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Research Article

Stability Consideration of the Voltage Switched Charge-pump Phase Locked Loop using Linear Approach

[1]Ehsan Ali, [2]Abdul Sattar Sand, [3]Farida Memon, [2]Ghulam Mustafa Bhuto and [1]Nasreen Nizamani
[1, 2]Department of Electronic/Electrical Engineering, QUEST, Nawabshah, Pakistan
[3]Department of Electronic Engineering, MUET, Jamshoro, Pakistan

Abstract: The aim of this study to provide possible criteria to consider the stability of Voltage Switched Charge-Pump Pump Phase Locked Loop (VSCP-PLL). Stability of the conventional CP-PLL architecture is often determined by Gardner's stability plane, when PLL system is locked and near the fixed point. In the latest work it has been shown that, this stability criterion is not conservative enough when considering initial condition far from the fixed point. However, Gardner's criterion is based on the PLL operating with a Current Switched Charge Pump (CSCP), which provides an ideal pump current during switching states. Nevertheless, a Voltage Switched Charge Pump (VSCP) is often used in many commercially available CP-PLL circuits (like 4046 family). The VSCP offers design simplicity but generates a peculiar effect related to the non-constant current flowing through the electrical load of Loop Filter (LF) w. r. t. operating point (average input voltage) of the Voltage Controlled Oscillator (VCO). This non-linearity may complicate the PLL characteristics making it difficult to analyze the stability of the system, since there is no any particular criterion developed to study the stability of a VSCP-PLL. In this study, a procedure is proposed for PLL designers to apply similar Gardner's stability criterion developed for the CSCPPLL to a VSCP-PLL. The stability simulations are performed using an efficient discrete time Event-Driven (ED) model. The stability condition is observed for the initial operating conditions far from the fixed point by simulating the transient domain of the VSCP-PLL in the Gardner's plane considering symmetrical and asymmetrical pump current.

Keywords: Event driven technique, linear modelling, stability, voltage switched charge pump PLL

INTRODUCTION

The Charge Pump Phase Locked Loop (CP-PLL) (Fig. 1) is widely used synchronizing technique for various application found in wireless communication and coherent system (Best, 1984; Banerjee, 2006; Gardner, 1979) The mixed-signal nature of a CP-PLL is constraint to study and analyze the system behavior using general theory of control system, since it constitutes analog and digital components (Antao *et al.*, 1996). Different modeling methods are used to characterize the non-linear transient behavior of the PLLs. Most often CP-PLL designers verify the system parameters assuming the linear model of the system when CP-PLL is locked (Gardner, 1980). Using this early linearization approach, non-linear and non-ideal effects cannot be included in the linear model. Furthermore, the influence of the non-linear and the non-ideal effect in the steady state of system is significant. The CP-PLL is highly non-linear during acquisition process, since the Phase and Frequency Detector (PFD) is a edge-triggering device and reacts

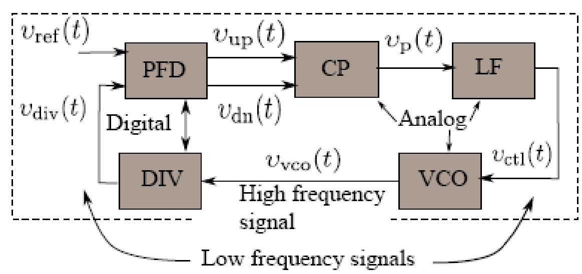


Fig. 1: The basic blocks of the CP-PLL for frequency synthesis

on the phase error (Paemel, 1994). Based on the characteristic curve of the PFD given in Best (1984) and Gardner (1980), it can be observed that the PFD is not able to differentiate between phase errors smaller and greater than 2π. Behavioral level models and Spice simulation at transistor level are often required to simulate transient domain of CP-PLL to finalize the design process. These techniques are time and computer resources consuming (Demir *et al.*, 1994; Hedayat *et al.*, 1997). To overcome these challenges, a

Corresponding Author: Ehsan Ali, Department of Electronic/Electrical Engineering, QUEST, Nawabshah, Pakistan

more rigorous and powerful modeling tool is necessary to characterize the on-locking and off-locking behavior of the mixed-signal PLLs. For these reasons the Event-Driven (ED) approach applied in Hedayat *et al*. (1997, 1999) and Wiegand *et al*. (2011) are more efficient and powerful in analysis than linear model and achieves speed-factor of Nx1000 comparing transistor level simulations (Ali *et al*., 2015). Since this ED model is based on simplification of the phase equations of the reference and feedback signal, It provides a powerful analysis tool to explore the non-linear behavior of the CP-PLL (Hedayat *et al*., 1997). When the CP-PLL is locked, its local dynamics can be further divided into two small scale ranges, the first part where the system is still ringing while having synchronized with input frequency having phase error variations $|\varphi_{\mathrm{err}}| < 2\pi$. Secondly, when the system is completely settled (near the steady state) with ideally constant or zero phase error (i.e., phase locking) (Gardner, 1980; Ali *et al*., 2013). Steady-state stability is a prime object in the PLL design process. By locating the roots of characteristics equations in the s-plane is often used to predict the stability of system (Mansukhani, 2000), whereas the CP-PLL is a sampled system which may go unstable with a small change in the internal state. Thus, it is important to consider the sampling effect of the non-linear pulse width modulated system in stability analysis. For this reason, Gardner's approximated criterion presented in Gardner (1980) is traditionally considered as the most useful stability limit established for 2^{nd} order current switched charge pump PLL (CSCP) but it not enough conservative criteria when system's initial conditions are far from fixed-point as reported later in Daniels and Farrell (2008) and Hangmann *et al*. (2014). Furthermore, Hangmann *et al*. (2014) shows that even the rule of thumb has some problems to correctly predict the stability of a CSCP-PLL. Since the design of an ideal CSCP which delivers ideally constant pump current during one switching period is highly challenging. In many commercially available PLL chips a VSCP delivering a constant voltage is preferred (Fairchild, 1984; Cleon *et al*., 2000 and Morgan, 2003). The advantages of utilizing a VSCP are the cheaper realization costs and the design simplicity. However, a Voltage Switched Charge Pump (VSCP) introduces a highly peculiar effect (Margaris and Petridis, 1985), since the current during one sampling period is not constant due to electrical load of LF circuit. This effect results in a varying (current) gain of the control systems and thus the tracking ability of the VSCP-PLL is significantly affected (Ali *et al*., (2015). To the best of our knowledge, there is no any particular criterion that exists for stability consideration of this typical architecture of the CP-PLL. Since it is not obvious to derive analytically the stability condition, only simulations are used to characterize the stability of the VSCP-PLL. Thus, it might be possible that the stability of the VSCP-PLL have similar problems as addressed in Hangmann *et al*. (2014) for a CSCP-PLL. The event switched macro-model of the VSCP-PLL was used to explore the stability limit of this system by setting initial conditions "near the fixed point" and Gardner's boundary was shown enough conservative (Ali *et al*., 2013). However, in practice PLL operating initial condition are not always near the fixed point. Therefore, in this study Gardner's stability limit is investigated in all condition of a VSCP-PLL, using symmetrical and asymmetrical pump currents and setting initial conditions far from fixed point.

MATERIALS AND METHODS

To investigate the PLL stability analytically, it is necessary to linearize the switching nature of the CP-PLL. Considering the linear transfer characteristics of the VCO and small variation in the phase error signal, closed loop transfer function based on the reference and divider signal can be derived by assuming linear characteristics of CP-PFD block. The obtained linear model of the PLL can be approximated as a Quasi-Time Continuous (QTC) model, predicts the macroscopic average behavior of the system (Gardner, 1980). The linearized model of the CP-PLL is represented as:

$$H(\mathrm{s}) = \frac{K_{\mathrm{v},\omega} K_{\varphi} H_{\mathrm{LF}}(\mathrm{s})}{Ns + K_{\mathrm{v},\omega} K_{\varphi} H_{\mathrm{LF}}(\mathrm{s})} \tag{1}$$

where $H_{\mathrm{LF}}(\mathrm{s})$ is the transfer function of the LF, $K_{\mathrm{v},\omega}$ is the linear gain of the VCO, $K_{\varphi} = \mathrm{I_p}/2\pi$ represents the current gain of the CP-PFD and 1/N represents the frequency divider ratio. This is the linear model based on QTC-approximation. However, due to switching nature of the PFD circuit, the CP-PLL is a highly stochastic system. Therefore, when CP-PLL is locked, the system follows very small phase and frequency variation resulting in very small phase error between the reference and the divider signal and considering the sampling effect of the loop, the stability condition for the second order CSCP-PLL was derived in Gardner (1980) which is given by:

$$K' = \frac{1}{\frac{\pi}{x}\left(1 + \frac{\pi}{x}\right)} \tag{2}$$

where, $x = \omega_{\mathrm{ref}}\tau_1$ and $K' = K\tau_1$ is called normalized loop gain of the system and ω_{ref} is the angular reference frequency (in Eq. (2)). Similar boundaries had been obtained by post-linearized autonomous

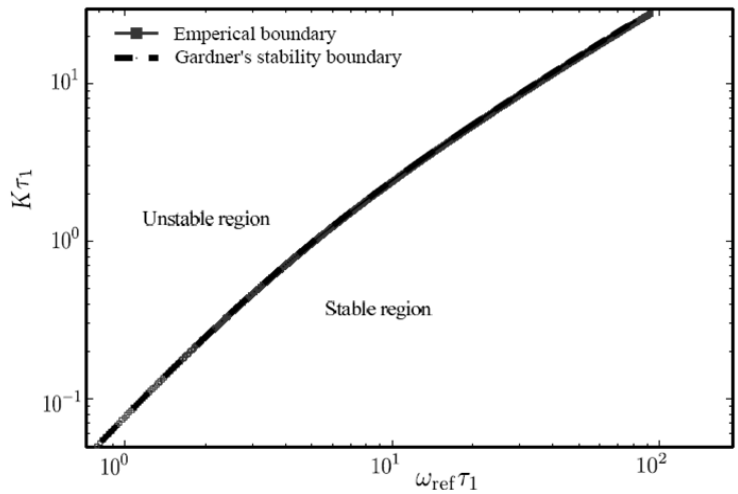


Fig. 2: Gardner's stability boundary and empirical boundary

expressions considering the locked state of the system in Paemel (1994). Another method is to use the *Empirical* condition given in Encinas (1993) is:

$$\frac{\omega_{\mathrm{ref}}}{\omega_{\mathrm{n}}} = a \tag{3}$$

If the estimated loop bandwidth (ω_{n}) of the CP-PLL is small comparing the frequency of the reference signal or more precisely the ratio a = 10. This condition regulates the stable operation of the system according to its sampling ratio $\omega_{\mathrm{ref}}/\omega_{\mathrm{n}}$ (Encinas, 1993). Typically, utilizing the Gardner's boundary (2) and using the empirical stability (3) in (2) is simulated as shown in Fig. 2. It can be stated that, the *Empirical* stability boundary is nearly the same as the Gardner's discrete-time approximation.

The linear model related to the voltage witched CP-PLL is presented and compared in the next section.

Linear model of the VSCP-PLL: The VSCP offers simplified design, but leads to a highly non-linear phenomenon of non-constant pump current $i_{\mathrm{p}}(t)$. The corresponding pump current during each PFD transition cycle is then represented as:

$$i_{\mathrm{p}}(t) = \frac{\upsilon_{\mathrm{p}}(t) - \upsilon_{\mathrm{C1}}(t)}{R_0 + R_1} \tag{4}$$

The relation (4) divulge an asymmetrical $i_{\mathrm{p}}(t)$ except if operated at the middle range of charge-pump supply. This typical topology of the VSCP produces an unbalanced current $i_{\mathrm{p}}^{\mathrm{up}} \neq i_{\mathrm{p}}^{\mathrm{dn}}$ throughout the acquisition and in the locking region. Figure 3 it can be noted that, the slope α^{vs} of the υ_{ctrl} is not constant with VSCP due to the decaying current $i_{\mathrm{p}}(t)$, on contrary α^{cs} is constant in the CSCP topology:

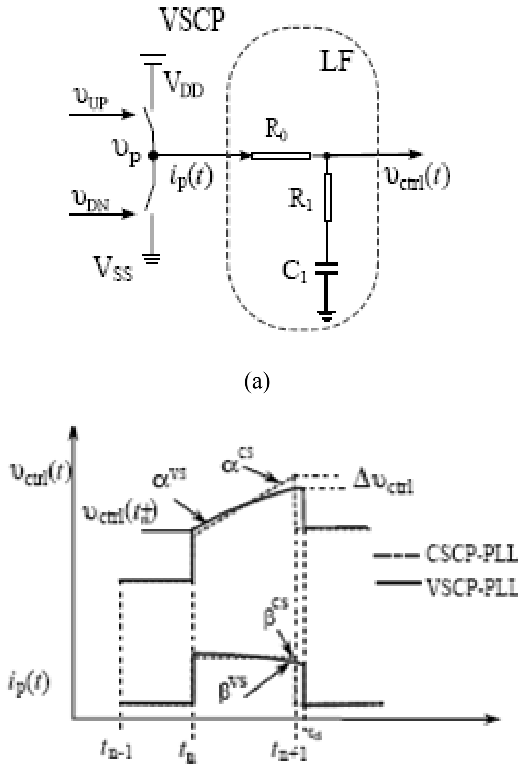
$$\alpha^{\mathrm{vs}} = \frac{i_{\mathrm{p}}(t)e^{\frac{-\Delta t}{\tau}}}{C_1} \text{ and } \alpha^{\mathrm{cs}} = \frac{i_{\mathrm{p}}(t)}{C_1} \tag{5}$$

$$\beta^{\mathrm{vs}} = i_{\mathrm{p}}(t)(1 - e^{\frac{-\Delta t}{\tau}}) \text{ and } \beta^{\mathrm{vs}} = i_{\mathrm{p}}(t) \tag{6}$$

where, $\Delta t = t_{\mathrm{n+1}} - t_{\mathrm{n}}$ and $\tau = (R_0 + R_1)C_1$. The non-linear terms appearing in (5) and (6) makes the analysis very complicated and a more serious issue in frequency tracking applications. Since the VSCP-PLL is rising faster than CSCP-PLL when $\upsilon_{\mathrm{ctrl}} < \frac{V_{\mathrm{DD}}}{2}$ due to higher current and slowdowns the system when $\upsilon_{\mathrm{ctrl}} > \frac{V_{\mathrm{DD}}}{2}$. Due to this nonlinearity, it can be seen that, VCO falling edge occurs after a little delay (τ_d) and $\Delta\upsilon_{\mathrm{ctrl}}$ appears when voltage switch goes off (when comparing both models with CSCP and VSCP) as shown in Fig. 3b. However, if initial conditions are close to the target value then at $\frac{V_{\mathrm{DD}}}{2}$ both system behaves nearly the same. Considering only the symmetrical current condition (i.e. when the PLL is operating at middle range of the supply voltage) a linear model can be derived. The pole-zero loop filter has a transfer function:

$$F(s) = \frac{1 + s\tau_0}{1 + s(\tau_0 + \tau_1)} \tag{7}$$

where, $\tau_0 = R_0\ C_1$ and $\tau_1 = R_1\ C_1$. The root of the LF transfer function increases the complexity in the phase transfer function of the PLL. From the VSCP-PLL model, the phase transfer function $H^{\mathrm{vs}}(s)$ is derived as:



(a)

(b)

Fig. 3: (a): The VSCP-LF configuration; (b): The non-linear phenomenon of the VSCP comparing with the CSCP

$$H(s) = \frac{\Phi_{\text{div}}(s)}{\Phi_{\text{ref}}(s)} = \frac{\dfrac{K_{\text{v},\omega} K_{\text{d}}^{\text{vs}} F(\text{s})}{N}}{s + \dfrac{K_{\text{v},\omega} K_{\text{d}}^{\text{vs}} F(\text{s})}{N}} \tag{8}$$

where gain of the PFD is $K_{\text{d}}^{\text{vs}} = \dfrac{V_{\text{sat+}} - V_{\text{sat-}}}{4\pi}$ and $V_{\text{sat+}}$ and $V_{\text{sat-}}$ belongs to supply levels V_{DD} and V_{SS} respectively (Best, 1984):

$$H^{\text{vs}}(s) = \frac{\omega_{\text{n}}^{\text{vs}} s \left(2\zeta^{\text{vs}} - \dfrac{\omega_{\text{n}}^{\text{vs}}}{\dfrac{K_{\text{v},\omega} \text{V}_{\text{DD}}}{2N}} \right) + \omega_{\text{n}}^{\text{vs}2}}{s^2 + 2\zeta^{\text{vs}} \omega_{\text{n}}^{\text{vs}} s + \omega_{\text{n}}^{\text{vs}2}} \tag{9}$$

The damping factor ζ^{vs} and natural angular frequency $\omega_{\text{n}}^{\text{vs}}$ of the voltage switched system is represented as:

$$\zeta^{\text{vs}} = \frac{\tau_1 \omega_{\text{n}}^{\text{vs}}}{2} + \frac{\omega_{\text{n}}^{\text{vs}}}{2} \frac{1}{\dfrac{K_{\text{v},\omega} V_{\text{DD}}}{2N}}, \quad \omega_{\text{n}} = \sqrt{\frac{K_{\text{v},\omega} i_{\text{p}}}{2\pi N C_1}} \tag{10}$$

In the current switched architecture, ω_{n} and ζ are the unique due to mirrored up and down pump current sources (Gardner, 1980) but this is generally not the case when considering a VSCP-PLL (Margaris and Petridis, 1985). If the ζ^{vs} is compared with that derived from CSCP-PLL, then it can be seen that, first part in (9) is equal to ζ of the CSCP-PLL with a residual term. It can be noticed that when $R_1 = 0$, $\zeta = 0$ while $\zeta^{\text{vs}} = \dfrac{\omega_{\text{n}}^{\text{vs}} N}{K_{\text{v},\omega} V_{\text{DD}}}$. The parameters $\dfrac{K_{\text{v},\omega} V_{\text{DD}}}{2N}$ appearing in (9) is called loop gain (Best, 1984). In the next section, approximated pump current conditions are presented.

Pump current conditions: Since the assumptions mentioned in the previous section (operating in the middle range of the supply voltage and the high gain loop) are rarely fulfilled, it is necessary to investigate the stability of the VSCP-PLL in a general approach. To do so, assuming different current situations we have observed five case studies with one symmetrical and four in asymmetrical conditions. The pump current is decreasing when the voltage across the capacitor is increasing. When the target voltage is higher than $V_{DD}/2$ then the up-current is smaller than the down current and one can call that asymmetry.

Symmetrical condition: The VSCP-PLL behaves similar to the CSCP-PLL, since the current flowing in the up and down half-cycles is equal, *i.e.*, $i_{\text{p}}^{\text{up}} = i_{\text{p}}^{\text{dn}}$.

If the target control voltage $\upsilon_{\text{ctrl(tar)}} \neq \dfrac{V_{\text{DD}}}{2}$ the pump current in one half cycle is greater than other half cycle and two asymmetrical situations arises:

$$\upsilon_{\text{ctrl(tar)}} < \frac{V_{\text{DD}}}{2} \text{ then } i_{\text{p}}^{\text{up}} > i_{\text{p}}^{\text{dn}}$$

$$\upsilon_{\text{ctrl(tar)}} > \frac{V_{\text{DD}}}{2} \text{ then } i_{\text{p}}^{\text{up}} < i_{\text{p}}^{\text{dn}}$$

To use Gardner's boundary for the evaluation of the VSCPPLL stability, Ali *et al*. (2013) proposed to use the maximum value of the current when considering the PLL to be near the steady state. However in this study we are considering both maximum and minimum pump currents in each asymmetrical condition:

$$I_{\text{p}} = \max(i_{\text{p}}^{\text{up}}, i_{\text{p}}^{\text{dn}}) \text{ and } I_{\text{p}} = \min(i_{\text{p}}^{\text{up}}, i_{\text{p}}^{\text{dn}}) \tag{11}$$

Event driven modeling technique: As explained earlier, it is not obvious to predict the stable operation of the VSCP-PLL exactly with an analytic expression. Thus, a simulation model is used to characterize the stability of the non-linear system. Since the PLL

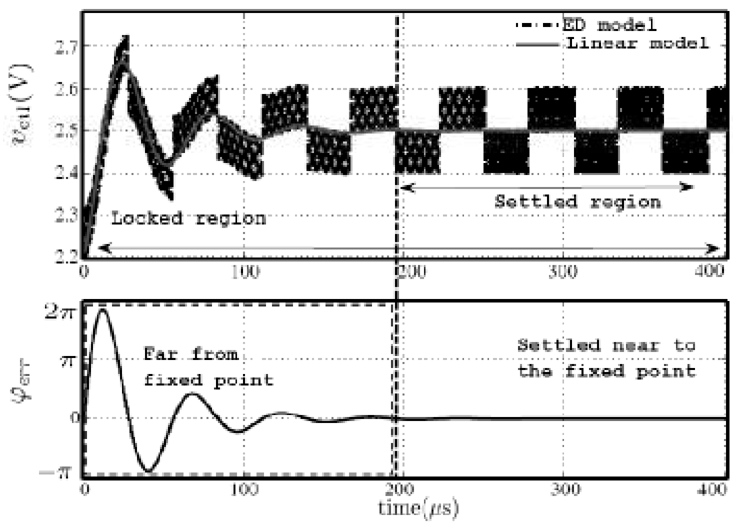


Fig. 4: Comparison of the ED and linear model of VSCP-PLL

exhibits a triggered behavior it is natural to use an Event-Driven model. The concept of ED technique is to calculate the commutation instants when incoming signals completes cycle using the phase equations as explained in Hedayat *et al*. (1997, 1999).

Considering the edge triggering nature of the system and integrating frequencies of the reference (ω_{ref}) and divider (ω_{div}) signals between an interval Δt, the phase of the reference (φ_{ref}) and the divider (φ_{div}) signals are calculated as:

$$\varphi_{\text{ref}}\left(t_{n+1}^{\text{ref}}\right) = \varphi_{\text{ref}}\left(t_n\right) + \int_{t_n}^{t_{n+1}^{\text{ref}}} \omega_{\text{ref}}(t')\,dt' = 2\pi \tag{12}$$

$$\varphi_{\text{div}}\left(t_{n+1}^{\text{div}}\right) = \varphi_{\text{div}}\left(t_n\right) + \int_{t_n}^{t_{n+1}^{\text{div}}} \omega_{\text{div}}(\upsilon_{\text{ctl}}(t'))\,dt' = 2\pi \tag{13}$$

where t_{n+1}^{ref} and t_{n+1}^{div} represents the time of the triggering edge occurring from the reference and divider signals. The effective triggering edge t_{n+1} which controls the system dynamics to switch the PFD into new state then corresponds to:

$$t_{n+1} = \min(t_{n+1}^{\text{ref}}, t_{n+1}^{\text{div}}) \tag{14}$$

Knowing the next triggering event, all signals at this time instant can be calculated as explained in Hedayat *et al*. (1997). As an example, the control voltage (using the LF topology of Fig. 3) is determined in the following way

$$\upsilon_{\text{ctrl}}\left(t_{n+1}\right) = \upsilon_{\text{ctrl}}\left(t_n^+\right) + \left(\upsilon_{\text{p}}(t) - \upsilon_{\text{ctrl}}\left(t_n^+\right)\right)\left(1 - e^{-\gamma_2\left(t_{n+1}-t_n\right)}\right) \tag{15}$$

where, $\gamma_2 = \frac{1}{(R_0+R_1)C_1}$ and the pump voltage $\upsilon_p(t)$ belongs to $\{V_{\text{DD}},\ \upsilon_{\text{ctrl}}(t),\ V_{\text{SS}}\}$ during PFD states $\{S_{+1}, S_0, S_{-1}\}$. The $\upsilon_{\text{ctrl}}\left(t_n^+\right)$ is the voltage step after the voltage pump is switched on and off (as shown in Fig. 3).

A comparison of the ED and linear model is shown in Fig. 4. Both models are locked and it can be seen that Event-Driven simulation is representing the switching behavior of the PLL and voltage jumps over the load of LF are obvious. However, the analytic model is predicting the approximated average behavior of the system. Furthermore, Fig. 4 (down part) demonstrates that, when the system is locked, initially the phase error is still ringing (until 200µs transient behavior), this part is referred as "far from fixed point". After this condition, the phase error of the system is nearly constant, since the system is settling to the target value. This part of the system is called "near to fixed point".

RESULTS AND DISCUSSION

To evaluate the validity of Gardner's stability boundary when applied to the VS-CPPLL a simulative stability characterization is presented in this section. The procedure of this simulative characterization of the stability condition is defined as follows. Initially, parameters of system are chosen in such a way that the transient behavior of the system is stable within the Gardner's in stability plane. Then a set of simulations (taking initial conditions far from fixed point) are performed decreasing successively the parameter $x = \omega_{\text{ref}}\tau_1$ by lowering the reference angular frequency ω_{ref}. To keep the target control voltage υ_{ctrl} at a constant level, the effect of declining ω_{ref} is systematically compensated by modifying proportionally $K_{v,\omega}$ and R_0.

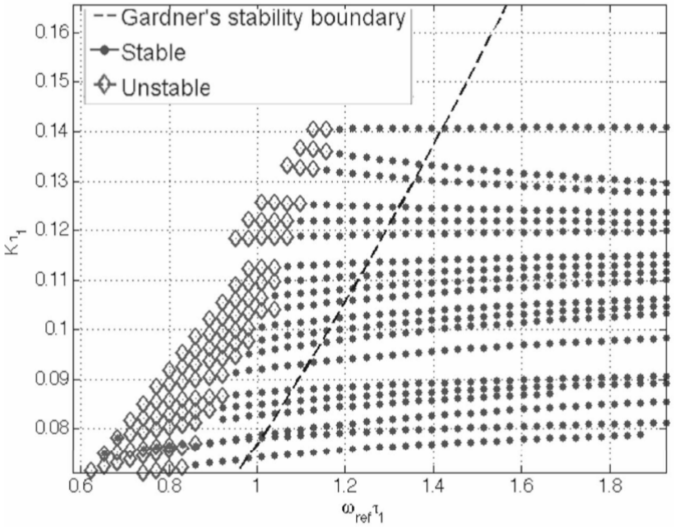


Fig. 5: The stability simulation results when $\upsilon_{ctrl} = V_{DD}/2$ with symmetrical current $i_p^{up} = i_p^{dn}$

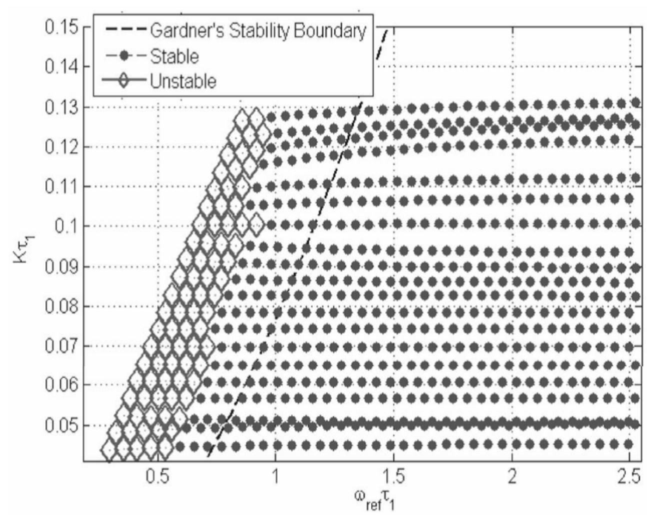


Fig. 6: The stability simulation results when $\upsilon_{ctrl} > V_{DD}/2$ with symmetrical current $I_p = i_p^{dn}$

This modification leads to a quasi horizontal line of transient simulations crossing of the Gardner's stability limit. These transient simulations were performed using the iterative ED model introduced in the previous section. Different notations are used for clarity in the results, where, • represents the stable system converging to the target voltage. ♦ represents the unstable system with static amplitude of the oscillation and x represents that the system still converging to a target voltage with a phase error less than 10-15 radians.

Considering the symmetrical pump current condition (equal pump currents in the up and down half-cycles) the simulation results depicted in Fig. 5 can be obtained. The system remains stable over the boundary vicinity. By penetrating deeper in the Gardner's stable zone, system finally finishes by becoming unstable. In the symmetrical condition, it can be stated that, the Gardner's stability is condition is enough conservative. Simulating the first asymmetrical condition $\upsilon_{ctrl} > V_{DD}/2$ the results of Fig. 6 and 7 can be observed. Here two different currents with higher and lower magnitudes are used in (2) to evaluate the validity of Gardner's boundary. Utilizing the higher magnitude of $I_p = i_p^{dn}$ it can be seen that Gardner boundary is a conservative condition, since unstable simulations occurs only on the left side of Gardner's stability limit. Different to the above simulations, Gardner's boundary is not conservative enough when considering lower value of $I_p = i_p^{up}$, since a lot of unstable simulations lies at the right side of Gardner stability limit. When simulating the second asymmetrical condition $\upsilon_{ctrl} < V_{DD}/2$, for the higher current $I_p = i_p^{up}$, the Gardner's boundary is more

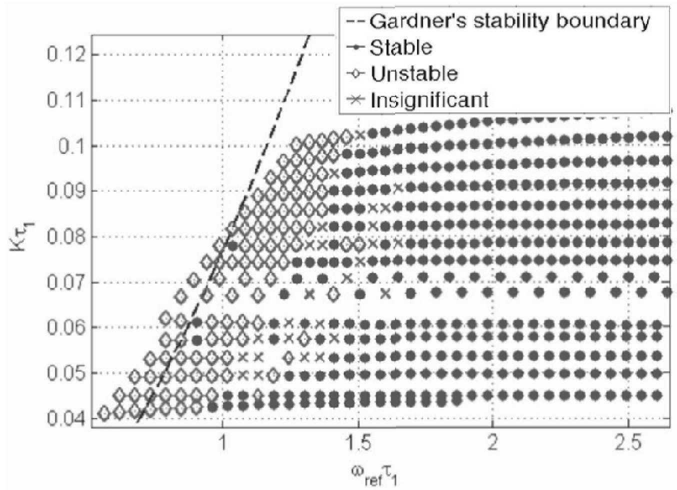


Fig. 7: The stability simulation results when $\upsilon_{ctrl}>V_{DD}/2$ with symmetrical current $I_p = i_p^{up}$

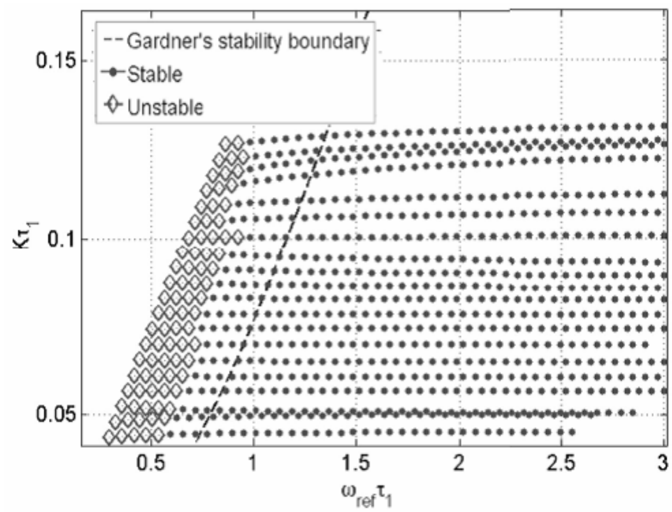


Fig. 8: The stability simulation results when $\upsilon_{ctrl}<V_{DD}/2$ with symmetrical current $I_p = i_p^{up}$

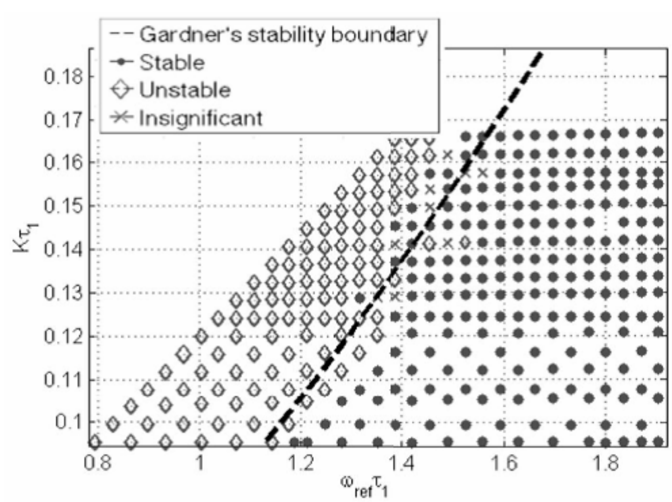


Fig. 9: The stability simulation results when $\upsilon_{ctrl}<V_{DD}/2$ with symmetrical current $I_p = i_p^{\mathrm{dn}}$

conservative as shown in Fig. 8. However utilizing the lower value of $I_p = i_p^{dn}$ makes some transient simulations unstable as shown in Fig. 9. These investigations show that it is sufficient to use Gardner's stability boundary when designing a conservative VSCP-PLL regarding stability when using the maximum pump current of the system. Since these investigations are performed using initial conditions far from the fixed point, it can be assumed that Gardner's boundary, as used in this study, is sufficient for all initial conditions.

CONCLUSION

In this study, the Gardner's stability plane derived for the CSCP-PLL was applied to the VSCP-PLL by assuming pump current conditions (given in section V). Using the simulative characterization it was shown that, Gardner's boundary is still more conservative for initial conditions far from fixed point when the higher magnitude of pump current is used. However the stability boundary is not enough conservative when using a lower magnitude of pump current and have problems similarly as investigated in Daniels and Farrell (2008) and Hangmann *et al*. (2014) for the CSCPPLL. Thus, Gardner's boundary can be used for the voltage switched CP-PLL considering all the initial operating conditions and considering the pump current described in this study, it is still useful to apply Gardner's plane.

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