

Research Article

Annual Maximum Exchange Rate in Southeast Asia Based on Methods of L-moment and Maximum Likelihood

¹Akbarizan, ²Muhammad Marizal, ²Rado Yendra, ³Kusaeri, ²Aripani Despina, ⁴Kamarulzaman Ibrahim and ⁵Ahmad Fudholi

¹Department of Law and Syari'ah,

²Department of Mathematics, Faculty of Science and Technology, Universitas Islam Sultan Syarif Kasim (UIN Suska), 28293 Pekanbaru, Riau,

³Department of Mathematics and Science Education, Faculty of Education and Teacher Training, Islamic State University of Sunan Ampel, Surabaya, Indonesia

⁴School of Mathematical Sciences, Faculty of Science and Technology,

⁵Solar Energy Research Institute, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor, Malaysia

Abstract: The main objective of this study is to determine the best fitting distribution to describe the annual series of maximum daily exchange rate from 1993 to 2013 for 4 countries in Southeast Asia based on L-moment and Maximum Likelihood (ML). Four three-parameter extreme-value distributions which are considered are Generalized Extreme Value (GEV), Generalized Logistic Distribution (GLD), Generalized Pareto Distribution (GPD) and Pearson III (P3). The estimation of parameters of these distributions is determined using the L-moment and maximum likelihood. The adequacy of the distributions based on parameter estimates computed using the two methods are evaluated using goodness-of-fit tests. When the goodness-of-fit results for these distributions are compared, it is found that, on the average, the performance of L-moment is better than the performance of maximum likelihood. Although the best fitting distribution may vary according to the method of estimation and country considered, in most cases, data for the majority of the several countries are found to follow the generalized logistic distribution.

Keywords: Annual maximum series, exchange rate, extreme exchange rate analysis, L-moment, maximum likelihood

INTRODUCTION

During the last several decades, exchange rate movement has become an important subject of macroeconomic analysis. However, despite its importance and much effort at constructing models, forecasting the exchange rate is still a challenge for academics and market practitioners.

The collapse of the Bretton Woods system of fixed exchange rates among major industrial countries marked the beginning of the Floating Exchange Rate Regime and exchange rate movement forecasting. However, empirical results stemming from various models in the literature, based on either fundamental economic principles or sophisticated statistical construction, could hardly satisfy and convince academics. Mussa (1979) argues that the spot exchange rate approximately follows a random walk process and most changes in exchange rates are in fact unexpected.

Besides, the seminal result of Meese and Rogoff (1983) shows that none of the structural exchange rate models used in their paper could significantly outperform a simple drift less random walk model in both short and medium terms. Even when the in-sample prediction of exchange rate performs well, the out-of-sample forecast is always disappointing when compared to that of random walk model. However, many researchers are focusing their research on a model that can predict the conditions that will come. Therefore, important to establish a model in the form of extreme distributions to analyze data exchange extremes that can assist in the study predict the movement of currency exchange at random.

In this study, an analysis of extreme exchange rate event has been undertaken using the annual series of maximum daily exchange rate data for the period of 20 years at 4 counties in Southeast Asia. The analysis involves fitting four extreme value distributions

Corresponding Author: Ahmad Fudholi, Solar Energy Research Institute, Universiti Kebangsaan Malaysia, 43600 Bangi, Selangor, Malaysia

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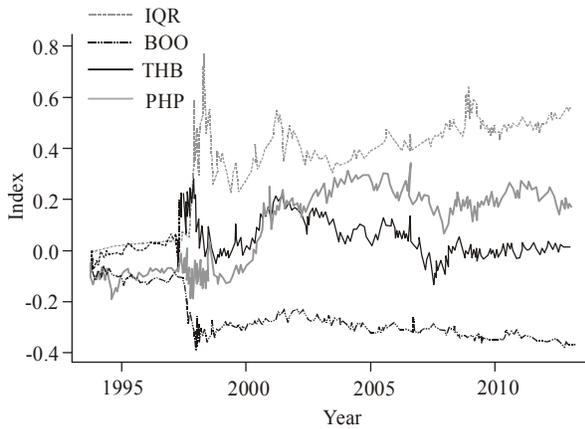


Fig. 1: Index exchange rate fourth county with Ringgit Malaysia in 1993-2013

which are Generalized Extreme Value distribution (GEV), Generalized Logistic Distribution (GLD), the Generalized Pareto Distribution (GPD) and Pearson distribution (P3). The estimation of parameters of these distributions is determined using the L-moment and Maximum likelihood. Comparison is made to determine the most suitable distribution to describe the extreme data for each country using several goodness-of-fit tests. Processing and simulation data was used the R programming language.

In the following section, we describe the data used along with the descriptive statistics involving mean, standard deviation and coefficient to variation for the annual extreme daily exchange rate. The next section describes the method of parameter estimation and goodness-of-fit procedures. The results of the analysis are discussed in a subsequent section and finally, a short discussion on the overall findings and conclusion is presented.

Data: The data consisting of annual maximum daily exchange rate from 4 countries in Southeast Asia from 1993 to 2013 have been obtained from www.oanda.com. In this website there are three types of data exchange among them is *aks*, bid and mid. *Aks* is based on the data type of offer. Bid is based on demand data. Furthermore mid is the data type of a value taken from the average of supply and demand. The data taken in this study is the data type of mid Malaysian Ringgit versus Singapore Dollar (SGD), Indonesian Rupiah (IDR), Thai Bath (THB) and Philipina Peso (PHP) (Fig. 1).

METHODOLOGY

Probability distributions for extreme exchange rate:

The probability density function (pdfs) and quantile function for each distribution that we consider are as given in Table 1 where y denotes the observed values of the random variable representing the event of interest, α is the scale parameter, μ is the location parameter and κ is the shape parameter of the distribution.

When we fit the models to the data, we consider two methods of parameter estimation which are the L-moments (LMOM) and Maximum Likelihood (ML). Details on these methods are given the following subsections.

L-moments: The L-moments are the summary statistics for probability distributions and data samples and are analogous to ordinary moments (Hosking, 1990). They provide measures of location, dispersion, skewness, kurtosis and other aspects of the shape of probability distributions or data samples. As described by Vogel

Table 1: List of distributions used in this study

Distribution	Probability Distribution Function	Cumulative Function	Quantile Function
GEV	$f(x) = \alpha^{-1} \exp\{-(1-\kappa)y - \exp(-y)\}$ with $y = \begin{cases} -\kappa^{-1} \log\{1 - \kappa(x - \varepsilon)/\alpha\}, & \kappa \neq 0 \\ (x - \varepsilon)/\alpha, & \kappa = 0 \end{cases}$ $-\infty < x < \varepsilon + \alpha/\kappa$ for $\kappa > 0$ $-\infty < x < \infty$ for $\kappa = 0$ $\varepsilon + \alpha/\kappa \leq x < \infty$ for $\kappa < 0$	$F(x) = \begin{cases} \exp\left(-\left(1 - \frac{\kappa}{\alpha}(x - \varepsilon)\right)^{\frac{1}{\kappa}}\right), & \text{if } \kappa \neq 0 \\ \exp\left(-\exp\left(-\frac{1}{\alpha}(x - \varepsilon)\right)\right), & \text{if } \kappa = 0 \end{cases}$	$Q(F) = \varepsilon + \frac{1}{\kappa} \left(1 - (-\ln(F))^{\kappa}\right)$
GPD	$f(x) = \frac{1}{\alpha} \left(1 - \frac{\kappa}{\alpha}(x - \varepsilon)\right)^{\frac{1-\kappa}{\kappa}}$ $\varepsilon \leq x < \infty$ for $\kappa \leq 0$ $\varepsilon \leq x \leq \varepsilon + \alpha/\kappa$ for $\kappa > 0$	$F(x) = 1 - \left(1 - \frac{\kappa}{\alpha}(x - \varepsilon)\right)^{\frac{1}{\kappa}}$	$Q(F) = \varepsilon + \frac{\alpha}{\kappa} (1 - (1 - F)^{\kappa})$
GLD	$f(x) = \frac{\alpha^{-1} \exp\{-(1-\kappa)y\}}{(1 + \exp(-y))^2}$ with $y = \begin{cases} -\kappa^{-1} \log\{1 - \kappa(x - \varepsilon)/\alpha\}, & \kappa \neq 0 \\ (x - \varepsilon)/\alpha, & \kappa = 0 \end{cases}$ $-\infty < x < \varepsilon + \alpha/\kappa$ for $\kappa > 0$ $-\infty < x < \infty$ for $\kappa = 0$ $\varepsilon + \alpha/\kappa \leq x < \infty$ for $\kappa < 0$	$F(x) = \left(1 + \left(1 - \kappa \left(\frac{x - \varepsilon}{\alpha}\right)^{\frac{1}{\kappa}}\right)\right)^{-1}$	$Q(F) = \varepsilon + \frac{\alpha}{\kappa} \left(1 - \left(\frac{1-F}{F}\right)^{\kappa}\right)$
P3	$f(x) = \frac{1}{\kappa(x - \varepsilon)\sqrt{2\pi}} \times \exp\left(-\frac{1}{2\kappa^2} (\ln(x - \varepsilon) - \alpha)^2\right)$	$F(x) = \Phi\left(\frac{\ln(x - \varepsilon) - \alpha}{\kappa}\right)$	$Q(F) = \varepsilon + \frac{\alpha}{\kappa} \left(1 - \exp(-\kappa\Phi^{-1}(F))\right)$

and Fennessey (1993), L-moment should be more preferable for small sample sizes due to its robust property.

For the random variable y_1, y_2, \dots, y_n of sample size n drawn from the distribution of a random variable Y with probability density function $F(y)$ and quantile function $Q(F)$. Let $y_{1:n} \leq y_{2:n} \leq \dots \leq y_{n:n}$ be the order statistics such that the L-moments of Y are defined by:

$$\lambda_t = \frac{1}{t} \sum_{i=0}^{t-1} (-1)^i \binom{t-1}{i} E[Y_{i:t}], \quad t=1, 2, \dots, n$$

where, t is the t th L-moment of a distribution and $E[Y_{i:t}]$ is the expected value of the i th smallest observation in a sample of size t . The first four L-moments of a random variable Y can be written as:

$$\lambda_1 = E[Y] \quad \lambda_2 = \frac{1}{2} E[Y_{2:2} - Y_{1:2}],$$

$$\lambda_3 = \frac{1}{3} E[Y_{3:3} - 2Y_{2:3} + Y_{1:3}],$$

And:

$$\lambda_4 = \frac{1}{4} E[Y_{4:4} - 3Y_{3:4} + 3Y_{2:4} - Y_{1:4}].$$

Hosking (1990) demonstrated the utility of estimators based on the L-moment ratios in hydrological extreme analysis. The second moment is often scaled by the mean so that a coefficient of variability is determined:

$$\tau = \frac{\lambda_2}{\lambda_1}$$

where, λ_1 is the measure of location. Similar to the definitions and the meaning of the ratios between ordinary moments, the coefficients of L-kurtosis and L-skewness are defined as:

$$\tau_t = \frac{\lambda_t}{\lambda_2}, \quad t \geq 3,$$

where, τ_3 is the measure of skewness (L- C_s) and τ_4 is the measure of kurtosis (L- C_k). Unlike standard moments, τ_3 and τ_4 are constrained to be between -1 and +1 and τ_4 is constrained by τ_3 to be no lower than -0.25. Because precipitation is nonnegative, τ is also constrained to the range from 0 to 1.

Maximum likelihood: Maximum likelihood method, which is a classical method, is very popular because it is the basic method for estimating the parameters. In most studies, the method of maximum likelihood get better compared to the other estimators. However, this method proved difficult and requires numerical solution and sometimes it fails to estimate the parameters,

especially when sample sizes are small or the distribution of more than three parameters (Hosking and Wallis, 1997). Therefore, the distribution used in this study using three parameters then this solution can only be done by numerical methods. Commonly used method for obtaining estimates of the distribution of the three parameters is the Newton-Raphson.

Goodness-Of-Fit (GOF): The performance between L-moments and ML method in estimating distributions extrem parameters of extreme exchange rate in several countries in Southeast Asia will be compared in this study. The selected GOF tests are Relative Root Mean Square Error (RRMSE), Relative Absolute Square Error (RASE) and Probability Plot Correlation Coefficient (PPCC). The first two methods involve the assessment on the difference between the observed values and expected values under the assumed distribution while the third method involves measuring the correlation between the ordered values and the associated expected values (Zawiah *et al.*, 2009). The formulas for the tests are:

$$RRMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n \left(\frac{y_{i:n} - \hat{Q}(F_i)}{y_{i:n}} \right)^2}$$

$$RASE = \frac{1}{n} \sum_{i=1}^n \left| \frac{y_{i:n} - \hat{Q}(F_i)}{y_{i:n}} \right|$$

And:

$$PPCC = \frac{\sum_{i=1}^n (y_{i:n} - \bar{y})(\hat{Q}(F_i) - \bar{Q}(F_i))}{\sqrt{\sum_{i=1}^n (y_{i:n} - \bar{y})^2 \sum_{i=1}^n (\hat{Q}(F_i) - \bar{Q}(F_i))^2}}$$

where, $y_{i:n}$ is the observed value for the i th order statistics of a random sample of size n , $\bar{Q}(F_i) = \frac{1}{2} \sum_{i=1}^n \hat{Q}(F_i)$ is the estimated quantile value

associated with the i -th Gringorton plotting position, $F1$. The smallest values of RRMSE and RASE will indicate the best method. In contrast, the value of PPCC that is closest to 1 will be considered as the best method for explaining the behaviour of extreme exchange rate in several countries in Southeast Asia.

RESULT

From Table 2, it can be concluded that based method L-moments, for each test Goodness-of-fit, extreme currency exchange in most countries follow the GLD and GPD distribution. This is in accordance with the opinion Gettinby *et al.* (2006) who found that the distribution of GLD is an appropriate distribution compared to the distribution GEV and GPD. Very different from the maximum likelihood method that for each test GOF, amaun extreme exchange rate in all countries to follow the GEV distribution. However,

Table 2: Results on the GOF tests with parameters estimated using L-moment and maximum likelihood

Country	Distribution	L-Moment			Maximum Likelihood		
		RRMSE	RASE	PPCC	RRMSE	RASE	PPCC
SGD	GEV	0.2446149	0.1847271	1.0000000	0.2534776	0.1865732	1.0000000
	GLD	0.2416998	0.1817697	1.0000000	0.2756091	0.1957962	0.9999999
	GPD	0.2488033	0.1915301	1.0000000	0.5986543	0.4642988	0.9999970
	PE3	0.2541037	0.2011030	0.9999958	0.3997562	0.3402639	-
IDR	GEV	0.2738668	0.1782760	0.9866996	0.2841819	0.1780645	0.9866420
	GLD	0.3249041	0.2099115	0.9832888	0.6853220	0.2858733	0.9658340
	GPD	0.1672612	0.1294422	0.9903065	0.3552942	0.2746300	0.9698952
	PE3	1.6995950	1.4330183	0.9685816	7.4664884	6.8203019	-
THB	GEV	0.1256168	0.0987004	0.9999993	0.1445324	0.1071660	0.9999991
	GLD	0.1234808	0.0967868	0.9999993	0.1666608	0.1165484	0.9999987
	GPD	0.1291772	0.1024742	0.9999993	0.4371383	0.4156324	0.9999947
	PE3	0.1236383	0.1003614	0.9999969	0.3198043	0.2559975	-
PHP	GEV	0.0878602	0.0724748	0.9999996	0.0894574	0.0717914	0.9999996
	GLD	0.0961349	0.0771494	0.9999995	0.1394943	0.0875837	0.9999988
	GPD	0.0715852	0.0607558	0.9999997	0.1043325	0.0836485	0.9999973
	PE3	0.4390502	0.4209943	0.9999980	-	-	-

Table 3: Selection of the best distribution for each state

Country	Distribution	$\hat{\mu}$	$\hat{\alpha}$	$\hat{\kappa}$
Dolar Singapura	GLD	0.46809	0.03076	-0.30083
Rupiah Indonesia	GPD	304.84740	6645.90716	2.20136
Bath Thailand	GLD	10.66035	0.44235	-0.24857
Peso Filipina	GPD	9.38019	11.99372	2.09835

when compared with the results of GOF tests between L-moment and ML methods, it appears that according to the RRMSE exams and obtained Rase LM method is better than the ML method. The same thing is also shown on the PPCC test, found that L -moment better than the ML method. So, the best classical method in estimating the parameters are L-moment method and the appropriate distribution for the data exchange rate for each country is the GLD and GPD distribution.

This study reviewed the extreme distribution corresponding to model the maximum data exchange rate for some countries in Southeast Asia. The review conducted with the parameter estimates for some distribution with several methods to estimate the parameters. Then choose the best method to estimate the parameters. The selection of the best distribution for each country is determined by the method of Goodness-of-fit.

From the results of studies that have been done shows that the best distribution for each country studied in modeling the maximum data exchange rate is shown in Table 3. While the best method to get the parameters is the classical method of L-moments as derived from the value Goodness- of-fit on the exam RRMSE and Rase, L-moments are the smallest and the PPCC test is that up to a value of 1.

CONCLUSION

The conclusion of this study suggests that the L-moment method is a method of estimation that is better than the method of maximum likelihood. It is found that, the performance of L-moment is better than the performance of maximum likelihood. Although the best fitting distribution may vary according to the method of estimation and country considered, in most cases, data

for the majority of the several countries are found to follow the generalized logistic distribution.

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