Research Journal of Applied Sciences, Engineering and Technology 12(10): 1011-1017, 2016
DOI:10.19026/rjaset.12.2820
ISSN: 2040-7459; e-ISSN: 2040-7467

Submitted: April 27, 2015 Accepted: May 10, 2015 Published: May 15, 2016

Research Article

Method to Defuzzify Groups of Fuzzy Numbers: Allocation Problem Application

[1,2]Jehan S. Ahmed, [2]Maznah Mat Kasim and [2]L. Majid Zerafat Angiz
[1]Department of Mathematics, Baghdad University, Baghdad, Iraq
[2]School of Quantitative Sciences, Universiti Utara Malaysia, 06010, Sintok, Kedah, Malaysia

Abstract: The defuzzification process converts fuzzy numbers to crisp ones and is an important stage in the implementation of fuzzy systems. In many actual applications, we encounter cases, in which the observed or derived values of the variables are approximate, yet the variables themselves must satisfy a set of relationships dictated by physical principle. When the observed values do not satisfy the relationships, each value is adjusted until they satisfy the relationships among observed data indicating their mathematical dependence on one another. Hence, this study proposes a new method based on the Data Envelopment Analysis (DEA) model to defuzzify groups of fuzzy numbers. It also aims to assume that each observed value is an approximate number (or a fuzzy number) and the true value (crisp value) is found in the production possibility set of the DEA model. The proposed method partitions the fuzzy numbers and the relationships among these observed data are observed as constraints. The paper presents the model, the computational process and applications in a real problem.

Keywords: Data envelopment analysis, defuzzification, groups of fuzzy numbers, observed data

INTRODUCTION

The modeling of complex systems is limited by incomplete knowledge and lack of information (Lai and Hwang, 1992). Hence, the fuzzy set theory developed by Zadeh (1965), along with its techniques, is an interesting and promising approach to address complex, real-world issues. In general, a fuzzy representation provides more information regarding a set than a crisp representation. However, this crisp representation remains necessary because it simplifies conception and clarification. Thus, the objective determination of the fuzzy structures of problematic systems is difficult. Thus, a crisp representation is typically easy to interpret and understand although it displays less information. To replace a fuzzy representation of sets with a crisp representation in fuzzy system applications, the process of defuzzification is applied (Leekwijck and Kerre, 1999; Mahdiani *et al*., 2013).

This definition enables the defuzzification of a set into a crisp subset of the original. Previous literature presents many defuzzification methods, but most of these methods generate fuzzy set results with the best information and composition. Furthermore, some of these methods lose their properties during actual observations of groups of related data. Meanwhile, defuzzification methods can generate similar results of a given data, with various relationships.

This study mainly presents a new method to defuzzify groups of fuzzy numbers with the tool Data Envelopment Analysis (DEA). The method is suitable when information about the original values is minimal, when the values are considered approximate or when we need to find estimation values based on the original values. Given a set of observed values and the basic relationships that they must satisfy, the method yields a set of adjusted numbers that are close to the original numbers and that also meets the relationship.

LITERATURE REVIEW

This section is highlight on the origin of the definition which commonly used and background of defuzzification methods.

Defuzzification: Defuzzification is an important fuzzy system stage that replaces fuzzy numbers with a representative crisp number (Esogbue *et al*., 2000; Mahdiani *et al*., 2013). Some common defuzzification techniques are center of area (COA), weighted average method and height method (Lee, 1990; Nurcahyo, 2014).

Related literature also described various defuzzification methods with different levels of complexity. For instance, Ma *et al*. (2000) brought forward a novel method to defuzzify fuzzy sets according to the metric distance between two symmetric and triangular fuzzy numbers. Similarly, Sladoje *et al*. (2011) demonstrated a novel defuzzification method for image processing. Their method determined the crisp set that is at a minimal distance from the fuzzy set by generating a family of

Corresponding Author: Jehan S. Ahmed, Department of Mathematics, Baghdad University, Baghdad, Iraq

distance functions. The distance between two fuzzy sets is expressed as a Minkowski distance.

Meanwhile, Naaz *et al*. (2011) proposed a simple model of the fuzzy load balancing algorithm in a distributed system and compared the effects of five defuzzification methods, namely, COA, bisector of area, Mean of Maximum (MOM), smallest of maximum and largest of maximum. Prior authors (Asady and Zendehnam, 2007; Saneifard and Ezatti, 2010) proposed defuzzification methods to rank fuzzy numbers. In the present study, we compare the proposed method with the Center Of Gravity (COG) method and with that proposed by Asady and Zendehnam (2007).

Center of Gravity (COG): The COG method was developed by Sugeno (1985) and is the most commonly used defuzzification method. This method calculates the position at which the left and the right areas are equal. COG refers to the centroid of the area and the defuzzification method can be expressed as:

$$x_{COG} = \frac{\int \mu_{\tilde{F}}(x).x\partial x}{\int \mu_{\tilde{F}}(x).\partial x},$$

where x_{COG} is the crisp value to the fuzzy number $\tilde{F}$

The method of Asady and Zendehnam: Asady and Zendehnam (2007) presented a defuzzification method based on the nearest point of a fuzzy number. The nearest point to the triangular fuzzy number $\tilde{F} = (x_0, \delta, \beta)$to be:

$$x_{A\&Z} = x_0 + \frac{\beta - \delta}{4},$$

where δ and β are the left and the right fuzziness values, respectively, $x_{A\&Z}$ is the crisp value to the fuzzy number $\tilde{F}$.

Data Envelopment Analysis (DEA): DEA is a recognized modern approach that stems from a Linear Programming (LP) model to evaluate the relative efficiencies of Decision Making Units (*DMUs*) with multiple inputs and outputs. DEA is a non-parametric technique and was initially proposed by Charnes *et al*. (1978) as a (*CCR*) model. This model was improved by other scholars, particularly in the form of the Banker *et al*. (1984) (*BCC*) model.

Assuming the inputs $x_{ij}(i = 1,2,..,m)$ and outputs y_{rj} $(r = 1,2,..,s)$ for DMU_j(j = 1, 2,…, n), the programming statement for the CCR model is formulated as follows:

Model (1):

$$\theta_p^* = min\,\theta_p$$
$$s.t.\ \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta_p x_{ip}\ \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rp}$$
$$\lambda_j \geq 0\ j = 1, \ldots, n$$
$$\theta\ free$$

where, λ_j is a non-negative value related to the $j^{th}DMU$. The vector $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)^t$ constructs a hull that covers all of the data points.

Model (1) is divided into three parts, namely, the left- and right-hand sides of the constraints and the objective function. The left-hand side generates the Production Possibility Set (PPS) and retouching this set changes the space. The right-hand side and the objective function lead *DMUs* to the frontier. Thus, the *DMUs* located on the efficiency frontier are considered the relative ideal points in DEA evaluation. That is, each inefficient *DMU* probes its own ideal *DMU* on the frontier. However, the question is whether the ideal points always lie on the efficiency frontier. In this research, we indicate that the ideal point can be probed within PPS.

Defuzzification of groups of fuzzy numbers: This section stresses on the origin of the premise underlying dependency. First, Kikuchi (2000) proposed the new defuzzification method that is capable of finding the most appropriate set of crisp numbers. The method assumes that each observed value is an approximate number (or a fuzzy number) and the true value is found in the support of the membership function. Although his method was validated for solving special kind of problems under assumption that inputs equal outputs in transportation problem and planning, it is quite notable that all the proposed methods in literature deal with no relationships on original data (observed data), which produce similar defuzzification results under various relationships. Literature is also rife with defuzzification methods emphasizing the transformation of individual fuzzy numbers into crisp (e.g., COG, MOM) and the method brought forward by Saneifard and Ezatti (2010).

However, because real application data is noted in groups that display some relationships and properties that emphasize their dependence, dependency takes significance. Therefore, in this study, a new defuzzification method that stresses on groups of fuzzy numbers is proposed. In other words, the present study is unique in that it addresses dependent data rather than what has been extensively examined in literature namely independent data.

To explain further, we refer to an example presented by Zerafat *et al*. (2009), where (G_1, G_2,.., G_n) indicates the supposed ranking places. Given a group of p experts (E_1, E_2,.., Ep) commenting on the weights of these places, the weights $w_{ik}(i = 1,2,..,n)(k = 1,2,..,p)$ can be aggregated into a group of fuzzy numbers. Thus, we first generate n fuzzy numbers. We then select the Triangular Fuzzy Numbers (TFN) from among the various shapes of fuzzy numbers because it is the most popular one. Therefore, the triangular fuzzy numbers are denoted by three points as follows:

$$\tilde{w}_i = (w_i^m, w_i^l, w_i^u)$$

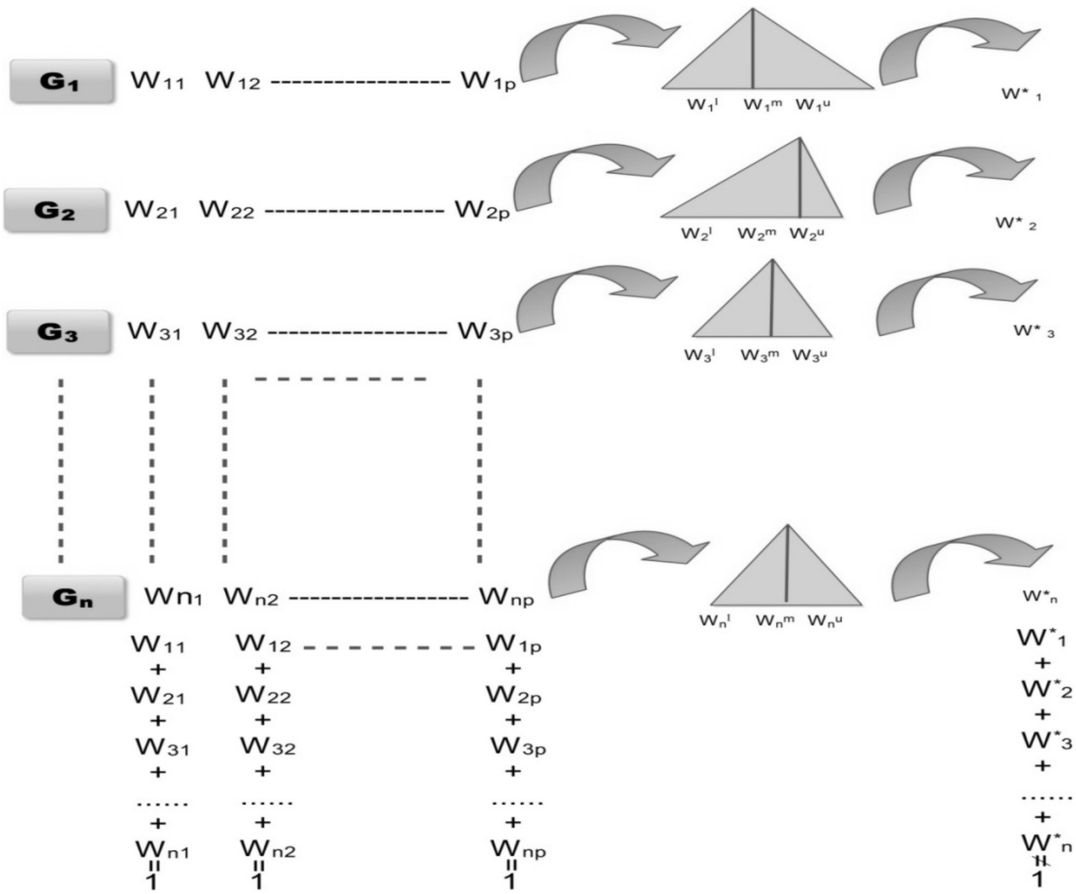

Fig. 1: Example to illustrate the shortcoming of existing defuzzification methods

In this case, we determine a representative of the fuzzy numbers given above $(w_1^*, w_2^*, .., w_n^*)$. This representative is established as the final weight of each G_i $(i = 1,2, \dots, n)$ and the sum of the representative weights must be one. However, this restriction may not be adhered to if a defuzzification method, such as COG or that developed by Asady and Zendehnam (2007), is employed because these methods do not have a condition that maintains these relations among the representative weights. The following diagram illustrates this matter (Fig. 1).

METHODOLOGY AND METHOD DEVELOPMENT

The method proposed to defuzzify groups of fuzzy numbers operates in six stages:

Stage 1: n triangular fuzzy numbers are generated based on the method proposed by Yeh and Chang (2009) as follows: $\tilde{x}_i = (x_i^m, x_i^l, x_i^u)$

$$x_i^m = \left(\prod_{k=1}^{p} x_{ik}\right)^{\frac{1}{p}}, \quad x_i^l = \min\{x_{i1}, x_{i2}, \dots, x_{ip}\} \quad and \quad x_i^u = \max\{x_{i1}, x_{i2}, \dots, x_{ip}\}$$

i =1,2,…,n is the number of fuzzy numbers, k =1,2,…,p is the number of observations, x_i^l is the lowest value, x_i^m is the geometric mean and x_i^u is the highest value.

This method displays the following membership functions:

$$\mu_{\tilde{x}_i}(x) = \begin{cases} L\left(\dfrac{x - x^l}{x^m - x^l}\right) & for \quad x^l \le x \le x^m \\ R\left(\dfrac{x^u - x}{x^u - x^m}\right) & for \quad x^m \le x \le x^u \end{cases}$$

Stage 2: T: The interval $[x^l, x^u]$ of each fuzzy number i is divided into m subintervals with equal width, with each subinterval being of width $\Delta x = \frac{x^u - x^l}{m}$. We label each element in these subintervals as shown $x_k = x^l + k * \Delta x$ k=1,2,..,m then the corresponding subintervals are;

$\{[x_i^l = x_{i0}, x_{i1}], [x_{i1}, x_{i2}], .. [x_{i(m-1)}, x_{im} = x_i^u]\}$.

Table 1: Illustration of inputs and outputs of DMUs

i j	i_1	i_2	.	.	i_{n-1}	i_n	o_1
DMU_0	$x_1^L = x_{10}$	$x_2^L = x_{20}$	.	.	$x_{n-1}^L = x_{(n-1)0}$	$x_n^L = x_{n0}$	1
DMU_1	x_{11}	x_{21}	.	.	$x_{(n-1)1}$	x_{n1}	1
DMU_2	x_{12}	x_{22}	.	.	$x_{(n-1)2}$	x_{n2}	1
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
DMU_{m-1}	$x_{1(m-1)}$	$x_{2(m-1)}$	.	.	$x_{(n-1)(m-1)}$	$x_{n(m-1)}$	1
DMU_m	$x_1^u = x_{1m}$	$x_2^u = x_{2m}$	.	.	$x_{n-1}^u = x_{(n-1)m}$	$x_n^u = x_{nm}$	1

Stage 3: With these subintervals, *mDMUs* are created. The PPS of these *DMUs* generate all of the possible solutions in the fuzzy interval. In other words, I $i = 1\, then x_1^l = x_{10}, x_{11}, .., x_{1(m-1)}, x_{1m} = x_1^u$ represents the input values of DMU_j ($j = 1, 2, \ldots, m$) that are used to produce PPS. The single output corresponding to DMU_j is assumed to be one. The inputs of each *DMUs* are illustrated in the following Table 1.

Stage 4: In this stage, we propose the following non-linear programming model based on the CCR model (1), but we replace the main objective with *n* objectives. The number of these objectives depends on the number of fuzzy numbers. Each objective gives an optimal solution $\bar{x}_i$ which has a minimum distance to all points in the fuzzy interval.

Model (2):

$$min \frac{\sum_{k=0}^{m} \mu_{\tilde{x}_i}(x_{ik})\, |\bar{x}_i - x_{ik}|}{\sum_{k=0}^{m} \mu_{\tilde{x}_i}(x_{ik})}\quad i = 1,2,\ldots,n$$

s.t. $\sum_{k=0}^{m} \lambda_k x_{ik} \le \bar{x}_i\; i = 1,2,..,n\; \sum_{k=0}^{m} \lambda_k \ge$ *1 Rxii=1,2,..n. xil≤xi≤xiu i=1,2,...,n λk≥0* $k = 0,1,2,\ldots,m$

In the proposed method, the relationships among these groups of fuzzy numbers are expressed as constraints $R(\bar{x}_i)$. The fourth constraint includes all of the intervals of the fuzzy numbers.

As shown in the model above, constraints including λ produce PPS that correspond to the CCR model. This model is solved only once unlike DEA evaluation, which requires the calculation of many models.

Stage 5: We assume that $z_{ik} = \bar{x}_i - x_{ik}$ and $|z_{ik}| = z_{ik}^+ + z_{ik}^-\; \forall(i,k)$. The multi-objective nonlinear programming Model (2) is then proposed as follows:

Model (3):

$$min \frac{\sum_{k=0}^{m} \mu_{\tilde{x}_i}(x_{ik})\, (z_{ik}^+ + z_{ik}^-)}{\sum_{k=0}^{m} \mu_{\tilde{x}_i}(x_{ik})}\quad i = 1,2,\ldots,n$$

$s.t.\; \sum_{k=0}^{m} \lambda_k x_{ik} \le \bar{x}_i\; i = 1,2,..,n\; \sum_{k=0}^{m} \lambda_k \ge 1$

$x_i^l \le \bar{x}_i \le x_i^u\; i = 1,2,\ldots,n$

$R(\bar{x}_i)\; i = 1,2,..n,$

$\bar{x}_i - x_{ik} - (z_{ik}^+ - z_{ik}^-) = 0\; i = 1,2,\ldots,n\; k = 1,2,\ldots,m$

$\lambda_k \ge 0\; k = 0,1,2,\ldots,m$

If $\mu_{\tilde{x}_i}(x_{ik})$ the membership functions of each fuzzy number and the relationship $R(\bar{x}_i)$ are linear, this model above is a Multi-Objective Linear Programming model (MOLP).

Stage 6: In order to determine the solution to Model (3) using weighted (Archimedean) goal programming model (WGP) it can be solved in the following way:

Model (4):

$$min \sum_{i=1}^{n} w_i d_i$$

$$s.t. \frac{\sum_{k=0}^{m} \mu_{\tilde{x}_i}(x_{ik})(z_{ik}^+ + z_{ik}^-)}{\sum_{k=0}^{m} \mu_{\tilde{x}_i}(x_{ik})} - d_i \le t`_i\; i = 1,2,\ldots,n$$

$\sum_{k=0}^{m} \lambda_k x_{ik} \le \bar{x}_i\; i = 1,2,..,n$

$\sum_{k=0}^{m} \lambda_k \ge 1\; k = 0,1,2,\ldots,m$

$R(\bar{x}_i)\; i = 1,2,\ldots,n$

$x_i^l \le \bar{x}_i \le x_i^u\; i = 1,2,\ldots,n$

$\bar{x}_i - x_{ik} - (z_{ik}^+ - z_{ik}^-) = 0\; i = 1,2,\ldots,n\; = 1,2,\ldots,m$

$\lambda_k \ge 0\; k = 0,1,2,\ldots,m$

In model (4), $w_i\; (i = 1,2,\ldots,n)$ denotes positive penalty weights. These weights can be determined through multi-criteria decision making techniques such as the Analytic Hierarchical Process (AHP) developed by Saaty (1980). However, we assume that each objective is equally important and allocate equal weight without losing generality. That is ($w_1 = w_2 = \cdots = w_n = 1/n$) allocated to each weight for this model $d_i\; (i = 1,2,\ldots,n)$ measures the over-achievement from the target point $t`_i\; (i = 1,2,\ldots,n)$ which is obtained by computing the MOLP model as a single objective *n* times (i.e., by considering each objective individually).

Table 2: The five inputs and unique output of j^{th} *DMU* (j= 0, 1, 2,.. 500)

im	i_1	i_2	i_3	i_4	i_5	o_1
DMU_0	$x_1^L = x_{1\,0}$	$x_2^L = x_{2\,0}$	$x_3^L = x_{3\,0}$	$x_4^L = x_{4\,0}$	$x_5^L = x_{5\,0}$	1
DMU_1	x_{11}	x_{21}	x_{31}	x_{41}	x_{51}	1
DMU_2	x_{12}	x_{22}	x_{32}	x_{42}	x_{52}	1
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
DMU_{500}	$x_1^u = x_{1\,500}$	$x_2^u = x_{2\,500}$	$x_3^u = x_{3\,500}$	$x_4^u = x_{4\,500}$	$x_5^u = x_{5\,500}$	1

Table 3: Results of the proposed method under different number of partitions

No. of fuzzy numbers i	Fuzzy numbers (x_i^l, x_i^m, x_i^u)	Crisp values m = 2	m = 9	m = 19	m = 20	m = 100	m = 250	m = 500
1	(14,29.2397,48)	31	29	28	28	28	28	28
2	(1,6.7661,19)	10	7	7	7	7	7	7
3	(4,15.5450,34)	19	14	15	16	16	16	16
4	(15,29.4486,51)	17	30	30	29	29	29	29
5	(30,51.8925,76)	53	50	50	50	50	50	50
Sum of the estimated no. of beds		130	130	130	130	130	130	130

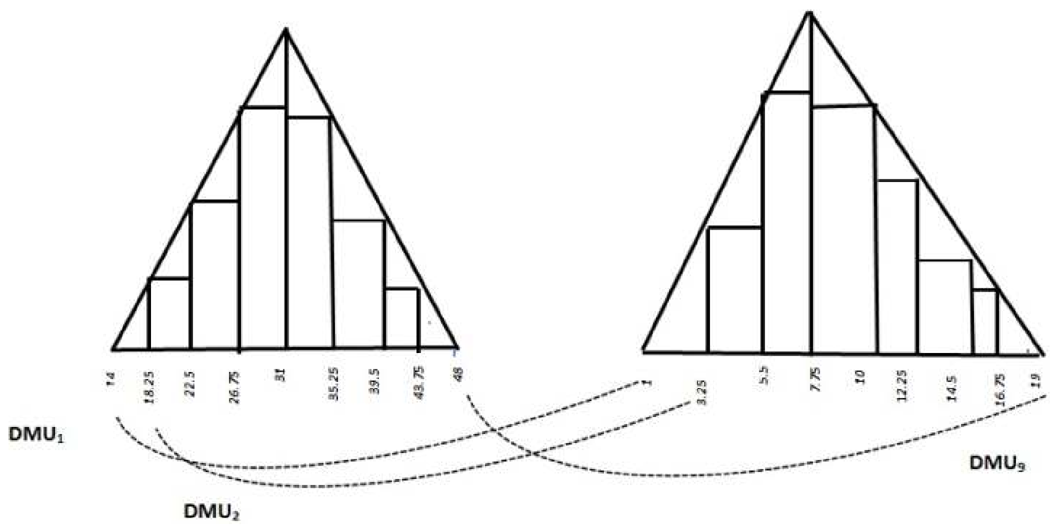

Fig. 2: The first and second inputs of DMU_1, DMU_2,.., DMU_9

Case study and data collection: In this section, we apply the proposed methodology in real life by estimating the required number of hospital beds for the different wards of a Malaysian hospital. We collected the data on the number of beds used by patients who were hospitalized over a period of 150 days from the hospital database of the Malaysian Ministry of Health. The hospital patients were divided into five categories based on age [toddler (T), schoolchildren (S), adult (A), old (O) and elderly (E)]. This case study aims to aid managers in determining the optimal number of beds to be allocated to each group because the number of available beds at this hospital is limited.

RESULTS AND DISCUSSION

The data is first compiled into a group of fuzzy numbers. Five groups are generated by using method proposed by Yeh and Chang (2009) in stage 1. The fuzzy number of each five groups is represented as the input of DMU_j. In DEA, the interval of each fuzzy number $i = 1,2,\dots 5$ is partitioned into $m = 500$ subintervals.

The starting point of each subinterval k = 0, 1, 2, …, 500 of fuzzy number is represented as (input1, input2, input3, input4, input5) for DMU_1 while the second points of each subinterval k = 0, 1, 2, …, 500 of fuzzy number is represented as (input1, input2, input3, input4, input5) for DMU_2 and so on until DMU_{500}. For better understanding, refer to Table 2 for details.

Therefore, the efficiency frontier produced by the starting points of the interval of fuzzy numbers is insignificant. In this case, optimal solution should occasionally be obtained from within the PPS rather than on the frontier, as demonstrated in the following example. Figure 2 indicates two groups of fuzzy numbers (Tand S). These groups correspond to DMU_j where $j = 1,2,\dots,m$ and $m = 9$.

The flexibility of this method enables the increase in different numbers of *DMUs* by increasing the

Table 4: Results of the proposed method when the total available beds is 200

No. of fuzzy numbers i	Fuzzy numbers (x_i^l, x_i^m, x_i^u)	Crisp values m = 2	m = 9	m = 20	m = 27	m = 100	m = 250	m = 500
1	(14,29.2397,48)	31	44	43	42	42	42	42
2	(1,6.7661,19)	10	15	16	16	16	16	16
3	(4,15.5450,34)	32	28	28	29	29	29	29
4	(15,29.4486,51)	51	43	44	45	45	45	45
5	(30,51.8925,76)	76	70	69	68	68	68	68
Sum of the estimated no. of beds		200	200	200	200	200	200	200

number of partitions to obtain the best solution. Hence, different numbers of partitions are introduced until we obtain a stable result.

Now, starting when $m = 0$ and from equation in stage 2 the element of the corresponding subinterval is $x_0 = x^l$. Then when $m = 1$ the elements of the corresponding subinterval are $x_0 = x^l$, $x_1 = x^u$ and the subinterval is$[x^l, x^u]$. This indicates that no result could be obtained that satisfy the relationship in these subintervals indicating that the results start to appear from $m = 2$ until $m = 500$.

Table 3 shows the group fuzzy numbers generated for each group of inputs (five groups of patients) over 150 days. For instance, when *i*=1, the first group of fuzzy numbers is described as follows as shown in column 2:

$$x_1^l = min\{x_{11}, x_{12}, .., x_{1\,150}\} = 14, x_1^m = \left(\prod_{k=1}^{150} x_{ik}\right)^{1/150} = 29.2397 \text{ and } x_1^u = max\{x_{11}, x_{12}, .., x_{1\,150}\} = 48$$

The results obtained using m = 500 different numbers of partitions ranged from m = 2 when the result start to appear as summarized in column 3 to 9. Then m = 9 to show different results for each groups, until m = 20 where the results start to be stabilized. In m = 100, 150 and 500, we get the same results as m = 20 indicating that the optimal numbers of beds determined for these five groups of patients under various partitions satisfies the relationship $(\sum_{i=1}^{5} \bar{x}_i = 130)$ and therefore, the total number of available beds is represented by$\bar{x}_1 = 28$, $\bar{x}_2 = 7, \bar{x}_3 = 16, \bar{x}_4 = 29, \bar{x}_5 = 50$.

In the next step, we assume that the available number of beds is 200 to show the validity of our method and to give the optimal solution that satisfies the relationship on the original data. Table 4 shows that the stabilized results start from m = 27 and in each m the proposed method that gives an optimal solution satisfied the relation $(\sum_{i=1}^{5} \bar{x}_i = 200)$, but we continue in our partitions until the result is stabilized and this appears in m = 27. This led us to be sure that no other optimal solution will appear under this relationship, in which case, the results under different numbers of partition starting from m = 27, 28 …., 500 are noted to confirm this matter.

Table 5: Results of COG and A&Z methods

No. of fuzzy numbers i	Fuzzy numbers (x_i^l, x_i^m, x_i^u)	COG	A&Z
1	(14,29.2397,48)	30	30
2	(1,6.7661,19)	9	9
3	(4,15.5450,34)	18	17
4	(15,29.4486,51)	32	31
5	(30,51.8925,76)	53	52
Sum of the estimated no. of beds		142	139

As mentioned above, most of the defuzzification methods deal with original data as individuals not as groups, with some relationships. For this matter, we used two defuzzification methods namely COG and the method developed by Asady and Zendehnam (A and Z) (2007) to defuzzify the five groups of fuzzy numbers. Table 5 shows the that results obtained under these methods did not satisfied the relationship on the original data as the summation of estimated number of beds for each group equals to the total number of available beds 130.

Now, we compare the results of the proposed method obtained in Table 6 with the results in Table 5. In this comparison, we ignore the constraint representing the relationship $(R\ (\bar{x}_i))$ in model (4). In other words, we apply this method when the original data has no relationships.

As shown in Table 6, the proposed method presents different results and different summation of the estimated number of beds under each partition. Then, the results starts stabilizing from m = 75. In this partition, the results obtained are the same as those of A and Z method.

Table 6: The results of the proposed method with no relationships in original data

No. of fuzzy numbers i	Fuzzy numbers (x_i^l, x_i^m, x_i^u)	Crisp values m = 2	m = 9	m = 66	m = 75	m = 100	m = 250	m = 500
1	(14,29.2397,48)	31	29	30	30	30	30	30
2	(1,6.7661,19)	10	9	9	9	9	9	9
3	(4,15.5450,34)	19	17	17	17	17	17	17
4	(15,29.4486,51)	33	31	31	31	31	31	31
5	(30,51.8925,76)	53	50	53	52	52	52	52
Sum of the estimated no. of beds		146	136	140	139	139	139	139

This finding, lead us to say that the proposed method gives the nearest point to the fuzzy numbers in case of no relationships and the optimum nearest point in case some relationships in the original data need to be satisfied in crisp values.

CONCLUSION

In this study, a new defuzzification method was developed to defuzzify groups of fuzzy numbers using the DEA model. The context of the proposed method with respect to some relationships in original data reveals that the crisp point maintains these relationships. The proposed method is unique because no other method in previous literature enabled crisp values to keep some relationships in the original data in a method that can be applied in real life problems not like a Kikuchi (2000) method. The example and case study confirm that the proposed method is applicable to both dependent and independent original data. To demonstrate the influence of the new approach on application, an allocation problem was presented. In this case study, the proposed method was utilized to estimate the optimal number of available beds in a hospital by categorizing patients according to ages.

For future research, the proposed method can also efficiently address nonlinear fuzzy numbers. In this case, many nonlinear membership functions can represent real problems to some extent, including the (hyperbolic and exponential) membership functions. This method can be followed by matching real problems to these functions using actual data or statistical techniques, such as regression, to get the actual function of these data.

ACKNOWLEDGMENT

This research is funded by the Ministry of Higher Education of Malaysia under Fundamental Research Grant Scheme (FRGS), 2014-2015.

REFERENCES

Asady, B. and A. Zendehnam, 2007. Ranking fuzzy numbers by distance minimization. Appl. Math. Model., 31(11): 2589-2598.

Banker, R.D., A. Charnes and W.W. Cooper, 1984. Some models for estimating technical and scale inefficiencies in data envelopment analysis. Manage. Sci., 30(9): 1078-1092.

Charnes, A., W.W. Cooper and E. Rhodes, 1978. Measuring the efficiency of decision making units. Eur. J. Oper. Res., 2(6): 429-444.

Esogbue, A.O., Q. Song and W.E. Hearnes II, 2000. Defuzzification filters and applications to power system stabilization problems. J. Math. Anal. Appl., 251(1): 406-432.

Kikuchi, S., 2000. A method to defuzzify the fuzzy number: Transportation problem application. Fuzzy Set. Syst., 116(1): 3-9.

Lai, Y.J. and C.L. Hwang, 1992. Fuzzy Mathematical Programming: Methods and Applications. Springer-Verlag, Berlin, Heidelberg.

Lee, C., 1990. Fuzzy logic in control systems: fuzzy logic controller. II. IEEE T. Syst. Man Cyb., 20(2): 419-435.

Leekwijck, W.V. and E.E. Kerre, 1999. Defuzzification: Criteria and classification. Fuzzy Set. Syst., 108(2): 159-178.

Ma, M., A. Kandel and M. Friedman, 2000. A new approach for defuzzification. Fuzzy Set. Syst., 111(3): 351-356.

Mahdiani, H.R., A. Banaiyan, M. Haji Seyed Javadi, S.M. Fakhraie and C. Lucas, 2013. Defuzzification block: New algorithms, and efficient hardware and software implementation issues. Eng. Appl. Artif. Intel., 26(1): 162-172.

Naaz, S., A. Alam and R. Biswas, 2011. Effect of different defuzzification methods in a fuzzy based load balancing application. Int. J. Comput. Sci. Issues, 8(5): 261-267.

Nurcahyo, G., 2014. Selection of defuzzification method to obtain crisp values for representing uncertain data in a modified sweep algorithm. J. Ilmiah Elektron, 5(2): 13-28.

Saaty, T.L., 1980. The Analytic Hierarchy Process. McGraw-Hill, New York.

Saneifard, R. and R. Ezatti, 2010. Defuzzification through a bi-symmetrical weighted function. Aust. J. Basic Appl. Sci., 4(10): 4976-4984.

Sladoje, N., J. Lindblad and I. Nyström, 2011. Defuzzification of spatial fuzzy sets by feature distance minimization. Image Vision Comput., 29(2-3): 127-141.

Sugeno, M., 1985. An introductory survey of fuzzy control. Inform. Sciences, 36(1-2): 59-83.

Yeh, C.H. and Y.H. Chang, 2009. Modeling subjective evaluation for fuzzy group multicriteria decision making. Eur. J. Oper. Res., 194(2): 464-473.

Zadeh, L.A., 1965. Fuzzy sets. Inform. Control, 8(3): 338-353.

Zerafat, A.L.M., A. Emrouznejad, A. Mustafa and A. Rashidi Komijan, 2009. Selecting the most preferable alternatives in a group decision making problem using DEA. Expert Syst. Appl., 36(5): 9599-9602.