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Research Article

Efficient Estimation of Multiple Parameters with Application in Localization

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Abstract: This study deals with a general procedure for efficient estimation of multiple parameters. The proposed technique is based on an original theorem named *EXtended Invariance Principle* (EXIP), which is established in the study. The resulting method allows obtaining in some non-linear cases an efficient estimation without iterative procedures. This new approach is applied to localize a target and estimate its speed in multistatic radar.

Keywords: Array processing, high signal-to-noise ratio, maximum likelihood estimator, statistical efficient estimation

INTRODUCTION

In signal processing, the Maximum Likelihood Estimator (MLE) is known by its good statistical properties. In fact, this estimator is asymptotically efficient and Gaussian when the noise power tends to 0 (Cramér, 1946; Stoica and Nehorai, 1990; Ottersten *et al*., 1993). But, it needs an iterative algorithm (e.g., Gauss-Newton) requiring an ad-hoc initializing. That is why; it would be interesting to derive efficient estimators which involve the iterative procedure to avoid the associated difficult calculation of ML, keeping its good statistical properties. A first contribution in this study is a theorem, which provides a general method to find that estimators and a second contribution is to apply this theorem to a problem of localizing in multistatic radar. The proposal theoretical method can be considered as a generalization of *EXtended Invariance Principle* (EXIP) (Swindlehurst and Stoica, 1998).

This study is organized as follows: Section 2 presents the materials, methods and applications in localization. To confirm our results, simulations are performed in section 3. Finally, section 4 gives our conclusions.

MATERIALS AND METHODS

Theoretical model: A noisy observation $\mathbf{m} \in \Re^N$ is related to a vector of unknown parameters $\boldsymbol{\theta} \in \Re^{P_\theta}$ by the relation:

$$\mathbf{m} = \mathbf{s}(\boldsymbol{\theta}_0) + \mathbf{n}$$

where, $P_\theta \leq N$, s(θ) describes the dependence between the non-noisy measurements and the vector of parameters, n is and additive Gaussian noise with zero-mean and a well-known covariance matrix $\mathbf{Q} = \sigma^2 \mathbf{Q}_n$.

The MLE $\hat{\theta}_{\mathrm{ML}}$ of θ_0 is obtained by minimizing the following function (Amar and Weiss, 2008):

$$f(\boldsymbol{\theta}) = \left(\mathbf{m} - \mathbf{s}(\boldsymbol{\theta})\right)^T \mathbf{Q}_n^{-1} \left(\mathbf{m} - \mathbf{s}(\boldsymbol{\theta})\right)$$

Except for the case where s(θ) is a linear function, there is no an explicit solution to minimize *f*(θ).That is why; we derive a new theorem which provides efficient estimators.

Theorem: The theorem uses two parametrizations. The First one, without constraint (non-constrained), it is composed of several parameters, which have some unused relations with each other (Bilodeau and Brenner, 1999). The second one, with constraint (constrained), it is only composed of parameters without any relations with each other. The objective of the theorem is to exploit the non-constrained estimation results, in order to easily obtain an efficient estimation with the second parametrization. The estimation result will be sharper, as the last parametrization uses the relations between the parameters. Indeed, let n be a Gaussian vector of a well-known covariance matrix $\mathbf{Q} = \sigma^2 \mathbf{Q}_n$ and a mean of **s**depending on a set of

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unknown parameters. Let $\mathbf{u} = (u_1 \cdots u_{P_u})^T \in \Re^{P_u}$ be the first parametrization (non-constrained) of the mean, denoted by Π_u with an exact value u_0 and $\mathbf{v} = (v_1 \cdots v_{P_u})^T \in \Re^{P_v}$ be a second parametrization (constrained) denoted by Π_v (exact value v_0), which is more restrictive in the sense where $P_v < P_u$. Let u(v) be the values of parameters of the first parametrization reached by the second one. A simple calculation shows that the expressions of the Fisher Information Matrix (FIM) F_u and F_v for $\sigma = 1$are (Messer, 2006):

$$\mathbf{F_u} = (\partial_\mathbf{u}\mathbf{s})^T_{\mathbf{u_0}} \mathbf{Q}_n^{-1} (\partial_\mathbf{u}\mathbf{s})_{\mathbf{u}_0} \quad (1)$$

$$\mathbf{F_v} = (\partial_\mathbf{v}\mathbf{u})^T_{\mathbf{v_0}} \mathbf{F_u} (\partial_\mathbf{v}\mathbf{u})_{\mathbf{v}_0} \quad (2)$$

where, $(\partial_\mathbf{u}\mathbf{s})$ and $(\partial_\mathbf{v}\mathbf{u})$ are the matrices of partial derivatives $N \times P_u$ and $P_u \times P_v$ with respect to u and v. Let $\hat{u}$ be a non-constrained efficient estimator asymptotically Gaussian of u_0:

$$\left\{ \frac{1}{\sigma}(\hat{\mathbf{u}} - \mathbf{u}_0) \xrightarrow[\sigma \to 0]{a.d.} N(\mathbf{0}, \mathbf{F}_\mathbf{u}^{-1}) \right. \quad (3)$$

Note that $\hat{\mathbf{F}}_u$ is a consistent estimator of the FIM (e.g. the FIM calculated with the values of estimated parameters û instead of u_0). Now, consider a function g(u, v) from $\Re^{P_u} \times \Re^{P_v}$ to $\Re^{P_u}$ and let $(\partial_\mathbf{u}\mathbf{g}) = [\partial_{\mathbf{u}_1}\mathbf{g}(\mathbf{u},\mathbf{v}) \cdots \partial_{\mathbf{u}_{Pu}}\mathbf{g}(\mathbf{u},\mathbf{v})]$.

Theorem: The minimization of the criteria $\widetilde{C}(\mathbf{v}) = \|\mathbf{g}(\hat{\mathbf{u}},\mathbf{v}) - \mathbf{g}(\mathbf{u}(\mathbf{v}),\mathbf{v})\|^2$ conducts to a consistent estimator $\tilde{v}$ of v0. The minimization of the criteria:

$$\begin{aligned} C(\mathbf{v}) = & \left(\mathbf{g}(\hat{\mathbf{u}},\mathbf{v}) - \mathbf{g}(\mathbf{u}(\mathbf{v}),\mathbf{v})\right)^T \\ & \left((\partial_\mathbf{u}\mathbf{g})^{-T}_{\hat{\mathbf{u}},\hat{\mathbf{v}}} \hat{\mathbf{F}}_u (\partial_\mathbf{u}\mathbf{g})^{-1}_{\hat{\mathbf{u}},\tilde{\mathbf{v}}}\right) \\ & \left(\mathbf{g}(\hat{\mathbf{u}},\mathbf{v}) - \mathbf{g}(\mathbf{u}(\mathbf{v}),\mathbf{v})\right) \end{aligned} \quad (4)$$

Conducts to an efficient and asymptotically Gaussian estimator $\hat{v}$ of v_0:

$$\left\{ \frac{1}{\sigma}(\hat{\mathbf{v}} - \mathbf{v}_0) \xrightarrow[\sigma \to 0]{a.d.} N(\mathbf{0}, \mathbf{F}_\mathbf{v}^{-1}) \right.$$

where, $\hat{v}$ is a consistent estimator of v_0 and $\hat{F}_v$ is the estimated FIM from $\hat{F}_u$:

$$\hat{\mathbf{F}}_\mathbf{v} = (\partial_\mathbf{v}\mathbf{u}(\mathbf{v}))^T_{\hat{\mathbf{v}}} \hat{\mathbf{F}}_\mathbf{u} (\partial_\mathbf{v}\mathbf{u}(\mathbf{v}))_{\hat{\mathbf{v}}} \quad (5)$$

Proof: Consider the function $f_w(\mathbf{u},\mathbf{v}) = (\mathbf{g}(\mathbf{u},\mathbf{v}) - \mathbf{g}(\mathbf{u}(\mathbf{v}),\mathbf{v}))^T \mathbf{W} (\mathbf{g}(\mathbf{u},\mathbf{v}) - \mathbf{g}(\mathbf{u}(\mathbf{v}),\mathbf{v}))$ where W is a positive defined matrix. That function reaches its minimal value for u = u_0 and v = v_0. Now, it is easy to show that for u = û, the theorem of implicit functions applied to the gradient of F_W provides the following relation between the value of v (which minimize f_w(û,v)) and û.

$$\Delta\mathbf{v} = \left((\partial_\mathbf{v}\mathbf{u}^T \partial_u \mathbf{g}^T \mathbf{W} \ \partial_u \mathbf{g} \partial_v u)^{-1} (\partial_\mathbf{v}\mathbf{u}^T \partial_u \mathbf{g}^T \mathbf{W} \ \partial_u \mathbf{g}) \Delta_\mathbf{u} + o(\Delta_\mathbf{u}) \right) \quad (6)$$

where $\Delta\mathbf{u} = \hat{\mathbf{u}} - \mathbf{u}_0$ and $\Delta\mathbf{v} = \hat{\mathbf{v}} - \mathbf{v}_0$.

∂_v u and ∂_u g are evaluated at v_0 and (u_0, v_0).

The consistency of v can be immediately deduced from the expression (6); in particular, the consistency of $\tilde{v}$ in the theorem corresponds to W = I. In addition, one can write for the term $o(\Delta u)$ of Eq. (6):

$$\left\| \frac{1}{\sigma} o(\Delta\mathbf{u}) \right\| = \left\| \frac{1}{\sigma} \Delta\mathbf{u} \right\| \frac{\|o(\Delta\mathbf{u})\|}{\|\Delta\mathbf{u}\|}$$

when σ→0 in the above expression, (1/σ) Δu converges in law to Gaussian random vector (hypothesis (3)) and $\frac{\|o(\Delta\mathbf{u}\|)}{\|\Delta\mathbf{u}\|}$ converges in probability to 0. Therefore, $\left\| \frac{1}{\sigma} o(\Delta\mathbf{u}) \right\|$ converges in probability to 0 and we conclude according to (6) that $(1/\sigma)\Delta\mathbf{v}$ is asymptotically Gaussian with zero-mean and a covariance matrix of:

$$\begin{aligned} & (\partial_\mathbf{v}\mathbf{u}^T \partial_u \mathbf{g}^T \mathbf{W} \partial_u \mathbf{g} \partial_v \mathbf{u})^{-1} (\partial_\mathbf{v}\mathbf{u}^T \partial_u \mathbf{g}^T \mathbf{W} \partial_u \mathbf{g}) \mathbf{F}_u^{-1} \\ & (\partial_u \mathbf{g}^T \mathbf{W} \partial_u \mathbf{g} \partial_v \mathbf{u})(\partial_\mathbf{v}\mathbf{u}^T \partial_u \mathbf{g}^T \mathbf{W} \partial_u \mathbf{g} \partial_v \mathbf{u})^{-1} \end{aligned}$$

The fact $\mathbf{W}_{opt} = (\partial_u \mathbf{g}^{-T}) \mathbf{F_u} (\partial_u \mathbf{g}^{-1})$ conducts to the value $(\partial_v \mathbf{u}^T \mathbf{F_u} \partial_v \mathbf{u})^{-1}$ of the above asymptotic covariance of $(1/\sigma)\Delta\mathbf{v}$: this is the Cramer-Rao Bound according to (2) of FIM. Therefore, the minimization of $F_{wopt}(\hat{\mathbf{u}},\mathbf{v})$ conducts to an efficient and asymptotic Gaussian estimation of v_0. Finally, we can replace W_{opt} by any estimator $\hat{\mathbf{W}} = (\partial_\mathbf{v}\mathbf{g})^{-T}_{\hat{\mathbf{u}},\hat{\mathbf{v}}} \hat{\mathbf{F}}_\mathbf{u} (\partial_\mathbf{u}\mathbf{g})^{-1}_{\hat{\mathbf{u}},\hat{\mathbf{v}}}$ without changing the conclusions: the resulting criterion function $F_{\hat{W}}(\hat{\mathbf{u}},\mathbf{v})$ is just the criterion C(**v**) of the theorem.

The originality of our theorem with respect to method EXIP (Swindlehurst and Stoica, 1998) is the introduction of the function g in the criterion: in fact, the method EXIP minimizes a criterion of type $\|\hat{\mathbf{u}} - \mathbf{u}(\mathbf{v})\|^2$. Our theorem is particularly used when the function $\mathbf{g}(\hat{\mathbf{u}},\mathbf{v}) - \mathbf{g}(\mathbf{u}(\mathbf{v}),\mathbf{v})$ is linear with respect to **v**, since the criteria $\widetilde{C}(\mathbf{v})$ and $C(\mathbf{v})$ can be exactly minimized in this case. For example, one can easily show that the efficient and asymptotic estimator developed by Chan and Ho (1994) to the sources localization (Wan and Peng, 2002; Grosicki, 2003) for applications in mobiles localization) is a direct consequence of our theorem. In this study, we give an original algorithm to localize a target using a multistatic radar.

Application in localization: Multistatic radar has N transmitters localized at $(x_i, y_i)_{1\le i\le N}$ and one receiver at (0, 0). Target is supposed to be displaced in a plane. Its exact coordinates are denoted by (x_0, y_0) and its speed coordinates (v_{x0}, v_{y0}). Our vector of measurements $\mathbf{m} = (\delta_1 \cdots \delta_N \, \dot{\delta}_1 \cdots \dot{\delta}_N)^T$ consists of N propagation distances between the receiver and the transmitters 1, 2, …, N and Nobtained derivatives of the propagation distance by the Doppler measurements. Let us introduce some notations:

$$r_i = \sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2} \quad (1 \le i \le N)$$

and $r = \sqrt{x_0^2 + y_0^2}$

$$d_i = \frac{v_{x0}(x_0 - x_i) + v_{y0}(y_0 - y_i)}{\sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2}} \quad (1 \le i \le N)$$

and $d = \dfrac{v_{x0}.x_0 + v_{y0}.y_0}{\sqrt{{x_0}^2 + {y_0}^2}} \quad (1 \le i \le N)$

The expressions of exact measurements (noiseless) for the transmitter *i* are:

$$\delta_i = r_i + r \quad \text{and} \quad \dot{\delta}_i = d_i + d$$

Assume a Gaussian noise with zero-mean and a well-known covariance matrix $\mathbf{Q} = \sigma^2 \mathbf{Q}_n$ is added to the measurements. The developed procedure below, can be considered as a multiple application of our theorem. In the different steps of the method, we use u, v, ũ, ṽ, û and v̂ with the exponent $^{(i)}$ for the parametrization *i*.

Step 1:

***Parametrization* $\mathbf{\Pi_u}$:** $\mathbf{u}^{(1)}$ is equal to the vector of measurements m (vector of $\Re^{2N}$). Its FIM is $\hat{\mathbf{F}}_{u^{(1)}} = \mathbf{Q}^{-1}$. The objective now is to find a linear relation between the parameters $(x_0, y_0, v_{x0}, v_{y0})$ and $\mathbf{u}^{(1)}$ which yields new unknown parameters. In this context, Chan and Ho (1994) gives the method to obtain the following relation for the transmitter *i*:

$$u_i^{(1)^2} - 2.u_i^{(1)}.r = x_i^2 + y_i^2 - 2.x_0.x_i - 2.y_0.y_i \tag{7}$$

Note that the first part of measurements (propagation distances) and some exact parameters (x_0, y_0) are related by a linear relation, if we consider *r* as an unknown parameter. For the measurements of derivatives of the propagation distance, we have the following relation:

$$r_i.d_i - r.d = -v_{x0}.x_i - v_{y0}.y_i$$

We can obtain another relation:

$$r_i.d_i - r.d = r_i.d_i + r.d_i - r.d - r.d_i$$
$$r_i.d_i - r.d = u_{i+N}^{(1)}.u_i^{(1)} - d.u_i^{(1)} - r.u_{i+N}^{(1)}$$

Combining the obtained two equations, we will obtain the following relation:

$$u_{i+N}^{(1)}.u_i^{(1)} - d.u_i^{(1)} - r.u_{i+N}^{(1)} = -v_{x0}.x_i - v_{y0}.y_i \tag{8}$$

Equation (8) is a linear relation between $u^{(1)}$ and the exact parameters if we consider *d* as an unknown parameter.

Parametrization $\mathbf{\Pi_v}$: v(1) is a vector of $\Re^6$. Its exact value $v_0^{(1)} = (v_{01}\, v_{02}\, v_{03}\, v_{04}\, v_{05}\, v_{06})^T$ has some components related to $(x_0, y_0, v_{x0}, v_{y0})$ by $v_{01} = x_0$, $v_{02} = y_0$, $v_{03} = v_{x0}$, $v_{04} = v_{y0}$, $v_{05} = r$ and $v_{06} = d$.

The left side of Eq. (7) and (8) allows to obtain $\mathbf{g}(\mathbf{u}(\mathbf{v}), \mathbf{v})$, while the right side gives $\mathbf{g}(\mathbf{u}, \mathbf{v})$. It is possible to minimize the criterion $\widetilde{\mathbf{C}}(\mathbf{v})$ and then to yield a consistent estimator $\widetilde{\mathbf{v}}^{(1)}$. Using this consistent estimator, we can calculate the matrix (4) $\partial_{\mathbf{u}}\mathbf{g}$:

$$\frac{\partial \mathbf{g}}{\partial \mathbf{u}} = \begin{pmatrix} 2u_1^{(1)} - 2\widetilde{r} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 2u_N^{(1)} - 2\widetilde{r} & 0 & \cdots & 0 \\ u_{N+1}^{(1)} - \widetilde{d} & \cdots & 0 & u_1^{(1)} - \widetilde{r} & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & u_{N+N}^{(1)} - \widetilde{d} & 0 & \cdots & u_{N+1}^{(1)} - \widetilde{r} \end{pmatrix}$$

We can obtain an efficient estimator $\hat{\mathbf{v}}^{(1)}$ of $\mathbf{v}_0^{(1)}$ by minimizing the criterion $C(\mathbf{v})$. Finally, we calculate the FIM of $\mathbf{v}_0^{(1)}$ (2) using the following matrix $\partial_{\mathbf{v}}\mathbf{u}(\mathbf{v})$.

$$\frac{\partial \mathbf{u}(\mathbf{v})}{\partial \mathbf{v}} = \begin{pmatrix} -2x_1 & -2y_1 & 0 & 0 & 2u_1^{(1)} & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -2x_N & -2y_N & 0 & 0 & 2u_N^{(1)} & 0 \\ 0 & 0 & -x_1 & -y_1 & u_{N+1}^{(1)} & u_1^{(1)} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & -x_N & -y_N & u_{N+N}^{(1)} & u_N^{(1)} \end{pmatrix}$$

At the end of this first step, the problem is over parametrized (there are relations between parameters) and the estimation can be improved by taking into account the relations between parameters. The last conduct to the objective of the following steps, which can be running by successive applications of the proposed theorem. For more clarity, one only gives the flow chart of different steps.

Step 2: This step is only a change of variable.

***Parametrization* $\mathbf{\Pi_u}$:** $u^{(2)}$ is equal to the estimate $\hat{v}^{(1)}$ (vector of $\Re^6$). It means that $\hat{\mathbf{F}}_{u^{(2)}} = \hat{\mathbf{F}}_{v^{(1)}}$. Let us introduce two new parameters: $z_x = x.v_x$ and $z_y = y.v_y$.

***Parametrization* $\mathbf{\Pi_v}$:** $v^{(2)}$ is a vector of $\Re^6$. Its exact value $v_0^{(2)} = (v_{01}\, v_{02}\, v_{03}\, v_{04}\, v_{05}\, v_{06})^T$ has some components related to $(x_0, y_0, v_{x0}, v_{y0})$ by $v_{01} = x_0$, $v_{02} = y_0$, $v_{03} = z_{x0}$, $v_{04} = z_{y0}$, $v_{05} = r$ and $v_{06} = d$.

The above relations allow to determine $\mathbf{g}(\mathbf{u}(\mathbf{v}), \mathbf{v})$ and $\mathbf{g}(\mathbf{u}, \mathbf{v})$ and then we have an efficient estimator $\hat{\mathbf{v}}^{(2)}$ of $\mathbf{v}_0^{(2)}$. Finally, we find the FIM $\hat{\mathbf{F}}_{v^{(2)}}$ of $\hat{\mathbf{v}}^{(2)}$.

Step 3:

***Parametrization* $\mathbf{\Pi_u}$:** $u^{(3)}$ is equal to the estimate $\hat{\mathbf{v}}^{(2)}$ (vector of $\Re^6$). It means that $\hat{\mathbf{F}}_{u^{(3)}} = \hat{\mathbf{F}}_{v^{(2)}}$.

***Parametrization* $\mathbf{\Pi_v}$:** $\mathbf{v}^{(3)}$ is a vector of $\Re^5$. Its exact value $v_0^{(3)} = (v_{01}\, v_{02}\, v_{03}\, v_{04}\, v_{05})^T$ has some components related $(x_0, y_0, v_{x0}, v_{y0})$ by $v_{01} = x_0$, $v_{02} = y_0$, $v_{03} = z_{x0}$, $v_{05} = r$ and $v_{05}.d = z_{x0} + z_{y0}$.

Then we obtain the efficient estimate $\hat{\mathbf{v}}^{(3)}$ and its FIM $\hat{\mathbf{F}}_{v^{(3)}}$.

Step 4:

***Parametrization* $\mathbf{\Pi_u}$:** $\mathbf{u}^{(4)}$ is equal to the estimate $\hat{\mathbf{v}}^{(3)}$ (vector of $\Re^5$). It means that $\hat{\mathbf{F}}_{u^{(4)}} = \hat{\mathbf{F}}_{v^{(3)}}$.

***Parametrization* $\mathbf{\Pi_v}$:** $\mathbf{v}^{(4)}$ is a vector of $\Re^4$. Its exact value $v_0^{(4)} = (v_{01}\, v_{02}\, v_{03}\, v_{04})^T$ has some components related to $(x_0, y_0, v_{x0}, v_{y0})$ by $v_{01} = x_0^2$, $v_{02} = y_0^2$, $v_{03} = z_{x0}$, $v_{04} = z_{y0}$ and $v_{01} + v_{02} = r^2$.

Then we obtain the efficient estimate $\hat{v}^{(4)}$ and $\hat{\mathbf{F}}_{v^{(4)}}$.

Step 5: this step is only a change of variable.

***Parametrization* $\mathbf{\Pi_u}$:** $\mathbf{u}^{(5)}$ is equal to the estimate $\hat{v}^{(4)}$ (vector of $\Re^4$). It means that $\hat{\mathbf{F}}_{u^{(5)}} = \hat{\mathbf{F}}_{v^{(4)}}$.

***Parametrization* $\mathbf{\Pi_v}$:** $\mathbf{v}^{(5)}$ is a vector of $\Re^4$. Its exact value $v_0^{(4)} = (v_{01}\, v_{02}\, v_{03}\, v_{04})^T$ has some components related to $(x_0, y_0, v_{x0}, v_{y0})$ by $v_{01} = x_0$, $v_{02} = y_0$, $v_{03} = v_{x0}$ and $v_{04} = v_{y0}$.

Then we obtain the efficient estimate $\hat{\mathbf{v}}^{(5)}$ (and then $\hat{\theta}$) and its FIM $\hat{\mathbf{F}}_{v^{(5)}}$.

SIMULATIONS RESULTS

Suppose that we have 6 transmitters localized at $\{(0, 0.5); (10, 0); (8, 7); (0, 3); (4, 9); (1, 2)\}$ km. Target is at (5, 5) km and its speed is (80, 80) m/s. A white

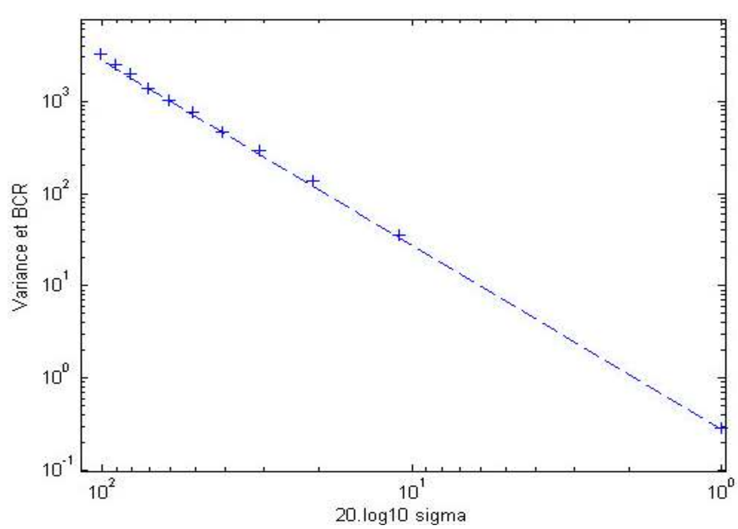

Fig. 1: CRB of $\hat{x}$ (-) and Variance of $\hat{x}$ (+)

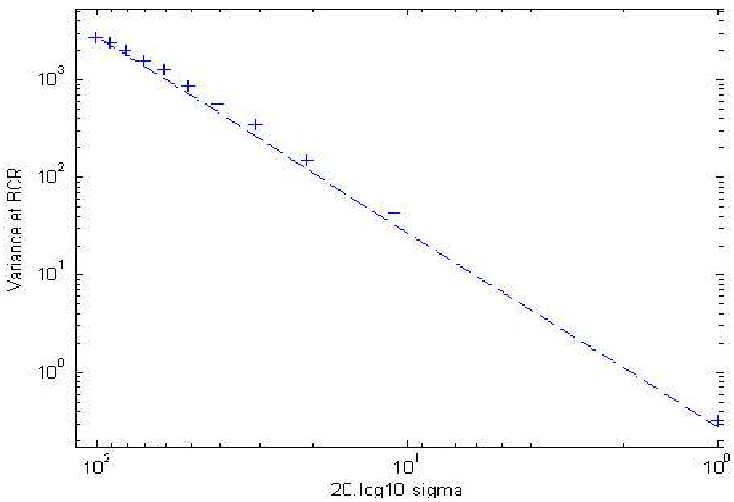

Fig. 2: CRB of $\hat{v}_x$ (-) and Variance of $\hat{v}_x$ (+)

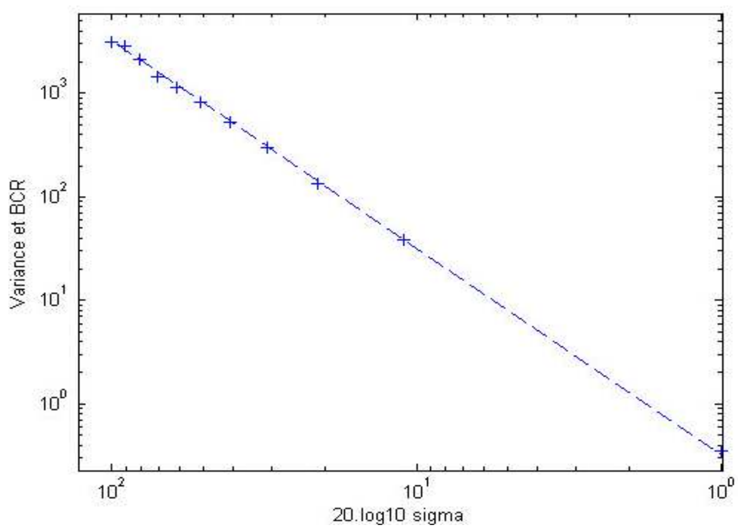

Fig. 3: CRB of $\hat{y}$ (-) and Variance of $\hat{y}$ (+)

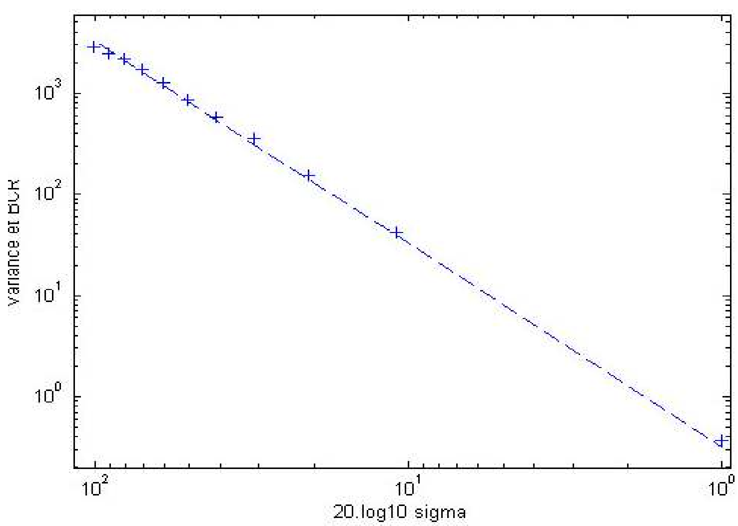

Fig. 4: CRB of $\hat{v}_y$ (-) and Variance of $\hat{v}_y$ (+)

Gaussian noise of covariance matrix $\mathbf{Q} = \sigma^2 \mathbf{I}_6$, is added to the measurements. Variances are estimated based on 1000 tries. They are compared to the Cramer-Rao Bound (CRB) for each parameter and sketched in terms of $20.\log_{10} \boldsymbol{\sigma}$. Figure 1 and 2 show us the results of $\hat{x}$ and $\hat{v}_x$, while the results for of $\hat{y}$ and $\hat{v}_y$ are shown on Fig. 3 and 4. Performances are very close to CRB and show that the criterion is asymptotically efficient by the proposed method.

CONCLUSION

The proposed method allows to obtain an efficient estimator without any iterative procedure. After the presentation of the theorem on which the method is based, we have applied to a problem of localization using a multistatic radar. Simulation results approve the theory.

Conflict of interest: Conflict of interest is in channel estimation, detection and array processing.

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