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Research Article

The Development of Mathematical Modeling for Solving Problem of Assessment of the Stability of Nonlinear Systems with Slowly Changing Parameters

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Abstract: The major objective of this study is to obtain mathematical models for evaluation behavior of non-linear system, parameters non-linear elements are taken to be time change. The problem is solved by transaction from description at non-linear non-stationary system in space of variable states to description of its behavior in space of parameter increments by means of apparatus of Sensitivity functions. Using this apparatus, we transit from non-linear differential equation, to describing behavior system as a space of variable states to linear description at the systems increment change at these parameters. For overcoming the generalized functions which appear invariably during obtaining the sensitivity function, we used describing function. The main study also is to develop a mathematical model to estimate the stability of electric drive relay action and to identify areas of its robust stability when exposed to uncontrolled parametric perturbations described by harmonic laws.

Keywords: Describing function, harmonic linearization, nonlinear non-stationary systems, stability, sensitivity functions

INTRODUCTION

The main problem that arises in the design automation of production facilities and processes is ensuring the stable operation of such systems, especially when exposed to perturbations that are poorly or not controlled, leading to changes in the numerical values of the parameters of such systems (Mokin and Yukhymchuk, 1999).

Problems of management, measurement and control in the real environment usually have to be solved by considering a degree of uncertainty in parameters relevant to facilities management, measurement and control. Disturbances external to them cannot be controlled. Therefore, when solving this class of problems, the researcher does not deal with the actual values of the majority of primary parameters and their estimations. An urgent scientific challenge is to identify these boundary changes of the primary parameters, which is the stability property of the class of systems under consideration (Mokin and Yukhymchuk, 1999).

The main contribution of this research is to develop a mathematical model to estimate the stability of electric drive relay action and to identify areas of its robust stability when exposed to uncontrolled parametric perturbations described by harmonic laws.

LITERATURE REVIEW

Continuously in the past, the nonlinear problem is received considerable attention and thus different kinds of solving methods are developed sufficiently (Amin *et al*., 2010; Homm and Rabenstein, 1997), but there is still not a complete solution system until now.

According to Merklin (2003), the stability of automatic control may be determined by evaluating the robustness of systems with respect to properties of stability, which have been investigated only for linear stationary systems. Methods have not yet been developed for nonlinear control systems to solve the problem analytically, determining the asymptotic stability for an arbitrary change of system parameters and the range of initial parameters under which stability is maintained.

In Mokin and Yukhymchuk (1993) an approach was proposed to solve the problem of estimating the stability in linear law change parameters of nonlinear non-stationary systems.

The authors of Blekhman and Fradkov (2001) and Polyak and Shcherbakov (2002) solved the problem of robust stability for a wide class of systems, but the results they obtained do not allow the investigation of the influence of initial parameters on the stability of the property (Yukhymchuk, 1997a).

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The authors of Mokin and Yukhymchuk (1999) developed a mathematical model of robust stability and sensitivity of nonlinear, non-stationary systems of automatic control. They solved the problem of estimating stability in the parameter increments space for some control systems with variable parameters and defined ranges of initial parameters of the system to ensure the stability of the property.

RESULTS AND DISCUSSION

Consider the problem of determining the stability of the example of the electric relay action when exposed to electromagnetic interference. In this case, in contrast to the results obtained in Mokin and Yukhymchuk (1993), the initial settings are changed by the harmonic laws.

According to Mokin and Yukhymchuk (1999), a block diagram of a simple variant of electric drive relay action is shown in Fig. 1, where N is the nonlinear part of the system, L is the linear part of the system, y is the output system error signal and x is a signal that is tracked.

The nonlinear part of the system is a relay without a hysteresis element and the output signal Z are given by the relation:

$$Z = \begin{cases} b_0, & \text{if } \varepsilon \geq 0 \\ -b_0, & \text{if } \varepsilon \leq 0 \end{cases} \qquad (1)$$

$$\varepsilon = X_0 - Y$$

The zone insensitivity of the relay element is selected based on the requirements for the electric resistance and the dynamic characteristics of the system.

The linear electric drive part is determined by a transfer function of the form:

$$w(p) = \frac{k}{s^2+1} \qquad (2)$$

where, K is the transfer coefficient of the executive motor and reducer.

We know that each relay actuator is implemented using electronic circuits whose parameters change under the influence of both external and internal uncontrollable parametric perturbations. These changes will inevitably lead to changes in the parameters of the output relay element included in the block diagram of the electrical circuit. This can lead to the instability of such systems. Therefore, it is of interest in the design to investigate the effect of changing the initial parameters of the relay element on the electric resistance and to identify useful changes in the primary parameters of the relay element, which provide the electrical resistance property.

To obtain numerical results, we assume that the parameters of the relay element change follow the law:

$$B(t) = b_0 \pm b\ \sin(\Omega t), b \ll b_0 \qquad (3)$$

where, $A \geq b_0$. These laws are characteristic changes upon exposure to electromagnetic disturbances, which are typical of industrial conditions.

Obviously, if (3), an analytical solution by known methods is difficult to obtain for the dynamics described by the nonlinear non-stationary electric differential equation. Therefore, to determine the impact of changing the primary parameters on the stability of the electrical circuit requires the application of a numerical methods approximation in parameter increments space.

Space increment settings were constructed with appropriate sensitivity functions, which use the results of generalization of the method described in (Yukhymchuk, 1997b).

To link this with the static characteristic form (1) under the laws of parameters (3) as in Yukhymchuk (1997a) and Batiha and Yukhymchuk (1994), we determined that its describing function is defined by the expression:

$$a(A,b,t) = \frac{4}{\pi A}\left(\int_0^{\frac{\pi}{2}}(b_0 + b.\sin(\Omega t)\sin(\omega t)d(\omega t)\right.$$

$$a(A,b,t) = \frac{4}{\pi A}\left(b_0 + \frac{b}{2}\left[\frac{1}{\left(\frac{\Omega}{\omega}-1\right)}\sin\left(\frac{\pi}{2}\left(\frac{\Omega}{\omega}-1\right)\right) - \frac{1}{\left(\frac{\Omega}{\omega}+1\right)}\sin\left(\frac{\pi}{2}\left(\frac{\Omega}{\omega}+1\right)\right)\right]\right) \qquad (4)$$

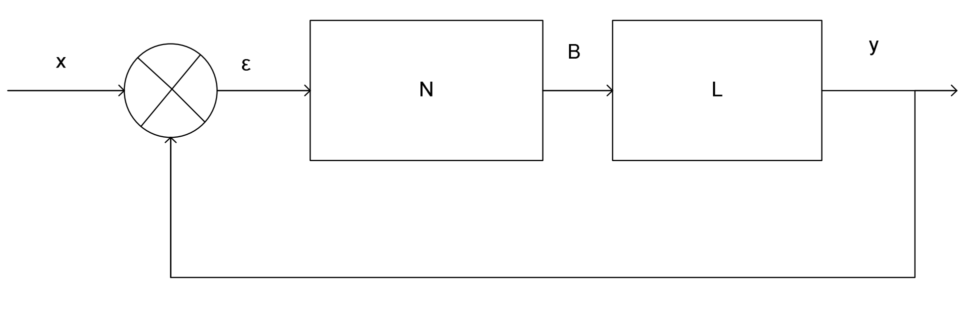

Fig. 1: Block diagram of electric relay

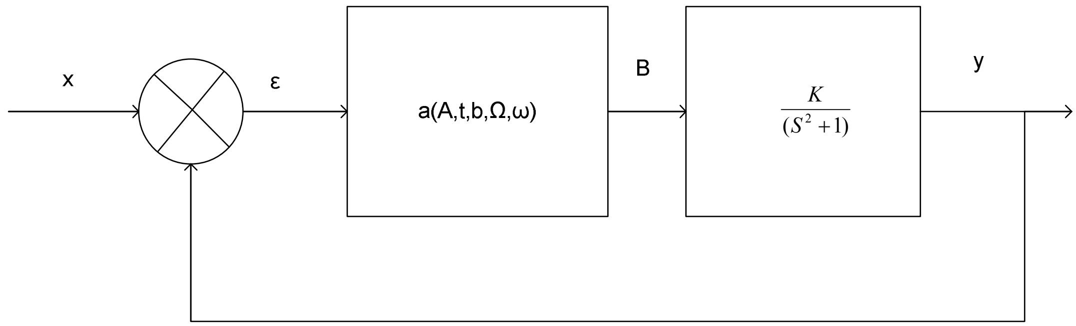

Fig. 2: Refined block diagram of electric relay

where, A is the amplitude of the input signal, the frequency of the input signal is ω and the range perturbation is Ω.

Using (4), the electric block diagram relay action is presented in the form shown in Fig. 2.

According to Mokin and Yukhymchuk (1999), the differential equation that relates the input x (t) and output y (t) electrical signals will look like:

$$\frac{d^2y}{dt^2} + y(t) + k.a(A,b,t).y(t) = k.a(A,b,t).x(t) \quad (5)$$

To assess the impact of the modified relay element on the output of the system, we proceed from the description (5) to describe the dynamics of the electric parameters in space increments, using algorithms developed in Yukhymchuk (1997b). To do this, using (5), we write the equation of dynamics relative to the sensitivity functions of the output signal to change the output signal B (t) in the relay link (Mokin and Yukhymchuk, 1991):

$$\frac{d^2}{dt^2}\left(\frac{\partial y}{\partial b}\right) + \frac{\partial y}{\partial b} + k\frac{\partial a(A,b,t)}{\partial b}\,y(t) + k.a(A,b,t)\frac{\partial y(t)}{\partial b} = k\frac{\partial a(A,b,t)}{\partial b}\,x(t) \quad (6)$$

Using the notation $\frac{\partial y}{\partial b} = U_b$, the expression (6) can be written as:

$$\frac{d^2U_b}{dt^2} + U_b + k.a(A,b,t)U_b = k\big(x(t) - y(t)\big)\frac{\partial a(A,b,t)}{\partial b} \quad (7)$$

The partial derivatives included in (7) subject to (4), have the form:

$$\frac{\partial a(A,b,t)}{\partial b} = \frac{2}{\pi A}\left\{\frac{1}{\left(\frac{\Omega}{\omega}-1\right)}\sin\left(\frac{\pi}{2}\left(\frac{\Omega}{\omega}-1\right)\right) - \frac{1}{\left(\frac{\Omega}{\omega}+1\right)}\sin\left(\frac{\pi}{2}\left(\frac{\Omega}{\omega}+1\right)\right)\right\} \quad (8)$$

To solve the problem of determining the general conditions of stability, consider the stability of free movement of the nonlinear system (with x (t) = 0). If we also consider free vibrations, which were established at the steady state point (y (t) = 0), the analysis of the relations (5) and (7) shows that the equation of free motion around the steady state point in the parameter space increases for the relay links have the same form as the equation of free motion (Mokin and Yukhymchuk, 1993):

$$\frac{d^2y}{dt^2} + y(t) + k.a(A,b,t).y(t) = 0 \quad (9)$$

Analysis of Eq. (10) shows that a necessary condition for absolute stability of the system is the following inequality:

$$a(A,t,b,c,\omega,\Omega) > 0 \quad (10)$$

Given (4), we analyze the inequality (10) for the laws for modified relay links, which are described in (3), if the relay element is changing such that $B(t) = b_0 + b\ \sin(\Omega t)$ then the solution of inequality (10) takes the form:

$$B(t) > b_0 - \frac{2.b_0.\sin(\Omega t)}{\frac{1}{\left(\frac{\Omega}{\omega}-1\right)}\sin\left(\frac{\pi}{2}\left(\frac{\Omega}{\omega}-1\right)\right) - \frac{1}{\left(\frac{\Omega}{\omega}+1\right)}\sin\left(\frac{\pi}{2}\left(\frac{\Omega}{\omega}+1\right)\right)} \quad (11)$$

If $B(t) = b_0 - b\ \sin(\Omega t)$ then the solution of inequality (11) has the form:

$$B(t) > b_0 + \frac{2.b_0.\sin(\Omega t)}{\frac{1}{\left(\frac{\Omega}{\omega}-1\right)}\sin\left(\frac{\pi}{2}\left(\frac{\Omega}{\omega}-1\right)\right) - \frac{1}{\left(\frac{\Omega}{\omega}+1\right)}\sin\left(\frac{\pi}{2}\left(\frac{\Omega}{\omega}+1\right)\right)} \quad (12)$$

We illustrate the inequality (12) by numerical examples. Suppose for $a(A,t,b,c,\omega,\Omega)$) that A = 4, ω = 5, b_0 = 2 and Ω = 50. For the set of parameters $B(t) = b_0 + b\ \sin(\Omega t)$, the stability region is shown in Fig. 3 and for the set of parameters $B(t) = b_0 - b\ \sin(\Omega t)$, the stability region is shown in Fig. 4.

Since Eq. (7) describe the change $\frac{\Delta y}{\Delta b}$, in time, the stability of solutions is necessary for a small change in the parameters of the nonlinear element b caused a small change in the speed of the initial value. Thus, if

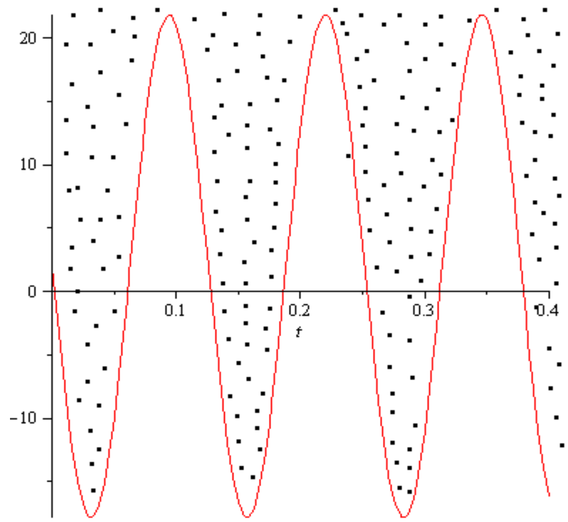

Fig. 3: Stability region of the electric relay with $B(t) = b_0 + b\ \sin(\Omega t)$

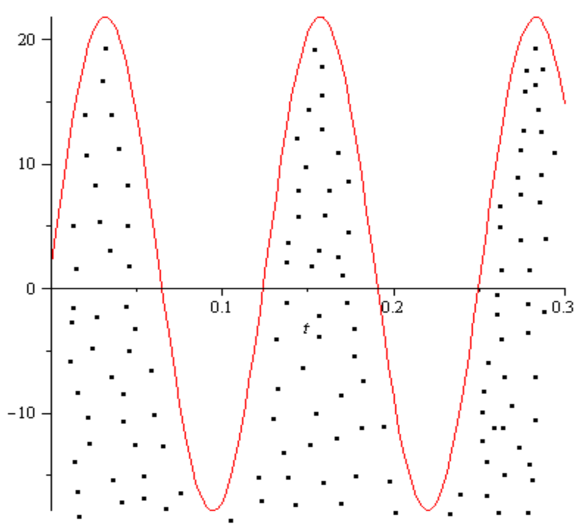

Fig. 4: Stability region of the electric relay with $B(t) = b_0 - b\ \sin(\Omega t)$

the system is stable relative to the initial value of the derivative, it will be stable in relation to the initial value (Yukhymchuk, 1997a).

Due to the fact that Eq. (7) is linear non-stationary differential equations, the definition of sufficient conditions for the stability of motion of a dynamical system in this case can be obtained by studying the asymptotic behavior of solutions of the corresponding system of equations for the components of the canonical solution of the equation of free movement (Yukhymchuk, 1997b).

In Mokin and Yukhymchuk (1999) it is shown that the performance obtained by the necessary conditions for the stability of solutions of equations of canonical components will be limited. That is a sufficient condition for the stability of the electric relay action performed.

CONCLUSION

Mathematical modeling was performed to determine the electric field of robust stability when exposed to uncontrolled parametric perturbations given by the harmonic laws. The model was then used to investigate the effect of changing the initial parameters of the relay element on electrical resistance.

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