpha i}s_{\gamma i}) - \\ &2RP_x - 2R(c_{\alpha i}c_{\gamma i} + s_{\beta i}s_{\alpha i}s_{\gamma i}) + r^2 + R^2 \end{aligned} \quad (22)$$

$$\begin{aligned} l_3^2 &= (P_x + rc_{\theta_1}(c_{\alpha i}c_{\gamma i} + s_{\beta i}s_{\alpha i}s_{\gamma i}) \\ &+ rs_{\theta_1}(-c_{\alpha i}s_{\gamma i} + s_{\beta i}s_{\alpha i}c_{\gamma i}) + c_{\theta_1}R + P_y + \\ &rc_{\theta_1}c_{\beta i}s_{\gamma i} + rs_{\theta_1}c_{\beta i}c_{\gamma i} + c_{\theta_1}R + P_z + \\ &rc_{\theta_1}(-s_{\alpha i}c_{\gamma i} + c_{\beta i}c_{\alpha i}s_{\gamma i}) + rs_{\theta_1}(s_{\alpha i}s_{\gamma i} + \end{aligned}$$

$$s\beta_i c\alpha_i c\gamma_i)). (P_x + rc_{\theta_1} (c\alpha_i c\gamma_i + s\beta_i s\alpha_i s\gamma_i) + rs_{\theta_1} (-c\alpha_i s\gamma_i + s\beta_i s\alpha_i c\gamma_i) + s_{\theta_1} R + P_y + rc_{\theta_1} c\beta_i s\gamma_i + rs_{\theta_1} c\beta_i c\gamma_i + s_{\theta_1} R + P_z + rc_{\theta_1} (-s\alpha_i c\gamma_i + c\beta_i c\alpha_i s\gamma_i) + rs_{\theta_1} (s\alpha_i s\gamma_i + s\beta_i c\alpha_i c\gamma_i)) \quad (23)$$

Hence, the length of the actuated legs can be determined by the square root of the Eq. (21), (22) and (23). The two possible solutions exist. Only positive values of l_i are taken further because a negative solution is not physically possible. A positive values means that the actuator will result in the inward leaning of the constant length leg l_i .

Mechanism Jacobian: The Jacobian matrix defined a transformation from the velocity of the moving platform in Cartesian space to the actuated joint velocities of the parallel manipulator (Joshi and Tsai, 2002). The relation between the joint motion q and the position of the moving platform, X , of the parallel manipulator is:

$$f(q, X) = 0 \quad (24)$$

where, f is an n dimensional implicit function of X and q . Differentiating Eq. (24) with respect to time results in the following relationship:

$$J_X \dot{X} = J_q \dot{q} \dot{q} = J_q^{-1} J_X \dot{X} \quad (25)$$

where, J_X is an $n \times n$ Jacobian matrix ($J_X = \frac{\partial f}{\partial X}$) and J_q is an $n \times m$ Jacobian matrix ($J_q = \frac{\partial f}{\partial q}$). Both Jacobian matrices are depended on configuration of the parallel manipulator. The conventional Jacobian formulation is based on the closed loop vectorial relationship. The closed loop vectorial relationship defines the plan of movement but does not involve the type of the joints. The 3-DOF parallel manipulator has a motion in Cartesian space and has three legs as given:

$$\dot{X} = [z\dot{\alpha}\dot{\beta}]^T ; \dot{L} = [l_1 \dot{l}_2 \dot{l}_3]^T \quad (26)$$

The vector loop system gives the following sum.

$$\overline{Op} + \overline{pb}_i = \overline{OA}_i + \overline{A}_i \overline{B}_i \quad (27)$$

Differentiating Eq. (27) with respect to time yields:

$$v_p + w_A \times b_i = lw_i \times s_i + l_i s_i \quad (28)$$

where, b_i and s_i denote the vector \overline{Pb}_i and a unit vector along $\overline{A}_i \overline{b}_i$, respectively. w_i denotes the angular velocity of the i^{th} leg. Now, by dot multiplying both side of Eq. (28) by s_i^T .

$$s_i^T \cdot v_p + (a_i \times s_i^T) \cdot w_A = \dot{l}_i \quad (29)$$

Equation (29) can be written three times of the three legs from which the two from Jacobian can be deduced by using relationship given:

$$J_x = \begin{bmatrix} s_1^T & (a_1 \times s_1)^T \\ s_2^T & (a_2 \times s_2)^T \\ s_3^T & (a_3 \times s_3)^T \end{bmatrix}_{3 \times 6}$$

and $J_q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3 \times 3}$ (30)

where, unit vector, s_i can be defined using the equation:

$$s_i |l_i| = q_i - A_i \quad (31)$$

Dexterity analysis of a parallel manipulator:

Dexterity is an important issue for design, trajectory planning and control of parallel manipulator. Dexterity is defined as the ability of the manipulator to make accurate movement and is a measure of position accuracy of the moving platform of the parallel manipulator. The position accuracy means that if the manipulator is command to stop at a particular point in the workspace, then the deviation from the desired stopping point will give the position accuracy of the manipulator. The dexterity of the manipulator is dependent on the condition index of the Jacobian matrix (Merlet, 2006a). The condition index of the manipulator is a measuring the amplification of the error in order to the kinematic transformation between the Cartesian space and joint space. The error amplification factor, called the condition index CI, defined as:

$$CI = \|J^{-1}\| \|J\| \quad (32)$$

where, $\|\cdot\|$ denotes to the norm. The norm of the Jacobian matrix and is defined as follows:

$$\|J\| = \sqrt{\text{tr}(\frac{1}{n} J J^T)} \quad (33)$$

where, n is the dimension of the square matrix J . When the condition index approaches to a value of 1, a matrix is said to be well-conditioned and the dexterity reaches to maximum. On the other hand, when condition index is equal to positive infinity, the matrix is said to be ill-conditioned. Dexterity is very sensitive to structural parameters because of Jacobain matrix dependence on them (Rao *et al.*, 2003).

There are different types of norms which may constitute in the definition of the condition index. Some norms or condition index definitions are given below inorder to study the performance check within the manipulators workspace.

- **Norm 2:** Norm2 of the matrix defined as the square root of the ratio of the largest (λ_{max}) to the smallest eigenvalues (λ_{min}) of the Jacobian matrix. The lengths of the maximum and minimum eigenvectors are considered as an image of the maximum and minimum error amplification factor. Condition number based on norm-2 is denoted by $C2$ (Lopes *et al.*, 2012):

$$C2 = \sqrt{\frac{\lambda_{max}}{\lambda_{min}}} \quad (34)$$

- **Determinant (C_{det}):** Yoshikawa (1985) defined another measure of the manipulator performance called kinematic manipulability. $J^T J$ and $J J^T$ form square Jacobian from the original rectangular Jacobian J . The squared matrix can be inverted and determinants can be taken easily in the space. The manipulability index is calculated (Rao *et al.*, 2003) using this formula:

$$C_{det} = \sqrt{\det(JJ^T)} \quad (35)$$

Equation (36) shows another measure of manipulability or dexterity indices as compared to the more frequently used condition number based on norm-2.

- **Frobenius norm:** The Frobenius norm condition number (Golub and Van Loan, 2013; Horn and Johnson, 2012), sometimes also called the Euclidean norm is defined as the square root of the sum of the absolute squares of its elements. Condition number based on Frobenius norm is denoted by C_{fro} :

$$\|J\|_{fro} = \sqrt{\sum_{i=1}^m \sum_{j=1}^n |a_{ij}|^2} \quad (36)$$

- **Norm 1:** Norm 1 is defined as the largest absolute column sum for a square matrix A:

$$\|A\|_1 = \max(\|col_1 A\|_1, \|col_2 A\|_1, \dots, \|col_n A\|_1) \quad (37)$$

where, A is JJ^T . Condition number based on norm-1 is denoted by $C1$.

- **Norm infinity:** Norm infinity is defined as the largest absolute row sum for a square matrix A:

$$\|A\|_\infty = \max(\|row_1 A\|_1, \|row_2 A\|_1, \dots, \|row_n A\|_1) \quad (38)$$

where, A is JJ^T . Condition number based on norm infinity is denoted by C_{inf} .

All types of condition numbers mentioned above can be compared for reliability and consistency to find the range of condition numbers in the evaluation of dexterity. The condition index depend on the Jacobian matrix are just local indices for a manipulator and global performance indices have been proposed to evaluate kinematic performance over the work space. Taking evaluation of a manipulator's dexterity in the workspace, for example, many scholars have applied the statistical method to dexterity performance evaluation. A Global Dexterity Index (GDI) can be used to evaluate a manipulator throughout its own workspace and so it can be used for the optimal design of a parallel manipulator. A global dexterity index was defined by Gosselin and Angeles (1991) and Kucuk and Bingul (2006) as:

$$GDI = \frac{\int (\frac{1}{CI}) dV}{V} \quad (39)$$

where, V is the total workspace volume. The GDI represents the uniformity of dexterity over entire workspace other than the dexterity at a certain configuration and can give a measure of kinematic performance independent of the different workspace volumes of the design candidates since it is normalized by the workspace size.

RESULTS AND DISCUSSION

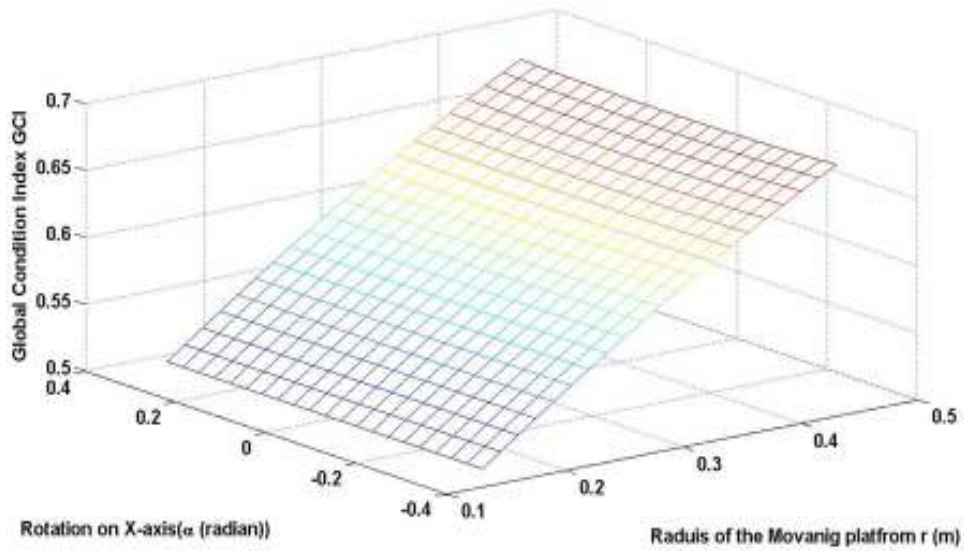
A program was written by MATLAB to evaluate dexterity. Table 1 shows the comparison of dexterity using different definitions of the condition index corresponding to several configurations. The condition index norm2 provides good index to identify structure singularities.

Figure 2 shows the results obtained for the simultaneous variations of the radius of the moving platform and rotation of the moving platform on the u and v axes with dexterity of the 3-DOF parallel manipulator. The parameters of design, such as size ratio of upper to lower platform radius ($R/r = 3/2$), height of the moving platform ($h = 0.15$ m) and joint location angle ($\theta_1 = -30, \theta_2 = 210$), are kept fixed. The radius of the moving platform (r) is increased from 0.15 m to 0.45 m. The global dexterity index increases with increased in the radius of the moving platform, while it has a little changes with varied rotation of the moving platform, α and β . The global dexterity index is maximal when the moving platform lies along the Z-axis and decreases when the manipulator approaching to the boundary workspace.

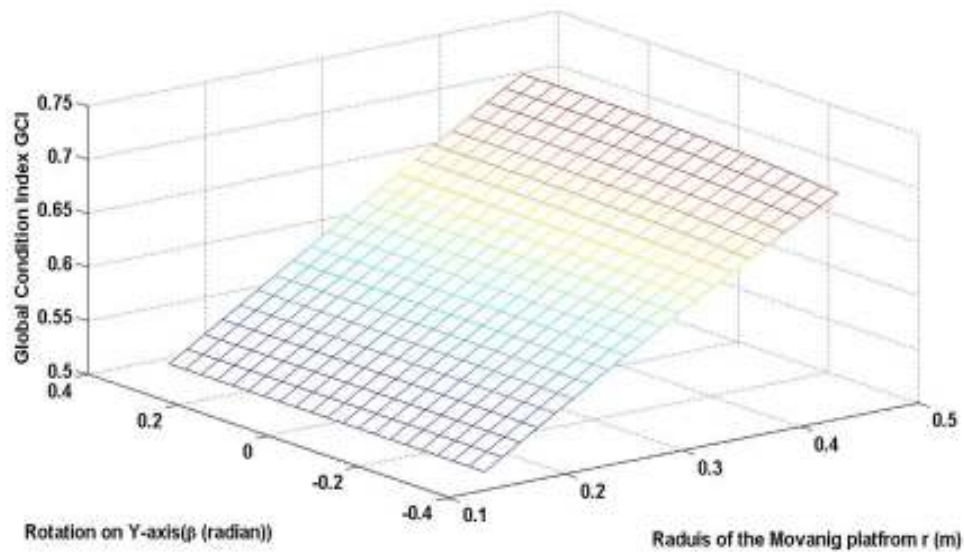
In Fig. 3, the platform size ratio (R/r) is varied for a selected configuration with dexterity of the manipulator. The manipulator height, joint location

Table 1: Comparison of dexterity using different definition of condition index

Configuration	Height	A	B	C_2	C_{det}	C_{fro}	C_1	C_{inf}
1	0.0500	0.0000	0.0000	1.7102	2.3097	1.8158	2.5588	2.0126
2	0.1000	0.0000	0.0000	1.7121	2.3113	1.8176	2.6020	1.9905
3	0.1500	0.0000	0.0000	1.7138	2.3127	1.8191	2.6395	1.9741
4	0.2500	0.0000	0.0000	1.7167	2.3146	1.8217	2.7009	1.9457
5	0.1500	0.0000	5.0000	1.7133	2.3152	1.8194	2.6473	1.9669
6	0.1500	0.0000	10.0000	1.7122	2.3184	1.8196	2.6520	1.9629
7	0.1500	0.0000	15.0000	1.7106	2.3220	1.8195	2.6540	1.9620
8	0.1500	0.0000	20.0000	1.7087	2.3260	1.8194	2.6533	1.9638
9	0.1500	5.0000	0.0000	1.7145	2.3122	1.8195	2.6472	1.9740
10	0.1500	10.0000	0.0000	1.7148	2.3123	1.8198	2.6563	1.9735
11	0.1500	15.0000	0.0000	1.7150	2.3131	1.8202	2.6668	1.9727
12	0.1500	20.0000	0.0000	1.7151	2.3144	1.8207	2.6785	1.9714
13	0.1500	10.0000	10.0000	1.7145	2.3145	1.8202	2.6650	1.9639
14	0.1500	10.0000	10.0000	1.7125	2.3094	1.8175	2.5991	2.0018
15	0.1500	20.0000	20.0000	1.7094	2.3066	1.8151	2.5430	2.0425
16	0.1500	20.0000	20.0000	1.7149	2.3149	1.8206	2.6778	1.9689

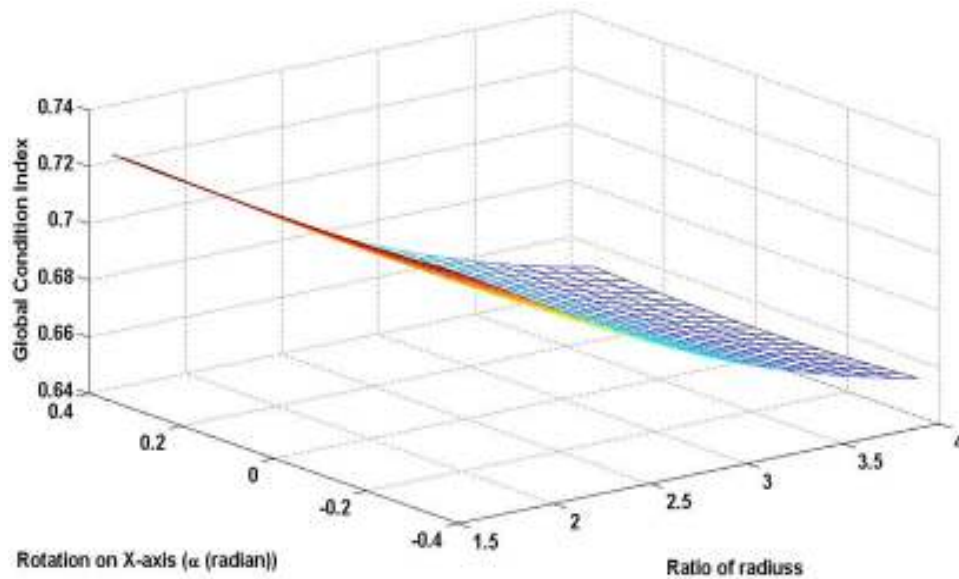


(a)

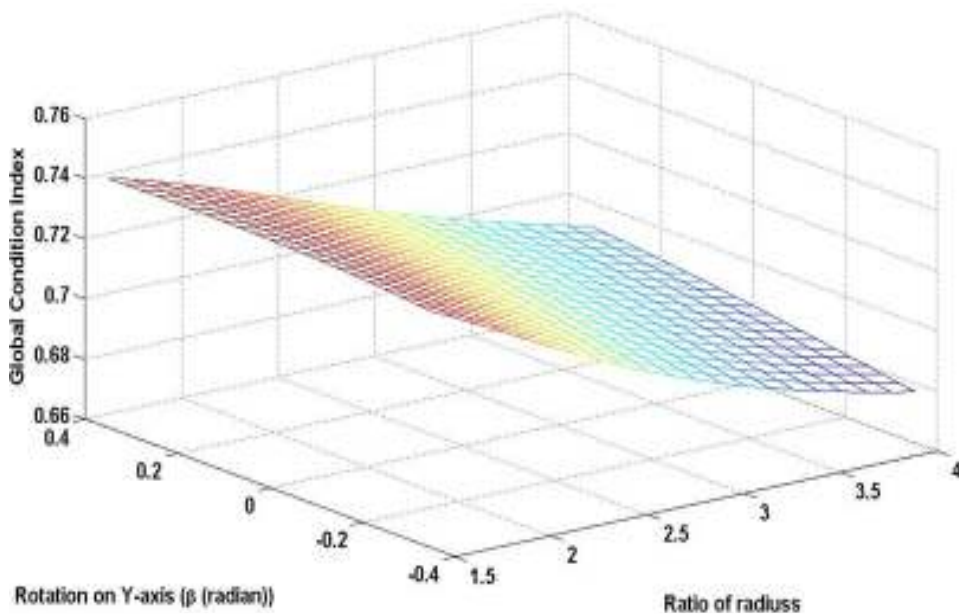


(b)

Fig. 2: Effect radius of the moving platform on the dexterity of the 3-DOF parallel manipulator a) along u -axis; b) along v -axis



(a)



(b)

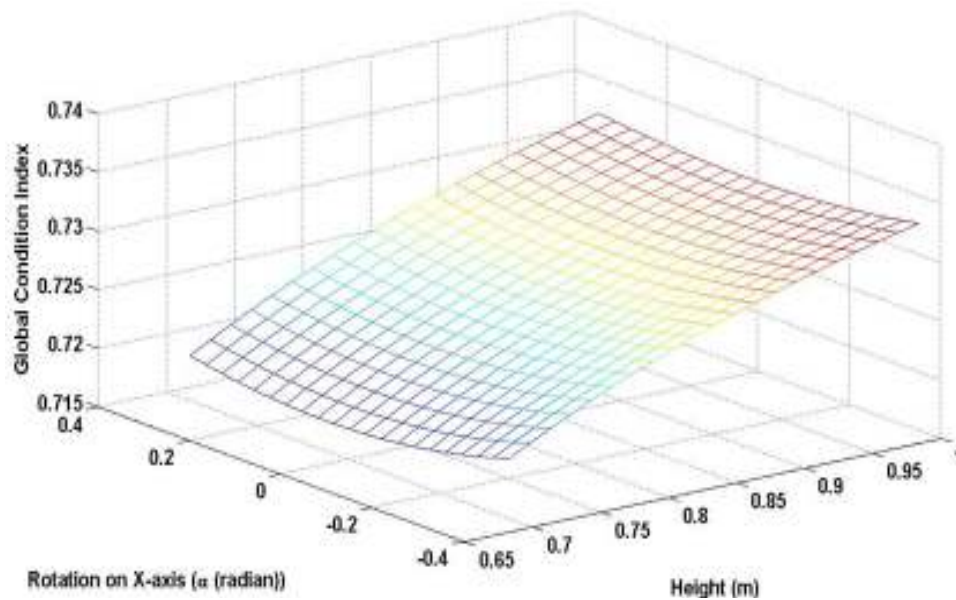
Fig. 3: Effect platform size ratio (R/r) on the dexterity of the 3-DOF parallel manipulator a) along u -axis; b) along v -axis

angle ($\theta_1 = -30, \theta_2 = 180$) and radius of the moving platform are kept fixed ($r = 0.35$ m). The varying the ratio (R/r) is varied from 1.5 to 3. The global dexterity index size decreases with increased in the ratio of radius (R/r) of the manipulator. The variation global dexterity index size is all symmetrical about plane α and β equal zero.

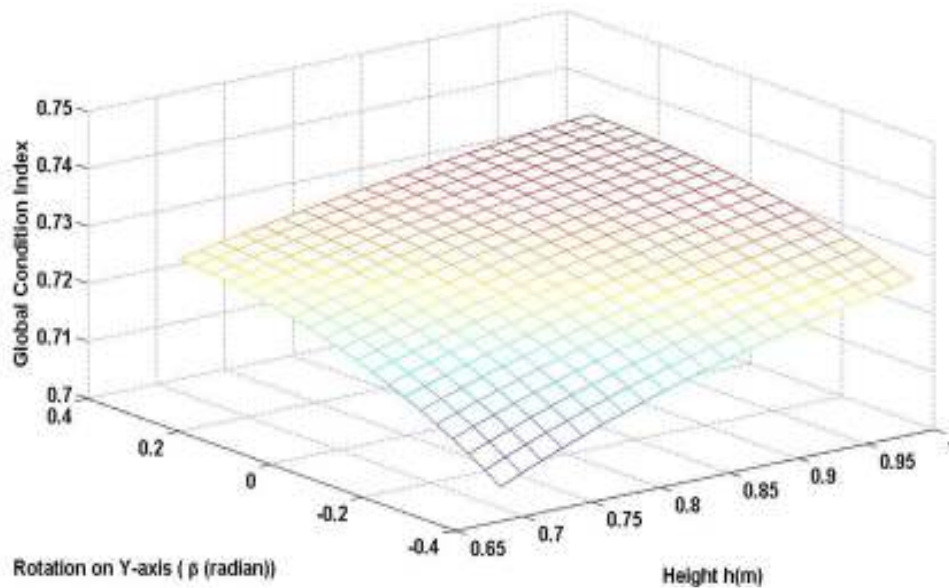
The global dexterity index of the parallel manipulator is varied along orientation of the moving platform, α and β , at different heights with fixed the other design parameters ($r = 0.35$ m, $R/r=3/2, \theta_1 =$

$-30, \theta_2 = 180$) as shown in Fig. 4. The increasing of height of the parallel manipulator, the global dexterity index increases gradually. The variation of the global dexterity index is little symmetrical with rotation on u -axis, while it increases with increased the rotation on v -axis.

The effect joint location is considered by changing locations of the joints on the moving platform as shown in Fig. 5. Design parameters, height, radius of the moving platform and platform size ratio, are fixed. The global dexterity index increases when the joint angle θ



(a)



(b)

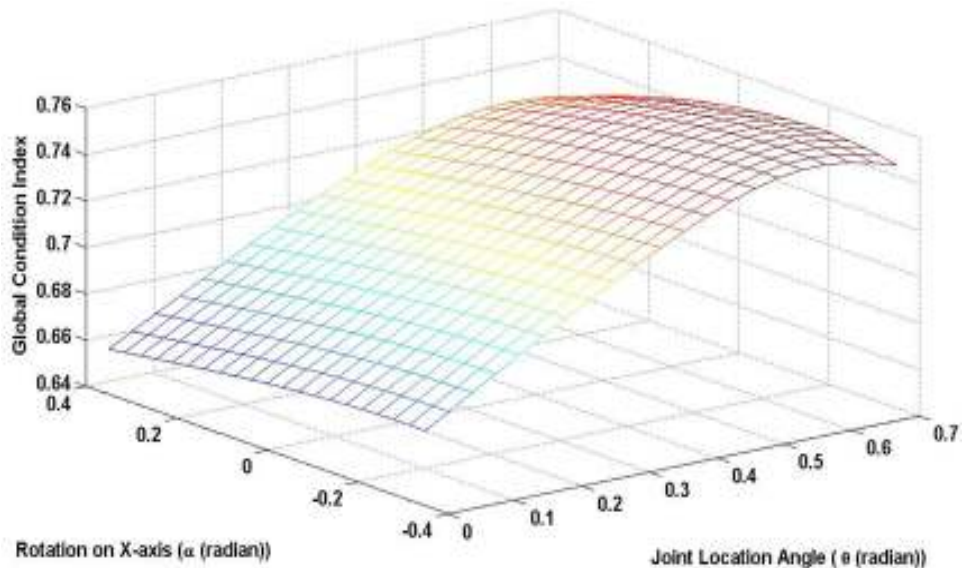
Fig. 4: Effect height on the dexterity of the 3-DOF parallel manipulator a) along u -axis; b) along v -axis

increased and then decreased after 40o, while it has a little change with varied rotation angles on u and v axes.

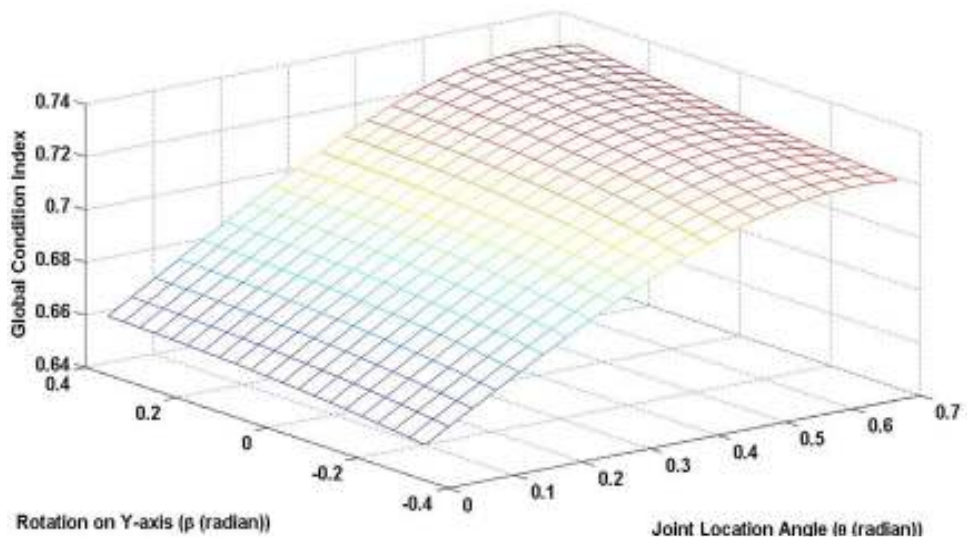
CONCLUSION

A new constraint 2RPU+RPS parallel manipulator with 3-DOF corresponding to one translation on z -axis and 2DOF rotation represented by α and β , rotation on x -axis and y -axis, respectively was discussed. The inverse kinematic and Jacobian matrix of the parallel manipulator were derived. The condition number based

on norm-2 was selected because it provides an index that is suitable for identifying the structure singularity. The global condition index is basically a weighted workspace wherein each elemental volume is weighted by the inverse of the condition number of the manipulator. Thus, the global condition index provides information about the quality of the manipulator workspace. The influence of the structural parameters of the parallel manipulator, such as the radius of the moving platform and base, height and joint location of the joints on the dexterity of manipulator were examined. The dexterity analysis of 3-DOF parallel



(a)



(b)

Fig. 5: Effect joints location on the dexterity of the 3-DOF parallel manipulator a) along u -axis; b) along v -axis

mechanism has significant value to its overall design. It not only helps to study the performance of parallel mechanism, but also offers theoretical basis in selecting the size of the components. The dexterity analysis provides necessary preconditions for the design and manufacture of its entity.

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