Research Article
Comparing Vector Autoregressive (VAR) Estimation with Combine White Noise (CWN) Estimation

[1,2]Ayodele Abraham Agboluaje, [1]Suzilah bt Ismail and [3]Chee Yin Yip
[1]School of Quantitative Sciences, College of Arts and Sciences, Universiti Utara Malaysia,
[2]Department of Mathematics and Computer Science, Faculty of Natural Sciences, Ibrahim Badamasi Babangida University, Lapai, Nigeria
[3]Department of Economics, Faculty of Business and Finance, Universiti Tuanku Abdul Rahman, Malaysia

Abstract: The purpose of this study is to compare one of the existing models, which is VAR model with the new Combine White Noise model. The VAR models have not been able to model the conditional heteroscedasticity and the leverage effect exhibited by the data. Likewise, GARCH family models cannot model leverage effect. The Combine White Noise (CWN) has proved more efficient and takes care of these weaknesses. CWN has the minimum information criteria and high log likelihood when compare with VAR estimation. The determinant of the residual covariance matrix value indicates that CWN estimation is efficient. It passes the Levene's test of equal variances. CWN has a minimum forecast errors which indicates forecast accuracy. All its outcomes outperform all the outcomes of VAR widely.

Keywords: Determinant residual covariance, EGARCH, error term, leverage, log likelihood, minimum forecast errors

INTRODUCTION

An error term is established when a stochastic model is not describing accurately, the correlation between dissimilar types of variables. The researchers have been establishing several models at different time to combat the unobservable variable (error term). The fact is that, the data time period (data size) with high data frequency have been making the data to exhibit different behaviors of errors when modeling, as the models demonstrate in the subsequent paragraphs (Sims, 1980; Engle, 1982; Bollerslev, 1986; Qin and Gilbert, 2001; McAleer, 2014; Al-Hagyan et al., 2015). Regard these challenges; Combine white Noise model is put in place to resolve the challenges.

Sims (1980) advocates the use of Vector Autoregressive (VAR) models as alternatives to econometric simultaneous equations modeling. A univariate autoregressive is a single equation single-variable linear model in which the current value of a variable is clarified by its lagged values. VAR is a linear model function of present and previous values of variables. VAR model offers an efficient way to confine the rich dynamics in multiple time series. VAR is easy to use and interpret. VAR provides a consistent and credible method to data description, forecasting, structural inference and policy analysis.

Cooley and LeRoy (1985) assume that if the interpretations of VAR models are non structural and are equivalent versions of the same model, which are invariant across observations. The interpretations of different causal have observationally equivalent versions of a given model. This invariance property has significant applications of VAR models. The invariant across observationally equivalent versions of a given model will be construction of reduced form VAR model for forecast. The same is true of Granger causality. The methods of atheoretical macro econometrics use for causality tests outcomes have the theoretical implications. Theorists will regard the outcomes of determination of the existence of Granger-causal orderings using VAR models. Thus, require subsequent enlightenment in terms of structural models. There is no objection to the principle, but to know whether the hypothesis is likely to be fruitful is an open question. Atheoretical macro econometrics has been credited for its use in analyzing causal orderings and policy interventions. The criticism depends on whether VAR models are interpreted as structural or non structural. If the models are structural in nature and interpreted as non-structural, the conclusions are not supported.

Corresponding Author: Ayodele Abraham Agboluaje, School of Quantitative Sciences, College of Arts and Sciences, Universiti Utara Malaysia, Malaysia

Blanchard and Quah (1993) assume that the unemployment and output dynamics provide two types of shocks, the effect of first type of shock on output is permanent, the second shock effect is temporary and the two shocks being interpreted as supply and demand shocks. Lippi and Reichlin (1993) argue that Blanchard and Quah's econometric work may be on the "wrong" side of the unit circle which leads to a moving average representation equal zeros. They advocate for an alternative moving average representations which is equivalent to a given estimated VAR.

Günnemann *et al*. (2014) present Robust Latent Autoregression (RLA) model to discover the users' base rating behavior and anomalies in rating distributions. They argue that the RLA results indicate that the highest error is shown in non robust VAR and Kalman Filtering. Since the unknown structure of the data cannot be identified, for their error increases rapidly for a high number of anomalies. RLA does better than the robust VAR method and RLA is more robust to the anomalies. When predicting the future rating distribution, any method with a high number of anomalies is more challenging. Since the non robust VAR is having the highest error, this indicates the weakness in VAR error terms. Equally, VAR cannot model the data that exhibit heteroscedasticity.

Forecasting models have serious challenges in terms of heteroskedastic errors. Engle (1982) proposes Autoregressive Conditional Heteroscedasticity (ARCH) model because of time varying volatility. The equations are non normal and in connection with change in stock market distribution and fat tail measuring effect and this effect is named ARCH. ARCH models are able to grip group errors and can withstand any changes made by economic forecaster. But ARCH cannot handle the abnormalities like crashes, mergers, news effect or threshold effects in the financial and economic sector. Bollerslev (1986) comes up with generalized ARCH to capture the volatility persistent that is flexible to the uplift of the weakness of ARCH model.

Vivian and Wohar (2012) and Ewing and Malik (2013) investigate that there are excess kurtosis and volatility persistence in GARCH.

Threshold GARCH and exponential GARCH capture the asymmetric effects of positive and negative shocks of the same dimension on conditional volatility in various ways (Nelson, 1991; Hentschel, 1995; McAleer, 2014; McAleer and Hafner, 2014; Al-Hagyan *et al*., 2015; Farnoosh *et al*., 2015). GARCH modelling the leverage is not possible because any restriction imposed will be positivity restriction which has no leverage effect (McAleer, 2014; McAleer and Hafner, 2014).

When the data size is becoming large and larger with high frequency data, the above models cannot have efficient and accurate results because of the exhibition of error terms that are unobservable in the

stochastic volatility time series (McAleer, 2014; McAleer and Hafner, 2014). These facilitate the new approach of Combine White Noise model to model the error terms for better estimation and to yield a better result.

In econometric estimation, an essential assumption is that the error term of the series should have equal variance (white noise) (Cuthbertson *et al*., 1992; Harvey, 1993), which is violated by heteroskedastic variances. Since heteroskedastic variances occur in error series which are unequal variances, then the error series are divided into subseries of equal variances (white noise series) to produce a Combine White Noise.

MATERIALS AND METHODS

The data is retrieved from the DataStream of Universiti Utara Malaysia library and is U.S. Real Gross Domestic Product (RGDP) quarterly data from 1960 to 2014 being applied in this study. The data indicates a behavior of non-stationary of an upward trend. Consider the autoregression model:

$$y_t = \phi y_{t-1} + \varepsilon_t, \tag{1}$$

Permit the stochastic approach of a real-valued time to be ε_t and the complete information through t time is I. The GARCH model is:

$$\varepsilon_t \mid I_{t-1} \sim N(0, h_t), \tag{2}$$

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i}$$
$$= \omega + A(L)\varepsilon_t^2 + B(L)h_t \tag{3}$$

The EGARCH specification is:

$$\log h_t = \alpha + \beta \mid z_{t-1} \mid + \delta z_{t-1} + \gamma \log h_{t-1}, \quad \mid \gamma \mid < 1 \tag{4}$$

where, $z_t = \varepsilon_t / \sqrt{h_t}$ is the standardized shocks, $z_t \sim iid$ (0, α). $|\gamma| < 1$ is when there is stability. The impact is asymmetric if $\delta \neq 0$ although, there is existence of leverage if $\delta < 0$ and $\delta < \beta < -\delta$ While both β and δ must be positive which the variances of two stochastic processes are, then, modeling leverage effect is not possible (McAleer, 2014; McAleer and Hafner, 2014).

The unequal variances (heteroscedastic errors) behaviors in the process of estimation being exhibited by GARCH models can be simplified into Combine White Noise models. The standardized residuals of GARCH errors which are unequal variances are decomposed into equal variances (white noise) in series to deal with the heteroscedasticity. The regression model

is employed to transform each equal variances series to model.

Moving average process is employed for the estimation of these white noise series which is called Combine White Noise:

$$Y_1 = \varepsilon_{1t} + \theta_{11}\varepsilon_{1,t-1} + \theta_{12}\varepsilon_{1,t-2} + \ldots \theta_{jq}\varepsilon_{j,t-q}$$

$$Y_2 = \varepsilon_{2t} + \Phi_{21}\varepsilon_{2,t-1} + \Phi_{22}\varepsilon_{2,t-2} + \ldots \Phi_{jq}\varepsilon_{j,t-q}$$

$$\vdots \ . \ Yj = \varepsilon_{jt} + \phi_{j1}\varepsilon_{j,t-1} + \phi_{j2}\varepsilon_{j,t-2} + \ldots \phi_{jq}\varepsilon_{j,t-q}$$

$$Y_{jt} = \sum_{j=1}^{q}\theta_j\varepsilon_{j,t-q} + \sum_{j=1}^{q}\Phi_j\varepsilon_{j,t-q} + \ldots \sum_{j=1}^{q}\phi_j\varepsilon_{j,t-q} \tag{5}$$

$$= A(L)\varepsilon_t + B(L)\varepsilon_t + \ldots = \varepsilon_t[A(L) + B(L) + \ldots] \tag{6}$$

$$= Q\varepsilon_t \tag{7}$$

$$= U_t ,$$

It can be written as:

$$Y_t = U_t , \ (U_t \sim N(0,\sigma_c^2) \tag{8}$$

where, $A(L) + B(L) + \ldots = Q$ which are the matrix polynomial, U_t is the error term of combine white noise model and σ_c^2 is the combination of equal variances. The combine variances of the combine white noise is:

$$\sigma_c^2 = \sigma_1^2 + \sigma_2^2 + \ldots \tag{9}$$

Considering the best two variances in the best two models produced by the Bayesian model averaging output. The combine variance follows:

$$\sigma_c^2 = \sigma_1^2 + \sigma_2^2 \tag{10}$$

The variance of errors, σ_c^2 in the combine white noise can be written:

$$\sigma_c^2 = W^2\sigma_1^2 + (1-W)^2\sigma_2^2 + 2\rho W\sigma_1(1-W)\sigma_2 \tag{11}$$

where, the balanced weight specified for the model is W. The least of σ_c^2 appearing, when the equation is differentiated with respect to W and equate to zero, obtaining:

$$W = \frac{\sigma_c^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2} \tag{12}$$

where, ρ is the correlation; intra-class correlation coefficient is used for a reliable measurement.

Table 1: Histogram-normality and ARCH tests

Normal test	Coefficient/value	Probability
Standard deviation	0.840452	
Skewness	-0.320441	
Kurtosis	4.515921	
Jarque-bera	24.71731	0.000004
Arch tests		
F-statistic	1.372665	0.2427
Obs*R-squared	1.376645	0.2407

RESULTS AND DISCUSSION

The data is transformed for proper investigation of the behaviour of the data. In returns series the data exhibit volatility clustering, long tail skewness and excess kurtosis which are the attributes of heteroscedasticity. The graph displays unequal variances that signify volatility (Al-Hagyan *et al*., 2015).

Table 1 reports that, there are left long tail skewness, excess kurtosis and Jarque-Bera test is significant, this portraits non-normality and the standard deviation is also less than one.

Table 1 shows the ARCH LM tests for the effect of heteroscedasticity in the data series; F-Statistic and Obs*R-squared prove that there is ARCH presence in the data.

The GARCH computations are more complex (Bollerslev, 1986; Nelson, 1991; McAleer and Hafner, 2014). No proposition has removed heteroscedasticity completely (White, 1980; Antoine and Lavergne, 2014; Uchôa *et al*., 2014). In GARCH modelling the leverage is not possible; any restriction imposed will be positivity restriction which has no leverage effect (McAleer, 2014; McAleer and Hafner, 2014).

Therefore, the standardized residuals graph of the GARCH model (GARCH errors) with unequal variances and zero mean are decomposed into equal variances series (white noise series). The equal variances graphs with mean zero are obtained from graphs of GARCH errors. These white noise series are fit into regression model to make each a model.

The implementation of Bayesian model averaging produces two best models (Asatryan and Feld, 2015; Merah and Chadli, 2015). For confirmations, fit linear regression with autoregressive errors with zero mean and variance one (Higgins and Bera, 1992). Therefore, the best two models are white noise models (Table 2).

The data of the two white noise series are used for the Combine White Noise model (Bates and Granger, 1969; Tayyab *et al*., 2015; Radhika and Arumugam, 2015). The computational results of Combine White Noise are in Table 3.

The US RGDP data with the data of the two white noise series are employed for Vector Autoregressive model. The VAR model results are in Table 3.

In Table 2 an independent samples test is conducted to test whether data set USRGDP-.06 and USRGDP 4.1

Table 2: Levene's test for equal variances independent samples test

		Levene's Test for equality of variances		t-test for Equality of means					95% Confidence interval of the difference	
		F	Sig.	t	do	Sig. (2-tailed)	Mean difference	Std. Error difference	Lower	Upper
B	Equal variances assumed	1.414	0.235	2.159	438	0.031	0.05909	0.02737	0.00530	0.11288
	Equal variances not assumed			2.159	255.236	0.032	0.05909	0.02737	0.00519	0.11299

Table 3: The summary of GARCH and Combine White Noise (CWN) models estimation and forecasting evaluation

Summary	CWN	VAR
Estimation		
Residual diagnostic		
Stability test (Lag structure)	Stable	Not stable
Correlogram (square) residual	Covariance stationary	Covariance stationary
Portmanteau tests	No autocorrelation	Autocorrelation
Histogram-Normality tests	Not normal	Not normal
ARCH Test	No ARCH effect	No ARCH effect
Dynamic forecast evaluation		
RMSE	0.482821	436.446
MAE	0.113995	363.6411
MAPE	1.387052	4.326751
Residual diagnostic		
Correlogram (square) residual	Stationary	Stationary
Histogram-Normality Tests	Not normal	Not normal
Serial Correlation LM Tests	No serial correlation	Serial correlation
Heteroscedasticity test	No ARCH effect	ARCH effect
Stability diagnostic		
Ramsey reset tests	Stable	Not stable
AIC	-0.523474	10.77502
BIC	-0.430622	10.96072
Log likelihood	63.32035	-1167.864
Determinant residual covariance	0.001923	8.600293

(i.e., the two data are for the two models from the output of Bayesian model averaging) have equal variances or not. The test reveals that the variability in the distribution of the two data sets is no significantly different with the value which is greater than the p-value 0.05. Therefore, they have equal variances (Lim and Loh, 1996; Boos and Brownie, 2004; Bast *et al*., 2015).

Table 3 reveals that Combine White Noise (CWN) appears to be the appropriate model for estimation and forecasting, when comparing the two models.

CWN fulfils the stability condition, is stationary, no serial correlation, no ARCH effect and has the minimum dynamic evaluation forecast errors. It fails the normality tests, but passes the Levene's test of equal variances. The Levene's test suggests that it has equal (constant) variance.

CWN has the determinant of the residual covariance matrix almost zero (that is, the determinant is equal to zero in two significant figures); this is an indication that CWN estimation is efficient. It has the minimum information criteria and high log likelihood.

CONCLUSION

Generally, GARCH models have been in use to model the data that exhibits the conditional heteroscedasticity. The GARCH models have not been able to model the leverage effect, because the positivity restriction imposed has no leverage effect (McAleer, 2014; McAleer and Hafner, 2014).

The standardized residual GARCH errors being decomposed into Combine White Noise (CWN) has proved to be more efficient and it takes care of GARCH weaknesses. The estimation of Combine White Noise model passes stability condition, stationary, serial correlation and the ARCH effect tests. It fails the histogram-Normality tests, but passes the Levene's test of equal variances.

From the results in Table 3: CWN model estimation yield better results with minimum information criteria and high log likelihood values than the VAR model estimation results which has high information criteria and low log likelihood values. CWN has the minimum forecast errors which are indications of better results when compare with the VAR model dynamic evaluation forecast errors (Ismail and Muda, 2006; Fildes *et al*., 2011; Lazim, 2013; Radhika and Arumugam, 2015).

All CWN outcomes outperform all VAR outcomes with wider margin. Therefore, CWN approach is recommended for modeling data that exhibit heteroscedasticity and leverage effect; since VAR cannot model heteroscedasticity and GARCH family cannot model leverage effect.

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