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Research Article

Construction and Researching Aircraft High Potential of Robust Stability Control System in the Form of Single-parameter Structurally Stable Mapping

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Abstract: The study presents a new approach to building control systems for objects with uncertain parameters in the form of single-parameter structurally stable mappings of catastrophe theory to synthesize highly efficient control system, which has an extremely wide field of robust stability.

Keywords: Control systems, lyapunov function, robust stability, single-parameter structurally stable mappings, stationary states, the aircraft angular motion, traffic control

INTRODUCTION

Control system design is one of the main tasks in automation of all branches of industry, including machine manufacturing, energy sector, electronics, chemical and biological, metallurgical, textile, transportation, robotics, aviation, space systems, high-precision military systems, etc. In these systems, the uncertainty can be caused by the presence of uncontrolled disturbances acting on an object control and ignorance of the true values of the parameters of control objects and unpredictable change them in time. The main goal in modern control system design is, in some sense, to provide the best protection against uncertainty in the knowledge of the system. The ability of a control system to keep stability in the conditions of parametrical or nonparametric uncertainty is realized as robust stability of system (Polyak and Sherbakov, 2002). Research of system robust stability consists in the indication of restrictions on control system parameters change (Polyak and Sherbakov, 2002; Dorato and Yedavalli, 1990).

The many papers research a problem of robust stability (Polyak and Sherbakov, 2002; Dorato and Yedavalli, 1990; Kuntsevich, 2006; Liao and Yu, 2008). In these works investigated the robust stability of polynomials, matrixes, within the linear principle of stability of continuous and discrete control systems, in works (Kuntsevich, 2006, Liao and Yu, 2008) are solving the problems of absolute robust stability. In the practical tasks, connected with development and creation of control systems in technology, economy, biology and other spheres, in the conditions of essential parametrical uncertainty, the increase in potential of robust stability is one of the key factors, which guaranteeing to a control system protection from entry in regime of determined chaos and strange attractors. And guarantees applicability of models and reliability of the designed control systems work.

At present it is conventional that, real control objects are nonlinear and one of the main properties of nonlinear dynamic systems is functioning in the mode of the determined chaotic traffic (Andrievsky and Fradkov, 1999; Nicols and Prigogine, 1989; Loskutov and Mikhaylov, 2007). In linear dynamic systems it is appear in the form of control system's zero steady state stability loss (Beisenbi, 2011b; Beisenbi and Erzhanov, 2002).

In this regard, in the conditions of uncertainty, there was a need for development of models and methods of design of control system with rather wide area of robust stability, which called control systems with the increased potential of robust stability (Beisenbi, 2011b; Beisenbi and Erzhanov, 2002). The concept of creation of a control system with the increased potential of robust stability is based on results of the catastrophes theory (Gilmore, 1984; Poston and Stewart, 2001), where the main structural-steady maps are received.

This study is devoted to design of control systems with increased potential of robust stability by dynamical objects with uncertain parameters in a class of the single-parameter structurally steady maps (Beisenbi, 1998, 2011a, 2011b; Beisenbi and Erzhanov, 2002; Ashimov and Beisenbi, 2000).

Researches of the recent years showed, that the method of Lyapunov functions can be (Barbashin, 1967; Krasovsky, 1959; Malkin, 1966) successfully

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used to analyse the robust stability of linear and nonlinear control systems. Usage of Lyapunov's functions method for the solution of a set of practical linear or nonlinear tasks is constrained by the lack of a general method for selecting or constructing Lyapunov functions and difficulties with their algorithmic representation (Barbashin, 1967; Malkin, 1966). An inappropriate choice of a Lyapunov function or the inability to construct one does not indicate instability of the system, only that a proper Lyapunov function has not been found.

The method of design of Lyapunov vector function (Voronov and Matrosov, 1987), on the basis of geometrical interpretation of asymptotic stability theorem and concepts of stability is offered. Therefore, the origin corresponds to a predetermined condition of the system, the unperturbed state and the equations of the state are formed concerning perturbations, i.e., in deviations of the perturbed motion from unperturbed (Malkin, 1966). Consequently, the state equations express the speed of change of a perturbations vector (deviations) and for steady system is directed toward the origin. And the gradient vector from required Lyapunov function, for stable system, will be always directed to the opposite side. It allows to present Lyapunov function in the form of a potential surface (Gilmore, 1984). Research of robust stability of the control system with uncertain parameters are based on ideas of a Lyapunov direct method.

MATHEMATICAL MODEL FORMULATION

Area of system stability: System can be written in expanded form:

$$\begin{cases} \dot{x}_1 = -b_{11}x_1^3 + (a_{11} + b_{11}k_1)x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \\ \dot{x}_2 = -a_{21}x_1 - b_{22}x_2^3 + (a_{22} + b_{22}k_2)x_2 + \ldots + a_{2n}x_n \\ \ldots \\ \dot{x}_n = -a_{n1}x_1 + a_{n2}x_2 + \ldots + b_{nn}x_n^3 + (a_{nn} + b_{nn}k_n)x_n \end{cases} \tag{1}$$

where, $x(t) \in R^n$ - control object state vector, $a_{ij}, i = 1,\ldots, n, j = 1,\ldots, n,$ $b_{ij}, i = 1,\ldots,n, j = 1,\ldots,n,$ - the elements of the control object.

The control law is described by a vector function in the form of single-parameter structurally steady maps (Gilmore, 1984; Poston and Stewart, 2001):

$$u_i = -x_i^3 + k_i x_i \quad i = 1,\ldots,n \tag{2}$$

The steady state $x_{is}^1 = 0,\ i = 1,\ldots,n$ of system (1) is defined by the solution of the equations:

$$\begin{cases} -b_{11}x_{1s}^3 + (a_{11} + b_{11}k_1)x_{1s} + a_{12}x_{2s} + \ldots + a_{1n}x_{ns} = 0 \\ -a_{21}x_{1s} - b_{22}x_{2s}^3 + (a_{22} + b_{22}k_2)x_{2s} + \ldots + a_{2n}x_{ns} = 0 \\ \ldots \\ -a_{n1}x_{1s} + a_{n2}x_{2s} + \ldots + b_{nn}x_n^3 + (a_{nn} + b_{nn}k_n)x_{ns} = 0 \end{cases} \tag{3}$$

From (3) we receive a steady state of system:

$$x_{is}^1 = 0,\ i = 1,\ldots, n \tag{4}$$

Other stationary states will be defined by solutions of the equation:

$$\begin{gathered} -b_{ii}x_{is}^2 + (a_{ii} + b_{ii}k_i) = 0 \quad x_{js} = 0 \quad i \neq 0 \\ i = 1,\ldots, n \quad j = 1,\ldots, n \end{gathered} \tag{5}$$

Great number of solutions of the Eq. (5) can be written as:

$$\begin{gathered} x_{is}^{2,3} = \pm\sqrt{\frac{a_{ii}}{b_{ii}} + k_i},\ x_{js} = 0,\ i \neq j,\ i = 1,\ldots,n; \\ j = 1,\ldots, n \end{gathered} \tag{6}$$

Here the system of the nonlinear algebraic Eq. (3) has the trivial decision (4) and uncommon decisions (6) when $\frac{a_{ii}}{b_{ii}} + k_i > 0$, $i = 1,\ldots, n$. At negative value $\frac{a_{ii}}{b_{ii}} + k_i < 0$, $i = 1,\ldots,n$ the Eq. (5) has imaginary decisions that can't correspond to any physically possible situation (Nicols and Prigogine, 1989). These decisions are joined with (4) when $\frac{a_{ii}}{b_{ii}} + k_i = 0$, $i = 1,\ldots, n$ and branch off from it when $\frac{a_{ii}}{b_{ii}} + k_i > 0$, $i = 1,\ldots, n$, i.e., in a point $\frac{a_{ii}}{b_{ii}} + k_i = 0$, $i = 1,\ldots, n$ bifurcation is happened. It is provided that the state (4) is globally asymptotically steady for all $\frac{a_{ii}}{b_{ii}} + k_i < 0$, $i = 1,\ldots, n$ and unstable at $\frac{a_{ii}}{b_{ii}} + k_i > 0$, $i = 1,\ldots,n$ states (6) also will be asymptotically steady, in other words, branches appears as a result of bifurcation while the state (4) loses stability and these branches are steady.

Verification of these statements is made on the basis of Lyapunov vector functions ideas (Voronov and Matrosov, 1987).

If Lyapunov function $V(x)$ is set in the form of vector function $V(V_1(x), \ldots, V_n(x))$, then components of speed vector will be equal (Beisenbi and Uskenbayeva, 2014a; Beisenbi and Yermekbayeva, 2013a; Beisenbi *et al*., 2015):

$$-\frac{dx_i}{dt} = \frac{\partial V_i(x)}{\partial x_1} + \frac{\partial V_i(x)}{\partial x_2} + \ldots + \frac{\partial V_i(x)}{\partial x_n},\ i = 1,\ldots,n \tag{7}$$

In the Eq. (7), substituting values of components of a vector of speed, we will get:

$$\frac{\partial V_i(x)}{\partial x_1} = -a_{i1}x_1, \ \frac{\partial V_i(x)}{\partial x_2} = -a_{i2}x_2, \ldots,$$
$$\frac{\partial V_i(x)}{\partial x_i} = b_{ii}x_i^3 - (a_{ii} + b_{ii}k_i)x_i, \ldots,$$
$$\frac{\partial V_i(x)}{\partial x_{n-1}} = -a_{i,n-1}x_{n-1}, \ \frac{\partial V_i(x)}{\partial x_n} = -a_{in}x_n \quad i = 1,\ldots,n \tag{8}$$

Full derivative on time from Lyapunov vector function V (x) taking into account the equation of a state (1), we can define as product of the gradient from Lyapunov vector function on a vector of speed (Beisenbi and Uskenbayeva, 2014b; Beisenbi and Yermekbayeva, 2013a, 2013b; Beisenbi *et al*., 2015), i.e.:

$$\frac{dV(x)}{dt} = -\sum_{i=1}^{n}\left(\sum_{j=1}^{n}\frac{\partial V_i(x)}{\partial x_j}\right)\frac{dx_i}{dt} = -\sum_{i=1}^{n}[a_{i1}x_1 + a_{i2}x_2 + \ldots - b_{ii}x_i^3 +$$
$$+ (a_{ii} + b_{ii}k_i)x_i + \ldots + a_{i,n-1}x_{n-1} + a_{in}x_n]^2$$
$$i = 1,\ldots,n \tag{9}$$

From here (10) follows that full derivate on time from Lyapunov function will be negative function.

From (9) for components of Lyapunov vector function we will get:

$$V_i(x) = -\frac{1}{2}a_{i1}x_1^2 - \frac{1}{2}a_{i2}x_2^2 -,\ldots,-\frac{1}{4}b_{ii}x_i^4 - \frac{1}{2}(a_{ii} + b_{ii}k_i)x_i^2 -$$
$$-\ldots-\frac{1}{2}a_{i,n-1}x_{n-1}^2 - \frac{1}{2}a_{in}x_n^2$$
$$i = 1,\ldots,n$$

We can present Lyapunov function in a scalar form in the view:

$$V(x) = \sum_{i=1}^{n}V_i(x) = \frac{1}{4}x_1^4 - \frac{1}{2}(a_{11} + b_{11}k_1 + a_{21} + a_{31} + \ldots + a_{n1})x_1^2 +$$
$$\frac{1}{4}x_2^4 - \frac{1}{2}(a_{12} + a_{22} + b_{22}k_2 + a_{32} + \ldots + a_{n2})x_2^2 +$$
$$\frac{1}{4}x_3^4 - \frac{1}{2}(a_{13} + a_{23} + a_{33} + b_{33}k_3 + \ldots + a_{n3})x_3^2 + \ldots$$
$$\ldots + \frac{1}{4}x_n^4 - \frac{1}{2}(a_{1n} + a_{2n} + a_{3n} + \ldots + a_{nn} + b_{nn}k_n)x_n^2 \tag{10}$$

Function (10) is Lyapunov function and conditions of positive definiteness are defined by inequalities:

$$\begin{cases} a_{11} + b_{11}k_1 + a_{21} + a_{31} + \ldots + a_{n1} < 0 \\ a_{12} + a_{22} + b_{22}k_2 + a_{32} + \ldots + a_{n2} < 0 \\ a_{13} + a_{23} + a_{33} + b_{33}k_3 + \ldots + a_{n3} < 0 \\ \ldots \\ a_{1n} + a_{2n} + a_{3n} + \ldots + a_{nn} + b_{nn}k_n < 0 \end{cases} \tag{11}$$

Thus, the area of system stability (1) for the established state (4) is defined by system of inequalities (11).

Research of stationary states (6) stability: The equations of a state (3) in deviations in relative steady state x_S^2 (7) can be written as (Beisenbi *et al*., 2015; Beisenbi, 2011a, 2011b; Beisenbi and Erzhanov, 2002):

$$\begin{cases} \dot{x}_1 = -b_{11}x_1^3 - 3b_{11}\sqrt{\frac{a_{11}}{b_{11}} + k_1x_1^2} + (a_{11} + b_{11}k_1)x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \\ \dot{x}_2 = -a_{21}x_1 - b_{22}x_2^3 - 3b_{22}\sqrt{\frac{a_{22}}{b_{22}} + k_2x_2^2} - 2(a_{22} + b_{22}k_2)x_2 + \ldots + a_{2n}x_n \\ \ldots \\ \dot{x}_n = -a_{n1}x_1 + a_{n2}x_2 + \ldots - b_{nn}x_n^3 - 3b_{nn}\sqrt{\frac{a_{nn}}{b_{nn}} + k_nx_n^2} - 2(a_{nn} + b_{nn}k_n)x_n \end{cases} \tag{12}$$

The full derivative from Lyapunov function V (x) taking into account the state equations in deviations (12) relative to the stationary state x_S^2 (6) is defined as:

$$\frac{dV(x)}{dt} = \frac{\partial V}{\partial x}\frac{dx}{dt} = -\sum_{i=1}^{n}(a_{i1}x_1 + a_{i2}x_2 + .a_{i3}x_3 + ..$$
$$- b_{ii}x_i^3 - 3b_{ii}\sqrt{\frac{a_{ii}}{b_{ii}} + k_ix_i^2} - 2(a_{ii} + b_{ii}k_i)x_i + \ldots + a_{in}x_n)^2 \tag{13}$$

Function (13) is negative function. We can find components of the gradient vector of Lyapunov function:

$$\frac{\partial V_i(x)}{\partial x_1} = -a_{i1}x_1, \ \frac{\partial V_i(x)}{\partial x_2} = -a_{i2}x_2,\ldots,$$
$$\frac{\partial V_i(x)}{\partial x_i} = b_{ii}x_i^3 + 3b_{ii}\sqrt{\frac{a_{ii}}{b_{ii}} + k_ix_i^2} - 2(a_{ii} + b_{ii}k_i)x_i,$$
$$\frac{\partial V_i(x)}{\partial x_{i+1}} = -a_{i,i+1}x_{i+1} \quad \frac{\partial V_i(x)}{\partial x_n} = -a_{in}x_n, \ i = 1,\ldots,n;$$

From here we receive Lyapunov function in a scalar form:

$$V(x) = \sum_{i=1}^{n}\left(\frac{1}{4}b_{ii}x_i^4 + b_{ii}\sqrt{\frac{a_{ii}}{b_{ii}} + k_ix_i^3}\right)$$
$$+ \frac{1}{2}\sum_{i=1}^{n}(-a_{1i} - a_{2i} - \ldots + (a_{ii} + b_{ii}k_i) - \ldots - a_{ni})x_i^2 \tag{14}$$

Function (14) on the beginning of coordinates addresses in zero, is continuous differentiable function and has the form of variables with odd degrees. Therefore on the basis of the Morse lemma (Gilmore, 1984; Poston and Stewart, 2001) function (14) around the steady state x_S^2 (6) can be represented as a quadratic form:

$$V(x) = \frac{1}{2}\sum_{i=1}^{n}(-a_{1i} - a_{2i} - \ldots + (a_{ii} + b_{ii}k_i) - \ldots - a_{ni})x_i^2$$

From here positive definiteness of Lyapunov function will be defined by an inequality:

$$(a_{ii}+b_{ii}k_i)>a_{1i}+a_{2i}+...-a_{ni}\quad i=1,...,n; \tag{15}$$

Let investigate stability of a steady state x_S^3 (6): The equation of a state (3) in deviations in relative steady state x_S^3 (6) can be written as (Beisenbi *et al*., 2015; Beisenbi, 2011a, 2011b; Beisenbi and Erzhanov, 2002):

$$\begin{aligned}&\dot{x}_i=a_{i1}x_1+a_{i2}x_2+.a_{i3}x_3+..-b_{ii}x_i^3+\\&+3b_{ii}\sqrt{\frac{a_{ii}+b_{ii}k_i}{b_{ii}}}x_i^2-2(a_{ii}+b_{ii}k_i)x_i+...+a_{in}x_n\\&i=1,...,n;\end{aligned} \tag{16}$$

Omitting formal actions for research of stability of stationary states of x_S^3 (6), similar for a steady state x_S (6) we will receive Lyapunov function in a scalar form:

$$\begin{aligned}V(x)&=\sum_{i=1}^{n}(\frac{1}{4}b_{ii}x_i^4-b_{ii}\sqrt{\frac{a_{ii}}{b_{ii}}+k_ix_i^3})\\&+\frac{1}{2}\sum_{i=1}^{n}(-a_{1i}-a_{2i}-...+(a_{ii}+b_{ii}k_i)-...-a_{ni})x_i^2\end{aligned}$$

On the Morse lemma we will lead (Gilmore, 1984; Poston and Stewart, 2001) Lyapunov function, by means of stability matrix, to a quadratic form (Beisenbi *et al*., 2015):

$$V(x)=\frac{1}{2}\sum_{i=1}^{n}(-a_{1i}-a_{2i}-a_{3i}-...+(a_{ii}+b_{ii}k_i)-...-a_{ni})x_i^2$$

Stability conditions of a steady state x_S^3 (6) it will be expressed by system of inequalities:

$$(a_{ii}+b_{ii}k_i)>a_{1i}-a_{2i}+...+a_{ni},i=1,...,n; \tag{17}$$

Thus, the control system constructed in a class of one-parametrical structural steady maps will be steady in indefinitely wide limits of change of uncertain parameters of the control object. The steady state x_S^1 (4) exists and is stable at change of uncertain parameters of object in area (11) and stationary statesx_S^2 and x_S^3 (6) appear at loss of stability of a state x_S^1 (4) and they are not simultaneously exist. Stationary states x_S^2 and x_S^3 (6) are stable when performing system of inequalities (15) and (17).

Thus, the control system constructed in a class of one-parametric structural stable maps will be stable in indefinitely wide limits of change of uncertain parameters of the control object. The steady state x_s^1 (4) exists and is stable at change of uncertain parameters of object in area (11). And stationary states x_s^2 and x_s^3(6) appear at loss of stability of a state x_s^1 (4) and they are not simultaneously exist. Stationary states x_s^2 and x_s^3(6) are stable when performing system of inequalities (15) and (17).

CASE STUDY

Description of dynamics of the aircraft angular motion: We investigate a task of traffic control of the aircraft by the pitch. Let consider that aircraft have constants, aprioristic-uncertain parameters, which values are located in the set area. We will notice that the similar situation can take place when aircraft flying on various modes, when height, the speed and loading of aircraft changes slowly in comparison with rate of angular motion. For the description of dynamics of the aircraft angular motion we use the following linearized equations (Andrievsky and Fradkov, 1999; Bukov, 1987):

$$\begin{cases}\dot{\alpha}(t)=q_z(t)+\alpha_y^{\alpha}(t)\alpha(t)+\alpha_y^{\delta_\beta}\delta_\beta(t)\\\dot{q}_z(t)=-\alpha_{m_z}^{\alpha}\alpha(t)-\alpha_{m_z}^{q_z}q_z(t)-\alpha_{m_z}^{\delta_\beta}\delta_\beta(t)\\\dot{\theta}(t)=q_z(t)\end{cases}$$

where,

$\theta(t),q(t)$	: The angle and the pitch rate
$\alpha(t)$	: The angle of attack
$\delta_\beta(t)$	: Angle of a deviation of rudder height
$\alpha_y^{\alpha}(t)$, $\alpha_y^{\delta_\beta}$, $\alpha_{m_z}^{\alpha}$, $\alpha_{m_z}^{\omega_z}$, $\alpha_{m_z}^{\delta_\beta}$	: Aircraft parameters

Their values depend on the factors stated above and can change over a wide range depending on height and the speed of flight. Exact values of parameters a priori not defined. Also we assume, that dynamics of executive body it is possible to neglect and consider that control is the deviation of rudder $\delta_\beta(t)$.

$$\begin{aligned}&x_1=\theta(t),\ x_2=q_z(t),\ x_3=\alpha(t),\ a_1=\alpha_{m_z}^{\alpha},\\&a_2=\alpha_{m_z}^{q_z},\ a_3=\alpha_{m_z}^{\delta_\beta},\ a_4=\alpha_y^{\alpha}(t),\ a_5=\alpha_y^{\delta_\beta}\\&u=\delta_\beta(t)\end{aligned}$$

Then the equation of the aircraft angular motion will assume the form:

$$\begin{cases}\dfrac{dx_1}{dt}=x_2(t)\\\dfrac{dx_2}{dt}=-a_1x_3(t)-a_2x_2(t)-a_3u\\\dfrac{dx_3}{dt}=x_2(t)+a_4x_3(t)+a_5u\end{cases} \tag{18}$$

As the control law we will choose:

$$u=-x_3^3+k_3x_3 \tag{19}$$

Thus, the system (18) with the control law (19) will assume the form:

$$\begin{cases} \dfrac{dx_1}{dt} = x_2(t) \\ \dfrac{dx_2}{dt} = -a_1 x_3(t) - a_2 x_2(t) - a_3(-x_3^3 + k_3 x_3) \\ \dfrac{dx_3}{dt} = x_2(t) + a_4 x_3(t) + a_5(-x_3^3 + k_3 x_3) \end{cases} \tag{20}$$

From the Eq. (20) we define the established conditions:

$$\begin{cases} x_{2S} = 0 \\ -a_1 x_{3S}(t) - a_2 x_{2S}(t) - a_3(-x_{3S}^3 + k_3 x_{3S}) = 0 \\ x_{2S}(t) + a_4 x_{3S}(t) + a_5(-x_{3S}^3 + k_3 x_{3S}) = 0 \end{cases} \tag{21}$$

The system (21) has the following stationary states:

$$x_{1S} = 0;\ x_{2S} = 0;\ x_{3S} = 0; \tag{22}$$

And other stationary conditions of system (21) are defined by the solution of the equations:

$$(a_3 - a_5)x_{3S}^2 - a_1 + a_4 - (a_3 - a_5)k_3 = 0$$

This equation has nonzero solutions in the form:

$$x_{1S} = 0;\ x_{2S} = 0;\ x_{3S} = \pm\sqrt{k_3 + \frac{a_1 - a_4}{a_3 - a_5}} \tag{23}$$

We investigate stability of system (20) in stationary points by the method of Lyapunov functions. Lyapunov function V (x) is set in the form of a vector function $V(V_1(x), \ldots, V_n(x))$, then from geometrical interpretation of the theorem of asymptotic stability we will get (Barbashin, 1967; Malkin, 1966):

$$\frac{\partial V_1(x)}{\partial x_1} = 0;\ \frac{\partial V_1(x)}{\partial x_2} = -x_2;\ \frac{\partial V_1(x)}{\partial x_3} = 0;\ \frac{\partial V_2(x)}{\partial x_1} = 0;$$
$$\frac{\partial V_2(x)}{\partial x_2} = a_2 x_2;\ \frac{\partial V_2(x)}{\partial x_3} = a_3 x_3^3 - (a_3 k_3 - a_1)x_3;$$
$$\frac{\partial V_3(x)}{\partial x_1} = 0;\ \frac{\partial V_3(x)}{\partial x_2} = -x_2;$$
$$\frac{\partial V_3(x)}{\partial x_3} = -a_5 x_3^3 + (a_5 k_3 + a_4)x_3$$

The full derivative on the time from Lyapunov vector function V(x) taking into account the equation of a state (20), is represented as product of the gradient vector from Lyapunov vector function on a vector of speed i.e.:

$$\frac{dV(x)}{dt} = -\sum_{i=1}^{n}\left(\sum_{j=1}^{n}\frac{\partial V_i(x)}{\partial x_j}\right)\frac{dx_i}{dt} = -(2 - a_2)x_2^2 + \\ + [a_3 x_3^3 - (a_3 k_3 - a_1)x_3]^2 - [a_5 x_3^3 - (a_5 k_3 + a_4)x_3]^2 \tag{24}$$

From the expressions (24) follows, that the full derivative on time from Lyapunov functions is always negative function.

On the basis of the Morse lemma we will present Lyapunov function in a scalar form in the following view:

$$V(x_1, x_2, x_3) = \frac{1}{2}(a_2 + 2)x_2^2 + \frac{1}{4}(a_5 - a_3)x_3^4 - \\ -\frac{1}{2}(a_3 k_3 - a_1 + a_5 k_3 + a_4)x_3^2$$

The conditions of (20) system stability in a steady state (22), we obtain, taking into account the negative definiteness of the functions (24) in the form of a system of inequalities:

$$a_2 > -2,\ a_3 > a_5,\ k_3 < \frac{a_1 - a_4}{a_3 + a_5} \tag{25}$$

Research of stationary states (23) stability: The equations of system state (20) with respect to deviations of the stationary state (23) is written:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -a_2 x_2 + a_3 x_3^3 + 3a_3\sqrt{k_3 + \dfrac{a_1 - a_4}{a_3 - a_5}}x_3^2 + 2(a_1 + a_3 k_3)x_3 \\ \dot{x}_3 = x_2 - a_5 x_3^3 - 3a_5\sqrt{k_3 + \dfrac{a_1 - a_4}{a_3 - a_5}}x_3^2 - 2(a_4 + a_5 k_3)x_3 \end{cases} \tag{26}$$

Full-time derivative of the Lyapunov function V (x) with the equation of state (26) with respect to the stationary state (23) is defined as:

$$\frac{dV(x)}{dt} = -(2 + a_2^2)x_2^2 - [a_3 x_3^3 + 3a_3\sqrt{k_3 + \frac{a_1 - a_4}{a_3 - a_5}}x_3^2 \\ + 2(a_1 + a_3 k_3)x_3]^2 - \\ [a_5 x_3^3 + 3a_5\sqrt{k_3 + \frac{a_1 - a_4}{a_3 - a_5}}x_3^2 + 2(a_4 + a_5 k_3)x_3]^2 \tag{27}$$

From the expressions (27) follows, that the full derivative on time from Lyapunov function will be a negative function. We find the gradient vector components from Lyapunov vector function:

$$\frac{\partial V_1(x)}{\partial x_1} = 0;\ \frac{\partial V_1(x)}{\partial x_2} = -x_2;\ \frac{\partial V_1(x)}{\partial x_3} = 0;\ \frac{\partial V_2(x)}{\partial x_1} = 0;$$
$$\frac{\partial V_2(x)}{\partial x_2} = a_2 x_2;$$
$$\frac{\partial V_2(x)}{\partial x_3} = -a_3 x_3^3 - 3a_3\sqrt{k_3 + \frac{a_1 - a_4}{a_3 - a_5}}x_3^2 - 2(a_1 + a_3 k_3)x_3$$
$$\frac{\partial V_3(x)}{\partial x_1} = 0;\ \frac{\partial V_3(x)}{\partial x_2} = -x_2;$$

$$\frac{\partial V_3(x)}{\partial x_3} = a_5 x_3^3 + 3a_5\sqrt{k_3 + \frac{a_1 - a_4}{a_3 - a_5}}x_3^2 + 2(a_4 + a_5 k_3)x_3$$

On a gradient we will construct Lyapunov's function:

$$V(x) = \frac{1}{2}(a_2+2)x_2^2 + \frac{1}{4}(a_5 - a_3)x_3^4 + \\ +(a_5 - a_3)\sqrt{k_3 + \frac{a_1 - a_4}{a_3 - a_5}}x_3^3 + \\ +(a_5 k_3 + a_4 - a_1 - a_3 k_3)x_3^2 \tag{28}$$

By the Morse lemma from the catastrophe theory we can replace Lyapunov function (28) with a quadratic form:

$$V(x) = \frac{1}{2}(a_2+2)x_2^2 + [(a_5 - a_3)k_3 + a_4 - a_1)]x_3^2 \tag{29}$$

The condition of positive definiteness of Lyapunov function (28) or (29) we will get in a view:

$$a_2 > -2,\ \ k_3 + \frac{a_1 - a_4}{a_5 - a_3} > 0 \tag{30}$$

Hence a necessary and sufficient condition for the stability of the stationary state (23) of (20) system is performance of an inequality (30).

SIMULATION RESULTS

Control law is designed for linearized model (18) and, we find sufficient conditions for the stability of the stationary state and positive definiteness of Lyapunov function.

For the equations of dynamics of the aircraft angular motion:

$$\begin{cases} \dot{\alpha}(t) = q_z(t) + \alpha_y^{\alpha}(t)\alpha(t) + \alpha_y^{\delta_\beta}\delta_\beta(t) \\ \dot{q}_z(t) = -\alpha_{m_z}^{\alpha}\alpha(t) - \alpha_{m_z}^{q_z}q_z(t) - \alpha_{m_z}^{\delta_\beta}\delta_\beta(t) \\ \dot{\theta}(t) = q_z(t) \end{cases}$$

$a_1 = \alpha_{m_z}^{\alpha}$, $a_2 = \alpha_{m_z}^{q_z}$, $a_3 = \alpha_{m_z}^{\delta_\beta}$,
$a_4 = \alpha_y^{\alpha}(t)$, $a_5 = \alpha_y^{\delta_\beta}$

The matrices of coefficients are defined as follows:

$$A = \begin{bmatrix} \alpha_y^{\alpha} & 1 & 0 \\ \alpha_{m_z}^{\alpha} & \alpha_{m_z}^{q_z} & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -1.2 & 1 & 0 \\ 3.9 & -0.85 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} \alpha_{m_z}^{\delta_\beta} \\ \alpha_y^{\delta_\beta} \\ 0 \end{bmatrix} = \begin{bmatrix} -0.16 \\ 9.5 \\ 0 \end{bmatrix}$$

According to the conditions of positive definiteness of Lyapunov function (29) we get gain of the system.

From (30) we define $k_3 > 0.5$, $(a_2 = -0.85) > -2$, $(a_5 = -0.16) < (a_3 = 9.5)$.

Figure 1 show the results of the simulation system with the parameters from Table 1.

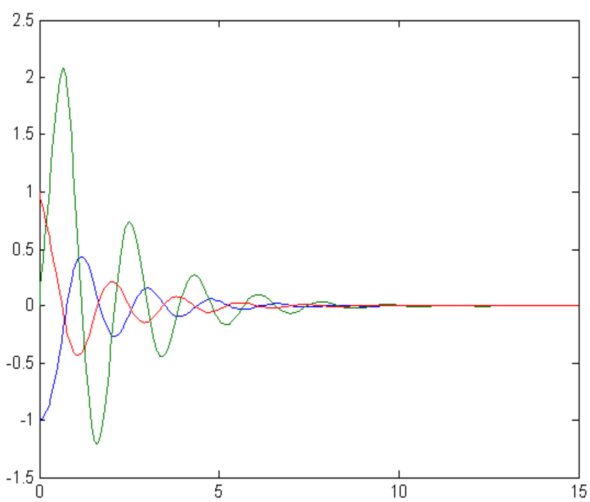
Figure 2 show the results of the simulation system with the parameters from Table 2.

Table 1: System parameters the varying gain parameter k_3.

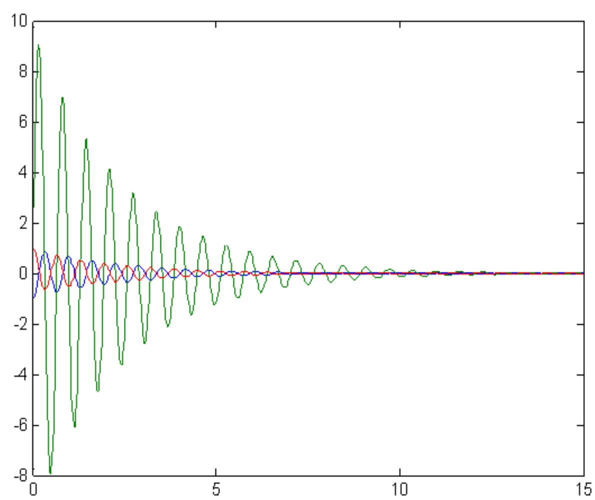
$\alpha_{m_z}^{\alpha}$	$\alpha_{m_z}^{q_z}$	$\alpha_{m_z}^{\delta_\beta}$	α_y^{α}	$\alpha_y^{\delta_\beta}$	k_3
3.9	-0.85	-0.16	-1.2	9.5	1
3.9	-0.85	-0.16	-1.2	9.5	10

Table 2: System parameters the gain parameter k_3 fixed

$\alpha_{m_z}^{\alpha}$	$\alpha_{m_z}^{q_z}$	$\alpha_{m_z}^{\delta_\beta}$	α_y^{α}	$\alpha_y^{\delta_\beta}$	k_3
3.9	-0.85	-0.32	-1.2	9.5	1
3.9	-0.85	-2.16	-1.2	9.5	1
3.9	-0.85	-0.16	-1.2	19.5	1
3.9	-0.85	-0.16	-1.2	29.5	1

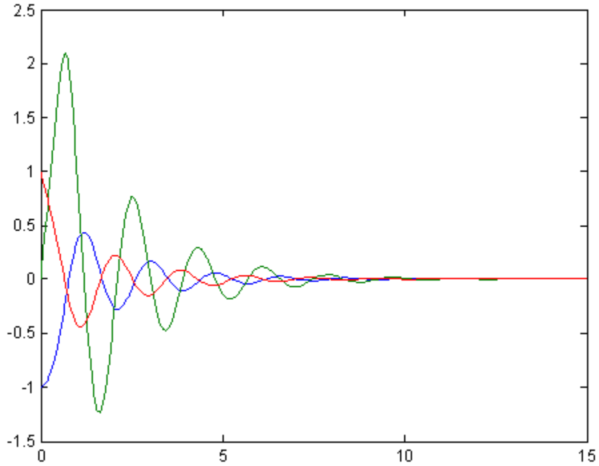


$k_3 = 1$

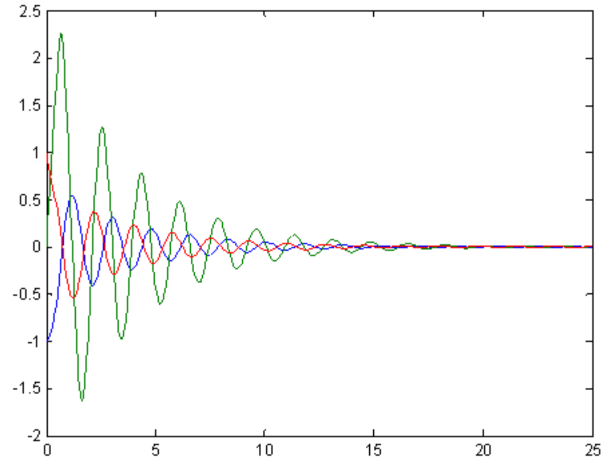


$k_3 = 10$

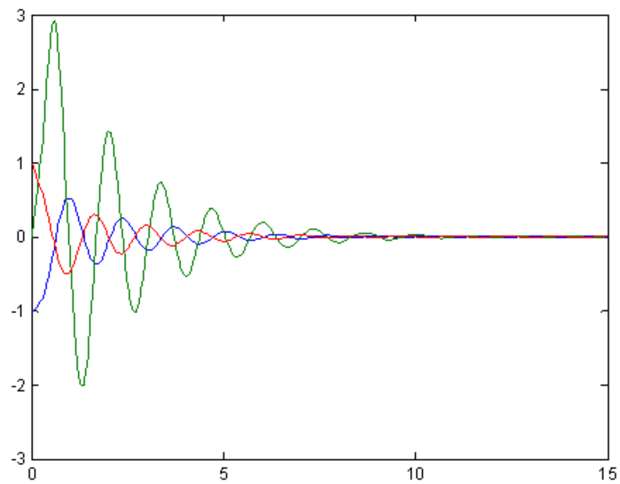
Fig. 1: Coefficient k_3 in the governing limits



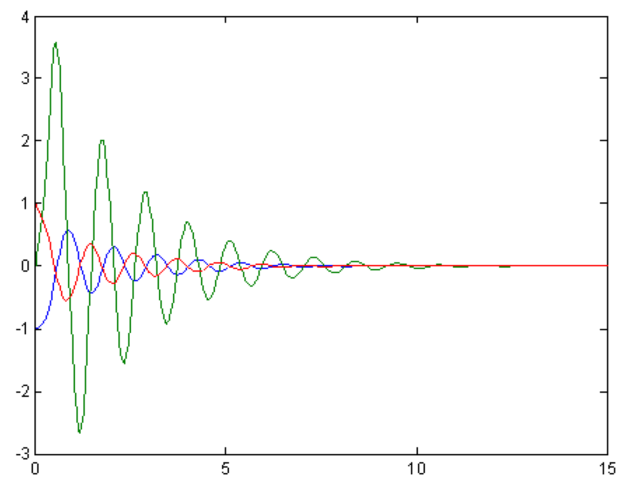
Parameter k_3 fixed, $\alpha_{mz}{}^{\delta}{}_{\beta} = -0.32$



Parameter k_3 fixed, $\alpha_{mz}{}^{\delta}{}_{\beta} = -2.16$



Parameter k_3 fixed, $\alpha_{y}{}^{\delta}{}_{\beta} = 19.5$



Parameter k_3 fixed, $\alpha_{mz}{}^{\delta}{}_{\beta} = 29.5$

Fig. 2: System with fixed parameters, $k_3 = 10$

CONCLUSION

Thus, the control system of aircraft motion with the increased potential of robust stability constructed in a class of single-parameter structurally steady maps provides stability for changes of uncertain parameters of the system.

It appears, the steady state (22) is globally asymptotically steady when performing conditions (25) and unstable at violation of conditions (25) and stability of a steady state (23) requires performance of conditions (30).

When $k_3 + \frac{a_1 - a_4}{a_3 - a_5} = 0$ there is a branching and there are new steady branches.

In other words, branches (23) appear as a result of bifurcation while the steady state (22) loses stability and these branches are steady. Stationary states (22) and (23) at the same time don't exist. It allows to increase the potential of robust stability of system in the conditions of uncertainty of parameters.

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