

## Research Article

### Optimal Fracturing Parameters Condition of Integrated Model Development for Tight Oil Sands Reservoir with 2D Fracture Geometry Using Response Surface Methodology

<sup>1</sup>Nguyen Huu Truong and <sup>2</sup>Wisup Bae  
<sup>1</sup>PetroVietnam University (PVU), Vietnam  
<sup>2</sup>Sejong University, Seoul, South Korea

**Abstract:** In this study, oil production in tight oil reservoir was declined due to high heterogeneity, complicated and complexity of reservoir of the low permeability reservoir ranges from 0.1 md to 5 md and reservoir porosity ranges from 10 to 18% lead to low fracture conductivity among the fractures. The challenge deal with this problem is to stimulate the reservoir of hydraulic fracturing for maximum oil production is necessary for the study. The application of the central composite design and response surface are the best tool in order to optimize the operating parameters based on the maximum net present value and through the series calculation, the optimal operating parameters of hydraulic fracturing have been determined of 46 bpm of injection rate, 88.5 min of the injection time and  $0.002 \text{ ft/min}^{0.5}$  of the leak-off coefficient and the maximum net present value of 46.5 \$mm. Finally, the integrated model development of hydraulic fracturing includes of the normal faulting stress regime, fracturing fluid selection and fluid model, proppant selection, fracture geometry, pressure model, material balance, fracture conductivity and simulation production of fractured well and unstimulated well that have been presented in this study. With the using two dimensional Perkins-Kern-Nordgren fracture geometry models coupled with carter leak-off as the 2D PKN-C has been used to account for leak-off coefficient, spurt loss in term of power law parameters to propagate the fracture half-length, fracture width. The result of the fracture conductivity of fractured well at the fracture half-length of 1,940 ft and average fracture width of 0.32 in by series calculated propped fracture concentration of  $1.63 \text{ lb/ft}^2$  and fracture closure pressure of 4,842.59 psi to fracture conductivity of 6,200 md-ft, which value is measured by the laboratory experiment. The post-fracture production has been shown the fold of increase oil of 18.7 and the oil production rate of fractured well demonstrated much rising compared to oil production rate of unstimulated case.

**Keywords:** 2D Fracture geometry, design of experiment, integrated model development, optimization of operating parameters, response surface methodology

## INTRODUCTION

Hydraulic fracturing stimulation is widely used in the petroleum industry today for rising oil production merits. The first fractured treatment application designed specially to stimulate the reservoir of production well that technology was conducted in the Hugoton gas field on July 1947, on Kelpper well was located in Grant County, KS. At this time due to equipment limitations, the limitations of techniques select the pay zone for hydraulic fracturing stimulation and pump horsepower limitations for fractured well at high reservoir depth, high fracture closure pressure. From 1950 to 1960, hydraulic fracturing stimulation was continuously developed of the facilities at which high pump power allowed to pump high injection rate, high pressure to reservoir at high well depth and high fracture closure pressure. At that time, the hydraulic fracturing fluid was continually developed of many

type polymers with high molecular weight of high viscosity is to high pressure and high temperature of the reservoir with high fluid efficiency. At the present, the hydraulic fracturing.

Technology has been continuously developed to stimulate reservoir with multistage fracturing in a horizontal well and multistage for multi-layered in a vertical well. Thus, the hydraulic fracturing technology has become a unique tool for stimulation reservoir in the petroleum industry today. In the previous literatures, many authors introduced the techniques to stimulate tight oil sandstone reservoir for maximum oil production to find optimal fracture half-length. Economides *et al.* (2002) presented the unified fracture design for optimal fracture geometry based on maximum net present value. With the optimal operating parameters of hydraulic fracturing of the injection rate, injection time and the leak-off coefficient were not yet studied of the field at which the operating parameters

have been followed in the field experience. Furthermore, tight oil reservoir is low permeability in the ranges from 0.1 md to 5 md and low porosity reservoir ranges from 10 to 18% and short effective wellbore radius that is low fracture conductivity as the oil production declined. The big challenge deal with the low fracture conductivity is to simulate the oil reservoir of hydraulic fracturing in order to enhance oil production. Due to tight oil reservoir is low permeability, low porosity, the two dimensions Perkins and KernNordgren Carter (2D PKN-C) fracture geometry model has been selected of the study account for leak-off coefficient, spurt loss in term of power law parameters. With the integrated model development of hydraulic fracturing includes of the normal faulting stress regime, fracturing fluid selection and fluid model, proppant selection, fracture geometry model, pressure model, material balance, fracture conductivity model and finally simulation production for fractured well and unstimulated well. In this study, the application of the central composite design of these variables and the Response Surface Method (RSM) with the response of the net present value is to find the optimal operating parameters of hydraulic fracturing of the injection rate, injection time and the leak-off coefficient for maximum production performance. The result of the optimal operating parameters is very advantage of the hydraulic fracturing stimulation in which the integrated model development of hydraulic fracturing applied to the tight oil reservoir in the detail.

**MATERIALS AND METHODS**

**Integrated model development and application:** In order to predict the fracture dimensions of fracture half-length and fracture width, treatment design parameters, the integrated model development of hydraulic fracturing of the tight oil reservoir using the 2D PKN-C fracture geometry model coupled with cater leak-off are required of the study. The integrated model has been shown in the Fig. 1 and discussed in detail of the research.

**Normal faulting stress regime:** During proppant slurry is pumped into the well with high pressure for producing the fracture width and fracture propagation

which are directed to the smaller of the principal stress. This is basically for design the net pressure at which the bottom hole pressure overcomes the closure pressure to form the net pressure with the positive. The value minimum *in-situ* stress is to proppant selection, selection pump power of hydraulic fracturing in the field. Thus, the good solution can be saved either the cost of the pump horsepower of the minimum in-situ stress or increase the net fracture pressure. This is lead to more fracture growth and increase the fracture conductivity among the fractures. Moreover, in a normal faulting stress regime, the vertical stress is usually maximized value one and the second value is the horizontal stress and the rest of minimum horizontal stress is smaller than other one.

**Fracturing fluid selection and fluid model:** Currently, the fracturing fluid selection is a very important part in order to success of proppant transport, porppant settling should not exceed of 10 ft/hr (Jiang *et al.*, 2003a, 2003b) and proppant settling is followed the Stoke’s Law of proppant slurry transport from the surface to the fractures. The idea fracturing fluid is compatible with the rock properties in the tight oil reservoir, sandstone reservoir and compatible with fluid flow of the reservoir, reservoir pressure and reservoir temperature. One the other hand, the fracturing fluid generated pressure to transport proppant slurry for open fracture and producing the fracture growth. Moreover, fracturing fluid should be minimized pressure drop in the pipe system for increase the pump horse power with the aim is increased the net fracture pressure. In fracturing fluid system, the breaker additive would be added to the fluid system to clean up the fractures after treatment. Due to the tight oil reservoir is high temperature and the rocks in the tight oil reservoir consists mainly of sandstones and conglomerate. Therefore, the Borate-Crosslink of 30 pptg Hydroxypropyl Guar (HPG) with 8 pptg Per sulfate Breaker of  $Na_2S_2O_8$  was selected for fracturing fluid to stimulate the tight oil formation. To depict precisely the fracture geometry during pumping, the power law fluid model would be selected in this study. Then the Non-Newtonian fracturing fluid model is given by:

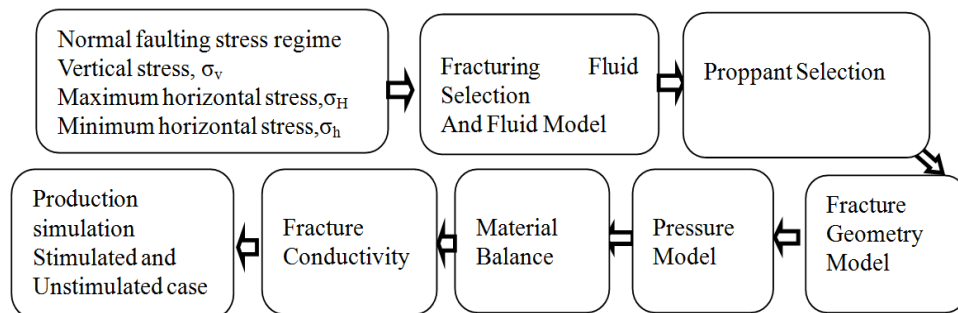


Fig. 1: Integrated model development of hydraulic fracturing for tight oil reservoir

$$\tau = K\gamma^n \tag{1}$$

where,  $\tau$ -shear stress,  $\gamma$ -shear rate, K-consistency coefficient index, n-flow behavior index of non-dimensional but related to the viscosity of the non-Newtonian fracturing fluid model (Refer to Valko and Economides, 1995; Economides and Martin, 2007), The power law model can be expressed by:

$$\begin{aligned} \text{Log } \tau &= \text{log } K + n \text{ log } \gamma \\ \text{Slope} &= [(N \sum XY) - (\sum X \sum Y)] / [(N \sum X^2) - (\sum X)^2] \\ \text{Intercept} &= (\sum Y - n \sum X) / N \end{aligned}$$

where,

- X = log  $\gamma$
- Y = log  $\tau$
- N = Data number
- Thus n = Slope
- K = Exp (intercept)

**Proppant selection:** Proppant characteristics such as proppant type, proppant size, proppant density, proppant porosity, proppant pack permeability and proppant sphere, half-sphere are very important role in order to decide the fracture conductivity and dimensionless fracture conductivity under fracture closure pressure of the field with the specific proppant fracture concentration inside fracture. To predict the fracture conductivity of the fractures of the fractured well, the proppant fracture concentration per unit fracture area in square feet and closure pressure of the fractures have been known in the previously. By laboratory experiment is to find the fracture conductivity or by using simulation fracture conductivity is to find the value of fracture conductivity. Usually, the high closure pressure should be selected the high strength proppant this means the strength of proppant is much more than closure pressure of the fracture. However, depend on the economics for selected proppant is based on the strength proppant which is slightly than closure pressure. If selected strength proppant is too much higher than closure pressure which may lost economic. In this study, the fracture closure pressures up to 4,842.59 psi, then this is basically selected strength proppant (Economides *et al.*, 2001). In this study, the intermediate strength proppant (ISP)-CARBO-Lite ceramics of 20/40 has been selected at which strength proppant is more than fracture closure pressure of 4,842.59 psi (Economides and Martin, 2007; Williams *et al.*, 1979).

**Fracture geometry model:** In this study, the 2D PKN fracture geometry model as (two dimension Perkins and Kern, 1961; Nordgren, 1972) has been used in order to predict the fracture dimensions of the fracture half-

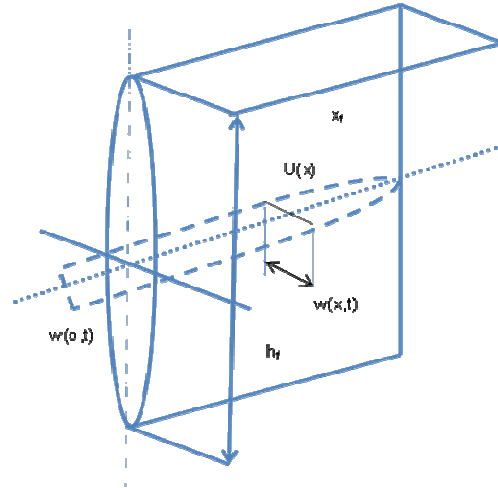


Fig. 2: The PKN fracture geometry

length and fracture width as presented in the Fig. 2. Due to tight oil reservoir is low permeability, low porosity, complicated reservoir, complexity, high heterogeneity. Thus, the 2D PKN-C fracture geometry mode is proper for hydraulic fracturing stimulation of the reservoir with the target of fracture half-length and more conductivity in the post fracture of the fractured well. Lately, the Carter solution II (Howard and Fast, 1957; Cinco-Ley *et al.*, 1978) account for total leak-off coefficient and spurt loss in term of the power law parameters of the flow behavior index and consistency index. Therefore form the new model as 2D PKN-C (Two dimension PKN; Perkins and Kern (1961) and Nordgren (1972)) coupled with Carter leak-off. In order to investigate precisely fracture dimension as fracture half-length and the near wellbore fracture width of power law fracturing fluid model has been presented here. The maximum fracture width model of the 2D PKN-C was presented as equal model below:

For the power law fracturing fluid model, the maximum fracture width at near wellbore in term of power law parameter can be computed as given:

$$w_{w,o} = 9.15 \frac{1}{2n+2} x^{3.98} \frac{n}{2n+2} \left( \frac{1+(n-1)n}{n} \right)^{\frac{n}{2n+2}} K^{\frac{1}{2n+2}} \left( \frac{(q_f/2)^n h_f^{1-n} x_f}{E'} \right)^{\frac{1}{2n+2}} \tag{2}$$

where,

$$E' = \frac{E}{1 - \nu^2}$$

where, n is the flow behavior index (dimensionless), K is the consistency index (Pa – sec<sup>n</sup>), E is the young modulus in psi, E' is the plain strain in psi,  $\nu$  is the Poisson's ratio. Moreover, the power law parameters are correlated with fluid viscosity of fracturing fluid (Rahman, 2008):

$$\begin{aligned} n &= 0.1756(\mu)^{-0.1233} \\ K &= 47.880 \times (0.5\mu - 0.0159) \end{aligned}$$

In which, E is the young modulus in psi,  $\nu$  is the Poisson's ratio,  $E'$  is the plain strain in psi.

The elliptical fracture geometry of the 2D PKN-C is expressed the average fracture width by proposing a shape factor of  $\pi/5$ . Usually, the average fracture width ( $\bar{w}$ ) along the fracture length is formed by  $w_{w,o} \times \pi/5$ . By using the Carter solution II, to solve the material balance at which the constant injection rate is equal to fracture volume plus with total fluid loss and pad volume injected to the well, therefore the fracture half-length is performed as equation below:

$$x_f = \frac{(2S_p + \bar{w}) q_i}{4\pi C_L^2 h_f} \left[ \exp(\beta^2) \operatorname{erfc}(\beta) + \frac{2\beta}{\sqrt{\pi}} - 1 \right] \quad (3)$$

where,

$$\beta = \frac{2C_L \sqrt{\pi t}}{w + 2S_p} P_{net} = \frac{E'}{2h_f} w_{w,o} \quad (4)$$

In Eq. (3) is presented the fracture half-length to present the fracture propagation account for the overall fluid leak-off, spurt loss in term of power law parameters of flow behavior index (n) and consistency index (K). From the close of the equation is either fracture half-length or injection time can be easily determined using a numerical root-finding method. To calculate the fracture half-length, the injection time, pump rate, leak-off coefficient, fracture height and spurt loss in term of power law parameters of flow behavior index (n) and consistency index (K) are previously known by using iterative method. To calculate one of the rest parameters of Eq. (3), all parameters have been known as power law parameters, pump rate ( $q_i$ ), fracture height ( $h_f$ ), plain strain modulus ( $E'$ ), the overall leak-off coefficient ( $C_L$ ) and spurt loss ( $S_p$ ). The net fracture pressure of the fracture is very important role to produce fracture dimensions of the fracture width, fracture half-length, fracture height. In the rest of the parameters are of fracture closure pressure, friction pressure losses in the pipe, pump rate are usually affected to pump horse power and net pressure. Usually, to reduce the friction pressure loss in the pipe system by adding the friction reducer of polymer concentration in the fracturing fluid is to increase the net pressure and prevent the erosion during proppant slurry pumping.

**Pressure model:** During proppant slurry is pumped into the well with high pressure and high flow rate for producing fracture growth and propagation the fractures. Usually, the fracture dimensions depend on the net fracture pressure in the fractures at which the net fracture pressure is the total of surface treating pressure plus with wellbore fluid pressure minus the total of the friction losses inside tubing, perforation, tortuosity and closure pressure. Accordingly, the fractures dimensions can be propagated when the bottom hole pressure overcomes the closure pressure of fractures or exceeding the closure pressure, which stresses starts to

act on the rock exceed the compressive or tensile failure of the rock. These stresses are related to the pump horse power and select the pump power required for enough injection pressure. In some case, if the fracture has high closure pressure with proppant slurry transport in high friction losses occasionally need to select the high pump power for hydraulic fracturing operation that spends a lot of the pump power for hydraulic fracturing injection not save the economics. In order to optimization design of pump power and reduce the pressure lost is impossible. The following net pressure model was expressed in detail of the model below (Rahman, 2008):

$$P_{net} = P_{inj} + P_{head} - P_{tubing\ friction} - \Delta P_{pf} - \Delta P_{tort} - P_c \quad (5)$$

$$HHP = \frac{(P_{tubing\ friction} + \Delta P_{pf} + \Delta P_{tort} + P_c - P_{head})}{40.8} \quad (6)$$

where,

- $P_{net}$  : The net pressure among the fractures
- $P_{inj}$  : The injection pressure as surface treating pressure
- $P_{head}$  : The wellbore fluid pressure due to its depth and its slurry concentration
- $P_{Tubing\ friction}$  : The tubing friction pressure lost due to the fracturing fluid effect on wellbore
- $\Delta P_{pf}$  : The pressure loss through the perforation
- $\Delta P_{tort}$  : The pressure loss due to tortuosity pressure effect

**Material balance:** The Cater solution for the material balance account for the leak-off coefficient, spurt loss, injection rate, injection time and power law parameters of flow behavior index and consistency index of n, K, respectively. During proppant slurry is pumped to the well to produce fracture growth and fracture propagation. In the particular, the material balance is presented as equation;  $V_i = V_f + V_l$ , where  $V_i$  is the total fluid volume injected to the well,  $V_f$  is the fracture volume that is required to stimulate reservoir and  $V_l$  is the total fluid volume losses to the fracture area in the reservoir. The fracture volume,  $V_f$ , is defined as two sides of the symmetric fracture by  $V_f = 2x_f h_f w_a$ , the fluid efficiency is defined by  $V_f/V_i$ . In 1986, Nolte proposed the relationship between the fluid volumes injected with pad volume and also proposed a model for proppant schedule. At the injection time t, the injection rate is entered into two wings of the fractures with q, the material balance is presented as the constant injection rate q is the sum of the different leak-off flow rate plus with fracture volume (Cinco-Ley *et al.*, 1978):

$$q = 2 \int_0^t \frac{C_L}{\sqrt{t-\tau}} \times \left( \frac{dA}{d\tau} \right) d\tau + 2S_p \times \frac{dA}{dt} + w \times \frac{dA}{dt} + A \frac{dw}{dt} \quad (7)$$

The fluid efficiency of fractured well at the time (t) is given by:

$$\eta = \frac{w_a h_f (w_a + 2S_p)}{4\pi C_L^2 h_f t} \times \left[ \exp(\beta^2) \operatorname{erfc}(\beta) + \frac{2\beta}{\sqrt{\pi}} - 1 \right] \quad (8)$$

**Fracture conductivity:** The value of fracture conductivity usually is measured from laboratory experimental (API standard) which is based on proppant type, proppant size, proppant shape, proppant damage factor, proppant permeability, proppant porosity are under closure pressure. The API standard for a test conductivity measures by linear flow through the proppant pack between steel plates under specific pressure is applied on it. Then the API standard conductivity test is usually tested at proppant fracture concentration of 2 lb/ft<sup>2</sup>. This is famous to measure conductivity which published and measured by Smith (1997) and Economides *et al.* (2002).

If the proppant pack permeability under closure pressure has been known of each proppant type was selected, then in-situ fracture conductivity can be calculated by:

$$\text{Fracture conductivity} = k_f \times w_p \quad (9)$$

For simulation fracture conductivity if the closure pressure, proppant fracture concentration in (lb/ft<sup>2</sup>) also can be calculated fracture conductivity, proppant permeability, proppant porosity under closure pressure.

**Dimensionless fracture conductivity:** The dimensionless fracture conductivity,  $F_{CD}$ , can be defined as Cinco-Ley *et al.* (1978) and Economides *et al.* (1994) is given by:

$$F_{CD} = \frac{k_f \times w_p}{k \times x_f} \quad (10)$$

In which:

- $k$  : The reservoir permeability in md and
- $x_f$  : The fracture length of fractured well in ft
- $k_f$  : The proppant permeability under closure pressure apply on the proppant laden
- $w_p$  : The propped fracture width at end of the job

**Transient production flow regime:** Based on the constant bottom hole pressure situation, the oil production from fractured well in transient flow regime can be calculated by Economides *et al.* (1994):

$$p_i - p_{wf} = \frac{162.6 q_0 B \mu}{k h} \left( \log t + \log \left( \frac{k}{\phi \mu c_t r_w^2} \right) + s_f - 3.23 \right) \quad (11)$$

In which,  $r_w'$  is the effective wellbore radius as given by:  $r_w' = r_w e^{-s_f}$ ,  $s_f$  is pseudo-skin is calculated by the relationship (Valko *et al.*, 1997):  $s_f = F - \ln \left( \frac{x_f}{r_w} \right)$ , where  $x_f$  is the fracture half-length and  $r_w$  is the wellbore radius. The F factor can be calculated by:

$$F = \frac{1.65 - 0.328u + 0.116u^2}{1 + 0.18u + 0.064u^2 + 0.005u^3} \quad (12)$$

where,  $u = \ln(F_{CD})$  and  $F_{CD}$  is the dimensionless fracture conductivity which is calculated by  $F_{CD} = \frac{k_f w_p}{k x_f}$ , also  $F_{CD}$  is related to proppant number which is along the penetration ratio ( $I_x = 2x_f/x_e$ ) and  $k_f w_p$  is the fracture conductivity which can be calculated by the laboratory experiment or conductivity simulation when knows a proppant fracture concentration of lb/ft<sup>2</sup> inside fracture under closure pressure apply on the proppant laden. Basically, the proppant number is defined by (Economides *et al.*, 2002):

$$N_{propp} = \left( \frac{2k_f}{k_{res}} \right) \times \frac{V_{prop}}{V_{res}} \quad (13)$$

where,

- $k_f$  = The effective proppant pack permeability
- $k_{res}$  = The reservoir permeability
- $V_{prop}$  = The propped volume in the pay zone (two wings, including void space between the proppant grains)
- $V_{res}$  = The drainage volume

Usually, in the transient oil production period is often short time oil production.

**Net Present Value (NPV) model:** Veatch (1986) presented a comprehensive the list of the various techniques. Meng and Brown (1987) were proposed to calculate the Net Present Value (NPV) as an approximately treatment optimization approach. Balen *et al.* (1988) presented a series of the parametric studies and the components of the NPV calculation. Net present value is defined as the revenue from the production forecast of the fractured well minus the production forecast of the unstimulated well and the total treatment cost in current dollars. In this research, the net present value of the future revenue can be calculated as below equation (Economides and Martin, 2007):

$$NPV = \sum_{j=1}^N \frac{(V_F)_j}{(1+i)^j} - \sum_{j=1}^N \frac{(V_0)_j}{(1+i)^j} - C_{tr} \quad (14)$$

$$C_{tr} = C_{pr} + C_{tfl} + C_{pu} + FC \quad (15)$$

In order to calculate total fluid cost, Rahman *et al.* (2003) presented to calculate total fluid volume without proppant as equation:

$$V_{tfl} = V_{pad} + V_{fl} \quad (16)$$

To compute the fracturing fluid volume in proppant slurry stage, then the amount fracturing fluid volume is only mixed with dry proppant in the slurry stage is given by:

$$V_{fl} = \frac{W_{pr}}{P_c} \quad (17)$$

In which,

$$\bar{P}_c = \eta P_c.$$

where,

- $C_{tr}$  = The total treatment cost
- $C_{pr}$  = The total proppant cost
- $C_{tfl}$  = The total fluid cost
- $C_{pu}$  = The total pumping horse power cost
- FC = The total fixed cost
- $\bar{P}_c$  = The proppant concentration in ppg
- $P_c$  = The proppant concentration end of the job (EOJ), finally
- $\eta$  = The fluid efficiency
- NPV = The net present value of a fractured well
- N = The number of year production
- $V_F$  = The fracture value production revenue of a fractured well reservoir
- $V_0$  = The fracture value production revenue of an unstimulated case reservoir
- i = The discount rate in %

**Response Surface Methodology (RSM) and Central Composite Design (CCD):** Response surface methodology is a tool very important to application to Design Of Experiment (DOE) to provide adequate and reliable measurement of the response and determine the optimization values of the independent variables in order to produce a maximum or minimum response (Cornell, 1990; Montgomery, 2001; Myers and Montgomery, 2002; Myers *et al.*, 2008). Then the single response modeled using the RSM corresponded to the independent variables. By using the RSM and application of Design of Experiment (DOE), a quadratic polynomial equation is developed to evaluate the response of products as a function of independent variables with interactions variables (Box and Draper, 1987). Currently, the response for the quadratic polynomial is expressed:

$$Y = \beta_0 + \sum_{i=1}^k \beta_i X_i + \sum_{i=1}^k \beta_{ii} X_i^2 + \sum_{i < j} \beta_{ij} X_i X_j + \varepsilon \quad (18)$$

In which,

- Y = The predicted response
- $\beta_0$  = Intercept coefficient
- $\beta_i$  = The coefficients of linear terms

- $\beta_{ii}$  = The coefficients of squared terms
- $\beta_{ij}$  = The coefficients of the interactions terms
- $X_i$  and  $X_j$  = Presented the coded independent variables

In this study, a second-order polynomial model is obtained by using the uncoded independent variables in which model would be formed such as:

$$\hat{Y} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_{11} X_1^2 + \beta_{22} X_2^2 + \beta_{33} X_3^2 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{23} X_2 X_3 \quad (19)$$

The coefficient of the model for the response is investigated by using multiple regression analysis technique involved of the RSM. Fit the quality of the model is judged from their coefficient of the correlations and determination (Meyer Fracturing Simulation, Mfrac Software).

The Design of Experiment (DOE) techniques commonly is used for process analysis and the models usually are the full factorial, partial factorial and central composite rotatable designs. An effective alternative to the factorial design is the Central Composite Design (CCD), which originally was developed by Box and Wilson and improved by Box and Hunter (1957). The CCD was widely used as a three-level factorial design, requires much fewer tests than the full factorial design and has been provided to be sufficient as describing the majority of steady state products of response. Currently, CCD is one of the most popular classes of design used for fitting second-order models. The total number of tests required for is  $2^k + 2k + n_0$ , including the standard  $2^k$  factorial points with its origin at the center,  $2k$  points fixed axially at a distance, say  $\beta$  ( $\beta = 2^{k/4}$ ), from the center to generate the quadratic terms and replicate tests at the center ( $n_0$ ), where k is the number of independent variables (Table 1 and 2) (Meyer Fracturing Simulation, Mfrac Software).

**Application to a sandstone reservoir:** The integrated model development of hydraulic fracturing has been presented in Fig. 1 for typical tight oil, sandstone reservoir having a reservoir permeable layer between the upper bound and lower bound of the reservoir, where the upper bound and lower bound have high stress. This is just taken from different sources

Table 1: Proppant selection data

Parameter	Value
Proppant type	20/40 CARBO-Lite
Specific gravity	2.71
Proppant strength	Intermediate strength
Proppant diameter	0.0287
Packed porosity	0.35
Conductivity damage factor	0.5
Fracture conductivity at closure pressure of 4,842.59 psi of proppant fracture concentration of 1.63 lb/ft <sup>2</sup>	6,200 md-ft

Table 2: Economic data

Parameter	Value
Fracturing fluid cost, \$/gal	1
Proppant cost, \$/lb.	1
Hydraulic horse power cost, \$/hhp	20
Fixed cost, \$	15,000
Revenue discount rate, i, %	10
Oil price, \$/bbl	100

Table 3: Reservoir parameters

Parameter	Value
Target fracturing depth, ft	10,000
Reservoir drainage area, acres	200
Reservoir drainage radius, ft	1,665.27
Wellbore radius, ft	0.328
Reservoir height, ft	75
Reservoir porosity, %	15
Reservoir permeability, md	0.5
Reservoir fluid viscosity, cp	1.5
Oil formation volume factor, RB/STB	1.1
Total compressibility, $\text{psi}^{-1}$	$1.00 \times 10^{-6}$
Initial reservoir pressure, psi	5,500
Flowing bottom hole pressure, psi	3,500
Closure pressure, psi	4,842.6

Table 4: Fracturing fluid parameters and optimal operating parameters

Parameter	Value
Fracture height, hf, ft	70
Sandstone Poisson's ratio	0.25
Leak-off coefficient, $\text{ft}/\text{min}^{0.5}$	$2.00 \times 10^{-3}$
Young's modulus, psi	$3.00 \times 10^3$
Pumping rate, bpm	46
Pumping time, minutes	88.5
Spurt loss, in	0
Proppant concentration end of the job, ppg	8
Flow behavior index, n	0.55
Consistency index, K, $(\text{lb} \cdot \text{s}^n / \text{ft}^2)$	0.04
Fracturing fluid type	Borate-Crosslink of 30 pptg HPG in 8 pptg Persulfate Breaker of $\text{Na}_2\text{S}_2\text{O}_8$
Proppant type	ISP, CARBO-Lite Ceramics 20/40, 169lb/ft <sup>3</sup>

(Economides *et al.*, 1994) to investigate the hydraulic fracturing treatment parameters. The sandstone layer has underlying and overlying shale layer is fractured in single stage with the reservoir depth range of 9,962.5 - 10,037.5 ft (Table 3 and 4).

**Design of experiment for operating parameters of hydraulic fracturing:** The injection rate of hydraulic fracturing for tight oil reservoir is followed the field experience at which the injection rate was considered in the range of 30 bpm to 50 bpm of the variable  $X_1$  (Rahman, 2008). Usually, the injection rate is much effect to the fracture half-length as when the injection

rate is increased; the fracture half-length is increased as more fracture conductivity as an increase in the Net Present Value (NPV). Whereas, when the decrease in the injection rate is to decrease the fracture half-length as decrease the fracture conductivity of course decrease the net present value (Yu and Rahman, 2012). Similarly, the effect of injection time on the fracture half-length was presented by Yu and Rahman (2012). The research was depicted that the increase in the injection time is to the fracture half-length is increased. Whereas, the decrease in the injection time leads to decrease in the fracture half-length due to the injection time is directly proportional to fracture half-length (Economides *et al.*, 2001). Thus, the injection time is a very important variable which is parameter much affect to the fracture half-length of course the Net Present Value (NPV). In many studies previously confirmed that injection time is directly proportional to net present value in the limited range of injection time of field experience. The injection time of hydraulic fracturing stimulation for tight oil reservoir is a considered of 60 min to 90 min of the variable  $X_2$  (Rahman, 2008). The effect of the leak-off coefficient on the net present value was presented by Economides *et al.* (1994), the presentation depicted that the total leak-off coefficient is increased to the net present value is decreased due to the more leak-off coefficient is usually more fluid volume loss and narrow fracture dimension as poor fracture conductivity of the post fracture. Thus, the high leak-off coefficient as low polymer concentration of the hydraulic fracturing is reduced oil production at the post fracture. Whereas, the low leak-off coefficient as high polymer concentration of fracturing fluid is more fracture dimensions during fracturing of longer fracture length and wider fracture width as more fracture conductivity. Thus, low leak-off coefficient is increased the net present value. Usually, the total leak-off coefficient is directly proportional to the mainly polymer concentration and fluid additive. In this study, hydraulic fracturing for the field of tight oil reservoir with the leak-off coefficient is considered in the range of  $0.002 \text{ ft}/\text{min}^{0.5}$  to  $0.004 \text{ ft}/\text{min}^{0.5}$  of the variable  $X_3$  as shown in the Table 5.

The reasonable experiment design of the central composite design is to investigate the effects of three operating parameters variables of hydraulic fracturing on the production performance with the net present value of the response. These operating parameters of the variables are namely of injection rate,  $X_1$ , injection time,  $X_2$ , leak-off coefficient,  $X_3$ , presenting the total

Table 5: Three independent variables and their levels for CCD

Variable Symbol		Low	Center	High
Injection rate (bpm)	$X_1$	30	40	50
Injection time, minutes	$X_2$	60	75	90
Leak-off coefficient, $\text{ft}/\text{min}^{0.5}$	$X_3$	0.002	0.003	0.004

Table 6: Independent variables and results of post fracture with simulation observed by central composite design (CCD)

Run	Coded level of variables			Actual level of variables			Responses (Stimulation observed)	
	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	bpm	Minutes	ft /min <sup>0.5</sup>	Cumulative Oil Production, (bbl)	NPV, \$mm
1	-1	-1	-1	30	60	0.002	1, 690, 000	35.55
2	1	-1	-1	50	60	0.002	1, 918, 700	41.24
3	-1	1	-1	30	90	0.002	1, 795, 400	38.73
4	1	1	-1	50	90	0.002	2, 024, 500	44.83
5	-1	-1	1	30	60	0.004	1, 475, 800	31.90
6	1	-1	1	50	60	0.004	1, 697, 600	37.60
7	-1	1	1	30	90	0.004	1, 560, 700	34.98
8	1	1	1	50	90	0.004	1, 800, 000	41.33
9	-1	0	0	30	75	0.003	1, 617, 700	37.38
10	1	0	0	50	75	0.003	1, 856, 000	43.71
11	0	-1	0	40	60	0.003	1, 686, 600	40.07
12	0	1	0	40	90	0.003	1, 795, 400	42.95
13	0	0	-1	40	75	0.002	1, 878, 300	45.47
14	0	0	1	40	75	0.004	1, 748, 200	43
15	0	0	0	40	75	0.003	1, 748, 200	43
16	0	0	0	40	75	0.003	1, 748, 200	43
17	0	0	0	40	75	0.003	1, 748, 200	43

number of test were required of the three variables of  $2^3+(2.3)+3 = 17$ . In this experiment design, the center point was set of 3 and the replicates of zero value. Therefore, the three independent variables of the operating parameters of the CCD were shown in the Table 5. The coded and actual levels of the dependent variables of each the experiment design in the matrix column is calculated in the Table 6. From the Table 6, the experiment of design is to conduct for the obtaining the response.

The net present value for 10 years of oil production was proposed to the result of the response in order to analyze the fractured well of the post fracture production. Therefore, the independent variables are correlated in them with the surface response. From the response of the net present value and the result of oil recovery, the operating engineers can control the proper the operating parameters of hydraulic fracturing. It can be observed the maximum net present value at which the operating parameters of hydraulic fracturing were determined.

The net present value model is to estimate based on the one discussed in the contractor drilling and production in offshore Viet Nam. A simple cash flow model in an Excel spreadsheet was calculated from the yearly income includes the depreciation regarding to a typical contractor fiscal regime in Viet Nam. These simulators cases were run over 10 years and the results include the oil production rate of the fractured well, injection time, leak-off coefficient of hydraulic fracturing, amount of proppant has been used, the amount of the fracturing fluid has been used. Thus, these input parameters for the net present value model consist of the average oil price of 100 \$/bbl and fracturing price per gallon of 1 \$/gallon, proppant price of 1 \$/lb, hydraulic horse power price of 20 \$/hhp, fixed price of 15,000 \$ and the discount rate of 10%.

**Experiment design for Hydraulic fracturing operating parameters:** The fractured well of the post fracture production has been presented by using the economic analysis with the net present value of the response for yearly a period of 5 years of oil production.

$$Y = 43.1614 + 3.01891X_1 + 1.64576X_2 - 1.95972X_3 - 2.7384X_1^2 - 1.77695X_2^2 - 0.343846X_3^2 + 0.133662X_1X_2 + 0.0326878X_1X_3 + 0.00588657X_2X_3 \quad (20)$$

## RESULTS AND DISCUSSION

The results analyzed for these variances with high quality fit model and the adequacy of the models are summarized in the Table 3. By the obtained of coefficient of  $R^2 = 0.995$  was shown on ANOVA in the Table 6 of the quadratic regression model, the model was demonstrated that only a little of the variations in the equation of 20 were not explained. Furthermore, the value of the adjusted coefficient was shown in  $R^2_{adjusted} = 0.988$ , this value depicted that the model very consistent (Table 7).

**Main and interaction effect plots:** The main effect plot is used as a tool to analyze the detailed main effect and interaction effect plots of the variances in the Design of Experiment (DOE). Figure 3 is presented the effect of these variables on the Net Present Value (NPV). In this graph also can be divided by two regions as clearly in Fig. 3 for the first region is below zero, where the coefficients of variation are presented with the negative coefficient factor of  $(X_3, X_1.X_1, X_2.X_2, X_3.X_3)$  as presented in Eq. (12) or in Fig. 3 are very consistent. Moreover, in the Fig. 3 shows the leak-off



Table 7: Regression coefficient of the predicted quadratic polynomial model

NPV	Coeff. SC	Standard Error	P	Confident interval ((±))
Constant	43.1614	0.174503	$4.66204 \times 10^{-15}$	0.41264
X1	3.01891	0.128962	$6.59018 \times 10^{-8}$	0.30495
X2	1.64576	0.128962	$4.20291 \times 10^{-6}$	0.30495
X3	-1.95972	0.128962	$1.28611 \times 10^{-6}$	0.30495
X1.X1	-2.7384	0.249146	$1.14374 \times 10^{-5}$	0.589145
X2.X2	-1.77695	0.249146	0.000188274	0.589145
X3.X3	-0.343846	0.249146	0.210021	0.589145
X1.X2	0.133662	0.144184	0.38476	0.340944
X1.X3	0.0326878	0.144184	0.827128	0.340944
X2.X3	0.00588657	0.144184	0.968574	0.340944

N = 17 Q<sup>2</sup> = 0.961 Cond. no. = 4.4382 DF = 7 R<sup>2</sup> = 0.995 Y-miss = 0 R<sup>2</sup><sub>Adj</sub> = 0.988 RSD = 0.4078 Confident level = 95%

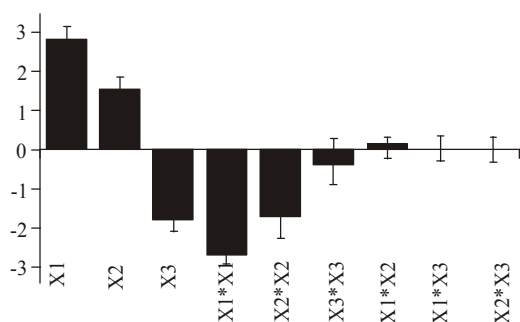


Fig. 3: The degree of factors of effects on the NPV

coefficient is much negative is decreased the net present value for this explanation when the leak-off coefficient increasing as high fluid volume loss through the fracture area during injection. This is reasonable decrease the fracture half-length with the factor of negative was presented in Eq. (20). The confirmed the leak-off coefficient is inversely proportional to the fracture half-length as presented in Eq. (3). In the field, the leak-off coefficient is often controlled by the three mechanisms the compression of the reservoir fluids, the thickness of the invaded zone, which is filled with the viscous fracture fluid and filter cake developed by Williams (1970) and Williams *et al.* (1979) in which only filter cake is controlled by the polymer concentration in fracturing fluid. Based on the maximum economics, the leak-off coefficient has been selected in  $0.002 \text{ ft}/\text{min}^{0.5}$  with 30 pounds per thousand gallons (pptg) of *Hydroxypropyl-Guar* (HPG) in 8 pptg  $\text{Na}_2\text{S}_2\text{O}_8$  Per sulfate Breaker additive. The second region is the above zero where these coefficients of the equation of 19 are presented the positive factors consists of ( $X_1$ ,  $X_2$ ,  $X_1X_2$ ,  $X_1X_3$ ,  $X_2X_3$ ). Figure 3 also demonstrated that when the injection rate ( $X_1$ ) is increased the net present value increased. This is because in the Eq. (3) shows the injection rate is directly proportional to the fracture half-length with more fracture half-length is more economical and this parameter is most likely affected to NPV. Figure 3 also depicted that when increasing the injection time as well increasing injection volume, which increase the fracture

half-length so longer injection time is longer fracture half-length of course more economics. Where  $X_1$  is the injection rate,  $X_2$  is the injection time and  $X_3$  is the leak-off coefficient.

**Optimization of operating parameters of Hydraulic fracturing for tight oil reservoir:**

The three dimensional and 2-D contour plots have been presented based on the model of the net present value (NPV) that was in the Eq. (20) which depicted the relationships between the independent variables and these dependent variables of the net present value. The maximum this value was at the red top of the ellipse of the contour diagram in the Fig. 4 and 5, respectively. In order to determine the optimal operating parameters of the hydraulic fracturing for tight oil reservoir is to the maximum net present value. These independents variables are on the smallest of the ellipse in the Fig. 5. Therefore optimal variables are 46 bpm of the injection rate, 88.5 min of the injection time and  $0.002 \text{ ft}/\text{min}^{0.5}$  of the leak-off coefficient.

**Proppant pump schedule:** At proppant schedule, the pad volume is only pumped the fracturing fluid with fluid viscosity without proppant into the well under high pressure in order to initiate open fracture. The slurry stage is pumped to the well as how proppant is added into the fracturing fluid system for mixing proppant slurry until proppant slurry reached the proppant concentration end of the job (EOJ) of 8 pptg. By series of the calculation from the material balance is given fluid efficiency of 32.44%. Thereafter, the pad volume is calculated of 87,220 gallons of 45 min for injection at a pump rate of 46 bpm. At the proppant slurry stage, the volume is required of 83761 gallons at 43 min for injection at 46 bpm. Proppant pumping schedule is shown in the Fig. 6.

**Production profile analysis:** In order to investigate the hydraulic fracturing efficiency, the post-fracture production is the one of the best tool to estimate the efficient hydraulic fracturing of the optimal operating parameters of 46 bpm of injection rate, 88.5 min of the

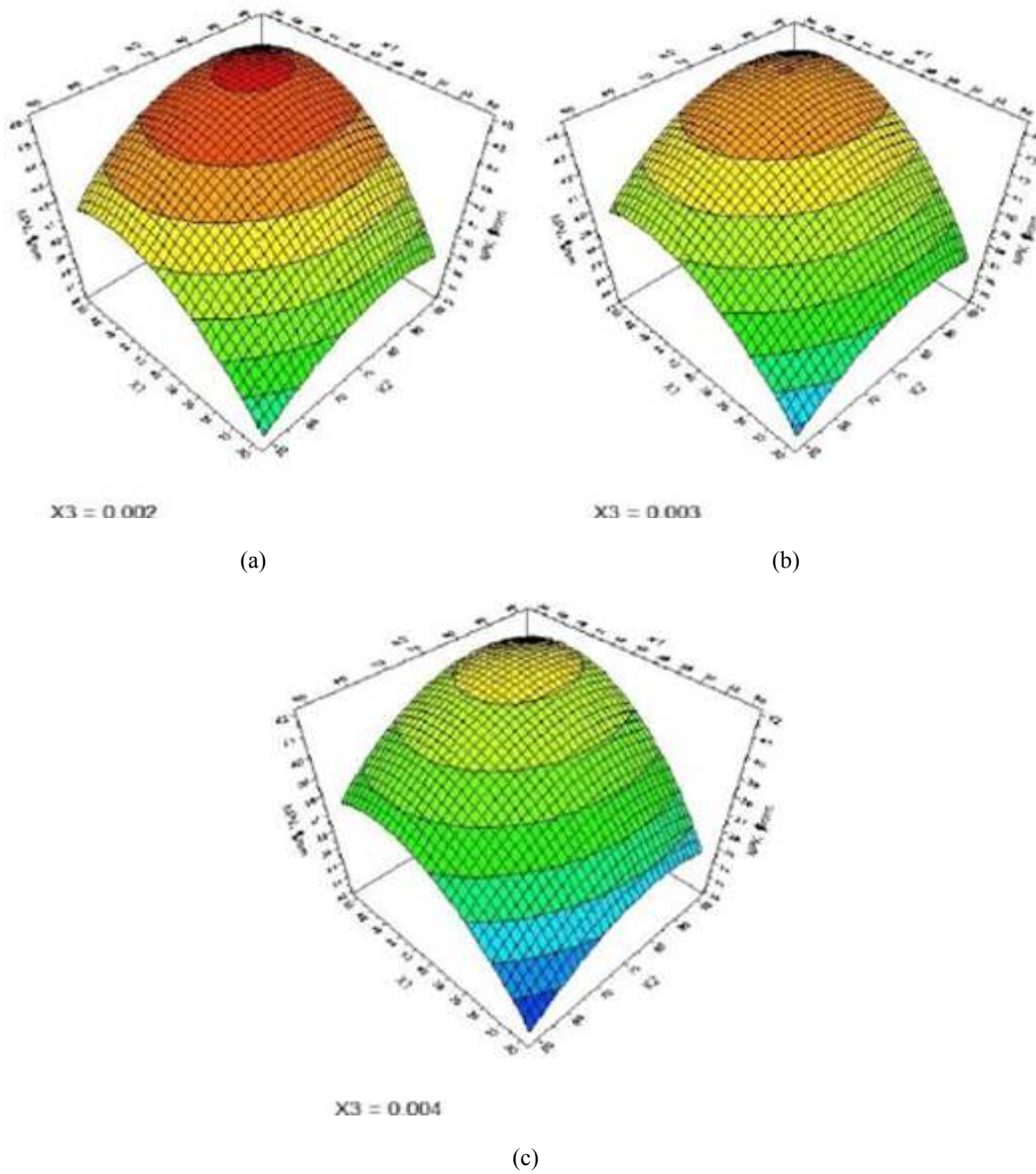
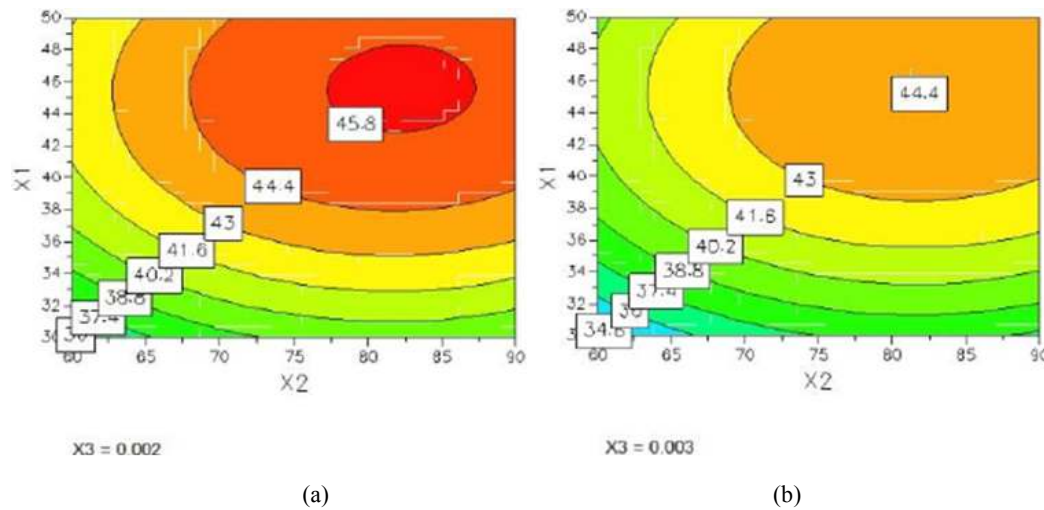
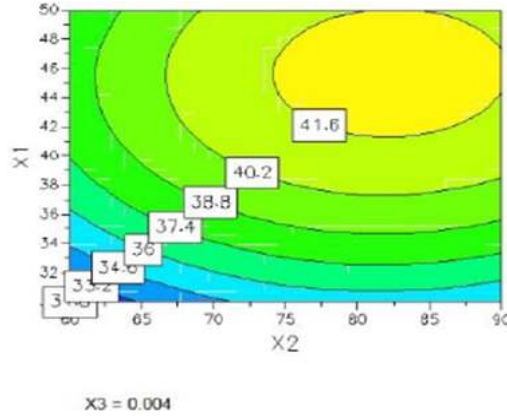


Fig. 4: Response surface plots (3-D) showing these effects of the variables on the net present value (NPV)





(c)

Fig. 5: Contour plots (2-D) showing the effects of the variables on the net present value (NPV)

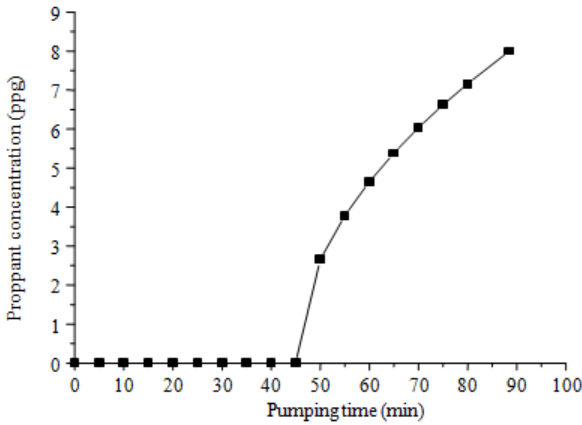


Fig. 6: Proppant concentration schedule versus pumping time

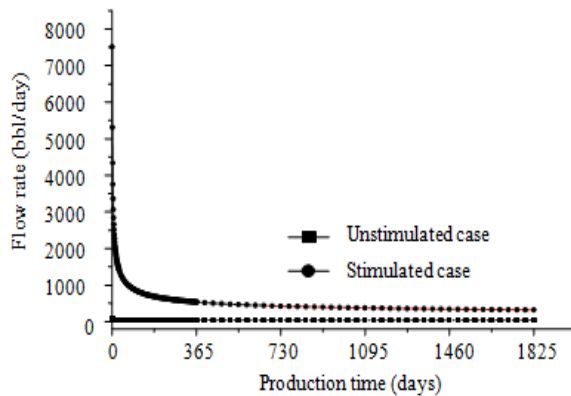


Fig. 7: The transient oil production rate of unstimulated case and fractured well at the optimal operating parameters of hydraulic fracturing

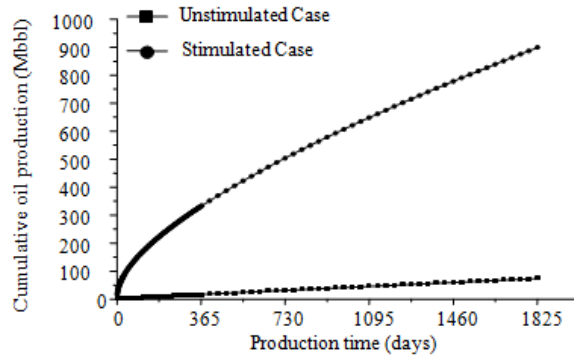


Fig. 8: The cumulative oil production of unstimulated case and stimulated case at optimal operating parameters of hydraulic

Table 8: Simulation production at optimal operating parameters of hydraulic fracturing for fractured well and unstimulated well

Parameter	Value
Closure pressure, psi	4,842.59
Proppant fracture concentration, lb/ft <sup>2</sup>	1.63
Fracture conductivity, mD-ft.	6,200
Dimensionless fracture conductivity, $F_{CD}$	3.2
Pseudo-skin, $S_f$	-7.6
Effective wellbore radius, ft.	650
Fold increase (PI)	18.7

constant fracture height of 70ft and the material balance with the series calculated proppant transport and proppant pumping schedule of the proppant concentration end of the job (8ppg) presented the fracture half-length of 1,940 ft and the average fracture width of 0.32 inch, the fluid efficiency of 32.44% and give proppant fracture concentration of 1.63 lb/ft<sup>2</sup>. The simulation fracture conductivity with fracture closure pressure of 4,842.59 psi versus proppant concentration of 1.63 lb/ft<sup>2</sup> was given fracture conductivity of 6,200 md-ft and dimensionless fracture conductivity ( $F_{CD}$ ) of 3.2, the pseudo skin of -7.6 which are given the effective wellbore radius of 650 ft. Finally, in Fig. 7 and 8 are demonstrated that the oil production rate and cumulative oil production of the

injection time and  $0.002 \text{ ft/min}^{0.5}$  of the leak-off coefficient at 30 pound per thousand gallons of Hydroxypropyl Guar (HPG) polymer. The optimal operating parameters of hydraulic fracturing was used of the 2D PKN-C fracture geometry model and the

fractured well is much greater than the oil production of unstimulated case. The Post-fracture production has been shown of 18.7- fold in the production increment (Table 8).

### CONCLUSION

In this study, the possibility of successful hydraulic fracturing of optimal operating parameter of hydraulic fracturing was determined based on maximum Net Present Value (NPV) and from the result of the research this leads us to conclude the following.

The Response Surface Methodology (RSM) and Central Composite Design (CCD) is the best tool to find optimal operating parameters of hydraulic fracturing in the field of tight oil reservoir with 46 bpm of injection rate, 88.5 minutes of injection time, 0.002 ft/min<sup>0.5</sup> of the leak-off coefficient.

This is a good lesson learned to optimize operating parameters of injection rate, injection time and leak-off coefficient for hydraulic fracturing stimulation.

The result of the maximum net present value of 46.5\$mm to find the leak-off coefficient was determined of 0.002 ft/min<sup>0.5</sup>. This is a new concept to find the HPG polymer concentration of 30 pptg with 8 pptg Na<sub>2</sub>S<sub>2</sub>O<sub>8</sub> persulfate breaker additive required of fracturing fluid.

The integrated model development of hydraulic fracturing of the tight oil reservoir is good potential to stimulate a layered tight oil reservoir.

The 2 D PKN-C fracture geometry model is sufficient to the fracture treatment design parameters of the tight oil reservoir.

The fractured well has been shown in the post-fracture production of the 18.7 folds of production incremental compared to the base case.

### ACKNOWLEDGMENT

The Authors would like to thank to Sejong University, Seoul, South Korea has been supported me during this study of the project. Also authors would like to thank all members of the petroleum engineering of Petrovietnam University has been help me and advised me for complete writing this study.

### REFERENCES

- Box, G.E.P. and N.R. Draper, 1987. Empirical Model-building and Response Surfaces. Wiley, New York.
- Box, G.E.P. and J.S. Hunter, 1957. Multi-factor experimental designs for exploring response surfaces. *Ann. Math. Stat.*, 28(1): 195-241.
- Cinco-Ley, H., V.F. Samaniego and A.N. Dominguez, 1978. Transient pressure behavior for a well with a finite-conductivity vertical fracture. *Soc. Petrol. Eng. J.*, 18(4): 253-264.
- Cornell, J.A. 1990. How to Apply Response Surface Methodology. 2nd Edn., American Society for Quality Control, Milwaukee, WI.
- Economides, M.J., A.D. Hill and C.E. Ehlig-Economides, 1994. Petroleum Production Systems. Prentice Hall, Upper Saddle River, NJ.
- Economides, M.J., N.P. Valko and X. Wang, 2001. Recent advances in production engineering. *J. Can. Petrol. Technol.*, 40(10): 35-44.
- Economides, M.J. and T. Martin, 2007. Modern Fracturing: Enhancing Natural Gas Production. ET Publishing, United States of America.
- Economides, M.J., R. Oligney and P. Valkó, 2002. Unified Fracture Design. Orsa Press, Alvin, TX.
- Howard, G.C. and C.R. Fast, 1957. Optimum fluid characteristics for fracture extension. *Drilling and Production Prac.*, API, pp: 261-270.
- Jiang, T., X. Wang, W. Shan and Y. Wang, 2003a. A new comprehensive hydraulic fracturing technology to minimize formation damage in low permeability reservoirs. Proceeding of SPE European Formation Damage Conference. The Hague, Netherlands, May 13-14.
- Jiang, T., Y. Wang, Y. Ding, W. Shan, X. Wang, X. Zhou and J. Liu, 2003b. The study of a new overall hydraulic fracturing mode in low permeable and unconventional reservoirs. Proceeding of SPE Eastern Regional Meeting. Pittsburg, PA, September 6-10.
- Montgomery, D.C., 2001. Design and Analysis of Experiments. 5th Edn., John Wiley and Sons, New York.
- Myers, R.H. and D.C. Montgomery, 2002. Response Surface Methodology: Process and Product Optimization Using Designed Experiments. 2nd Edn., John Wiley and Sons, New York.
- Myers, R.H., D.C. Montgomery and C. Anderson-Cook, 2008. Response Surface Methodology: Process and Product Optimization Using Designed Experiments. 3rd Edn., John Wiley and Sons, New York, pp: 13-135.
- Nordgren, R.P., 1972. Propagation of a vertical hydraulic fracture. *Soc. Petrol. Eng. J.*, 12(4): 306-314.
- Perkins, T.K. and L.R. Kern, 1961. Width of hydraulic fractures. *J. Petrol. Technol.*, 13(9): 937-949.
- Rahman, M.M., 2008. Productivity prediction for fractured wells in tight sand gas reservoirs accounting for non-darcy effects. Proceeding of the SPE Russian Oil and Gas Technical Conference and Exhibition. Moscow, Russia, October 28-30.
- Rahman, M.M., M.K. Rahman and S.S. Rahman, 2003. Optimizing treatment parameters for enhanced hydrocarbon production by hydraulic fracturing. *J. Can. Petrol. Technol.*, 42: 38-46.

- Smith, M.B., 1997. Hydraulic Fracturing. 2nd Edn., NSI Technologies, Tulsa, Oklahoma.
- Valko, P. and M.J. Economides, 1995. Hydraulic Fracture Mechanics. John Wiley and Sons, Chichester, England.
- Valko, P., R.E. Oligney and M.J. Economides, 1997. High permeability fracturing of gas wells. Gas TIPS (Fall), 3(3): 31-40.
- Williams, B.B., 1970. Fluid loss from hydraulically induced fractures. J. Petrol. Technol., 22(7): 883-888.
- Williams, B.B., J.L. Gidley and R.S. Schechter, 1979. Acidizing Fundamentals. SPE Monograph, Society of Petroleum Engineers of AJME, Richardson, TX. Vol. 6.
- Yu, H. and M.M. Rahman, 2012. Pinpoint multistage fracturing of tight gas sands: An integrated model with constraints. Proceeding of the SPE Middle East Unconventional Gas Conference and Exhibition, Abu Dhabi, UAE.