

Research Article

SimE: A Geometric Approach for Similarity Estimation of Fuzzy Sets

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Abstract: The characteristics of a fuzzy set are decided by its membership function. This work aims to provide a geometric approach for enhancing the design and performance of fuzzy systems. Similarity Estimator (SimE) evaluates the membership functions of fuzzy sets on Euclidean space based on geometric area. The overlapping regions between the sets are partitioned into geometric structures. The area of overlapping is computed by summing the area of polygons and integrating the area under curves. Similarity between fuzzy sets is directly proportional to the area of overlapping between them. SimE was tested over a range of real numbers with finite intervals. Fuzzy sets using different membership functions were created for the same data distribution. From the test results it can be inferred that fuzzy sets defined using triangular membership functions have a minimum overlapping area when compared to fuzzy sets defined using other membership function. Optimal overlapping area of fuzzy sets improves the semantic representation and the performance of the system. SimE can be used by knowledge engineers to design efficient fuzzy systems.

Keywords: Design automation, fuzzy systems, fuzzy set theory, geometric area, membership function, similarity measure

INTRODUCTION

Large amount of data are generated and manipulated everyday but often due to human errors and system failures the data may become noisy, ambiguous and redundant. Automated systems and computer-aided systems that use data for control and decision making have to use this incomplete, vague and imprecise data. Though data preprocessing techniques such as cleaning, transformation, normalization and reduction make the data suitable for use, the quality of the data can still be improved to enhance the reasoning and decision making ability of these systems. Fuzzy set theory and soft computing techniques are capable of handling imprecise, vague and incomplete data in automated tasks (Isermann, 1998).

The first fuzzy logic controller was developed in the mid-seventies (Mamdani and Assilian, 1975). It was used to represent the linguistic terms and the uncertainties involved in the control system, using a set of heuristic control rules, fuzzy sets and fuzzy logic. Ever since the birth of fuzzy theory, it is progressively applied increasingly in various fields of engineering and medicine (Soufi *et al.*, 2016; Jane *et al.*, 2016; Japhine Susmi *et al.*, 2015). Generally, a fuzzy logic

based system consists of four modules: a fuzzification interface, knowledge base (fuzzy rule base), inference engine and the defuzzification module.

There are no fixed rules or design patterns for the design and development of fuzzy systems. The efficiency and effectiveness of a fuzzy system depends on several factors such as the choice of membership functions, their input values, the area of intersection among them, the information content of the knowledge base and also the skill, domain knowledge and experience of the designer.

Membership Functions (MF) are easily identifiable when human expertise for the domain of application is available. However, for other applications it has to be identified and designed only by a system designer or knowledge engineer. The choice of a membership function and its design are important for any application of fuzzy set theory as they decide the overall performance of the fuzzy system (Zimmermann, 1996; Jang *et al.*, 1997). Better design of a fuzzy system leads to enhanced reasoning and performance.

The purpose of a fuzzy system is to handle uncertainty with human reasoning. The conjunctive (AND) and disjunctive (OR) reasoning model is used for combining the membership functions. However this

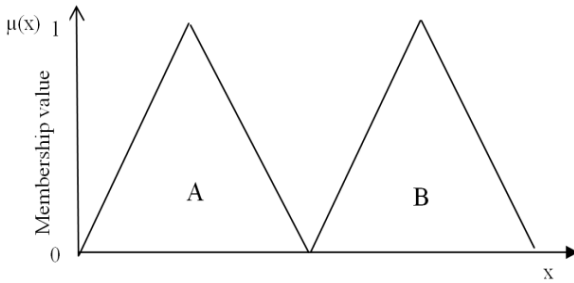


Fig. 1a: Fuzzy sets with no similarity

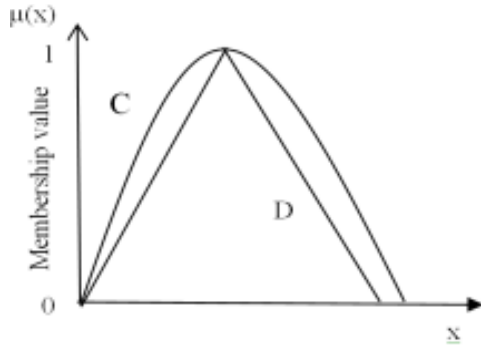


Fig. 1b: Similar fuzzy sets

model is not the general way of human reasoning in many situations. Furthermore, a fuzzy system is designed with same level of membership space $[0, 1]$ for all fuzzy sets. Equal importance is given to all the linguistic variables used in the design which is not the actual scenario in the real world.

This study proposes Similarity Estimator (SimE); an approach where fuzzy sets are said to be similar based on their overlapping region. Consider the following example: Let A and B are two fuzzy sets as shown in Fig. 1a. Both fuzzy sets are exactly same in shape but the Euclidean space enclosed by the sets is not similar, hence there is no overlapping between the sets. However, in Fig. 1b, C and D are different in shape but they are more similar as the region (Euclidean space) represented by the sets are similar. A fuzzy system with few efficiently designed fuzzy sets and few rules has lower computational demands. Therefore, the region of overlapping between the fuzzy sets is an important measure in designing any fuzzy system.

PRELIMINARIES

This section presents the basic concepts and mathematical models of fuzzy sets, Membership functions and fuzzy set similarity.

Fuzzy set: Fuzzy set is a set whose elements have a degree of membership. Fuzzy set 'A' on the universe of discourse X is characterized by a membership function, which associates with each element 'x' a real number in

the interval $[0, 1]$. A Fuzzy set can be defined mathematically as follows:

$$A = \{(x, \mu_A(x)) \mid x \in X\} \quad (1)$$

where, $\mu_A(x)$ is called Membership Function (MF) for the fuzzy set A. X is universe of discourse $X = \{x_j/j = 1, 2, \dots, m\}$ (Zadeh, 1965). The MF maps each element of X to membership grade ranging from 0 to 1: $0 \leq \mu_A(x) \leq 1$.

Membership functions: The membership grade of an element in a fuzzy set is determined by its membership function. There are different types of MFs namely Triangular, Gaussian, Trapezoidal, Bell-shaped, linear, sigmoidal and polynomial membership function (Jang *et al.*, 1997). This work considers four membership functions namely, Triangular, Gaussian, Trapezoidal and Bell-shaped membership functions for testing SimE.

Fuzzy set similarity: The fuzzy set similarity is a measure of approximate equality between fuzzy sets which describe the relationship among fuzzy sets. Similarity plays a major role in decision making, classification and clustering. It is a major criterion in deciding the number of fuzzy (Jager and Benz, 2000; Setnes *et al.*, 1998) sets for a fuzzy system design.

The following section reviews some of the works in literature that use different membership functions and fuzzy set similarity measures.

LITERATURE REVIEW

Zhao and Bose (2002) have evaluated triangular, Gaussian, trapezoidal, two sided Gaussian and bell shaped MF for a fuzzy control system. Sensitivity has been analyzed and the effect of different types of membership functions in the fuzzy control system for speed-controlled induction motor drive was compared. The result indicates that triangular MF gives better performance than other MFs and the next identified best performer is trapezoidal MF.

Hasuik *et al.* (2014) have developed a construction algorithm to obtain an appropriate MF. The construction approach is based on both fuzzy Shannon entropy with smoothing function and interval estimation derived from human cognitive behaviour and subjectivity. They have introduced natural assumptions for the fuzzy Shannon entropy and the smoothing function. Hence the approach proposed is suitable for constructing any type of membership functions. However their approach is not appropriate for continuous spaces.

Alikhademi and Zaiyanudin (2014) have developed a framework based on Particle Swarm Optimization (PSO) to obtain the MF from quantitative data. In their framework an appropriate MF is generated for each

input variable using PSO and it is optimized using S and Z fuzzy shapes. Using PSO, appropriate MF is obtained iteratively and the generated MF is used to transform quantitative data into fuzzy data. Their fuzzification approach is tested for typical data mining application (classification) to select an optimal set of association rules.

Bera *et al.* (2014) have determined MF for fuzzy random variable. Fuzzy random variable follows a normal distribution with imprecise definition of mean and standard deviation. Probabilistic distribution and cumulative density function both are used to generate the MF of fuzzy random variable. In their work, cumulative density function used to fix the lower and upper bound of MF. They have demonstrated the technique on fuzzy random variable (temperature change) for a temperature controller. They have taken alpha-cut value and percentile value as evaluation parameter. From the result they inferred that the triangular membership function misinterprets uncertainty compared to the trapezoidal membership function.

Jiménez *et al.* (2014) have computed the degree of similarity between the fuzzy sets by performing the fuzzy operations, such as union and intersection over the fuzzy sets. By means of determining degree of similarity between the fuzzy sets they have minimized the number of different fuzzy sets required to model the fuzzy classifier. They have taken the threshold value as 10 percent for the maximum similarity degree of fuzzy sets. The value was accepted in the scientific community in order to obtain more interpretable fuzzy classifier model. By minimizing number of fuzzy sets, number of rules is also minimized. They have tested the similarity measure technique with dataset collected from the Health Information System of the Incentive Care Burn Unit (ICBU) from 1999 to 2002. In the experiment they have used fuzzy sets with Gaussian membership functions. From their results they inferred that greater the similarity between the fuzzy sets, makes the system more ambiguous and the model is hardly interpretable by a human.

Several methodologies have been proposed in literature for determining MF for fuzzy systems. Generally, determination is based on data distribution, data intervals and elements of a set. The works of Alikhademi and Zaianudin (2014) emphasize on estimating the similarity between fuzzy sets based on elements of the sets. Moreover there are works that highlight geometrical operations on fuzzy sets (Bogomolny, 1987; Rosenfeld and Haber, 1985) but they have not used geometric approaches as a measure of similarity to evaluate fuzzy sets.

In this paper, a novel method called SimE, is devised to evaluate the design of fuzzy sets based on

the geometric area of intersection of fuzzy membership functions. The approach may be used as a measure to estimate the similarity between fuzzy sets.

SIMILARITY ESTIMATOR

Similarity Estimator (SimE), is an algorithm that uses the geometric area of overlapping for analyzing the similarity between the fuzzy sets. Different MFs are analyzed based on the region of overlapping between the fuzzy sets for a given data distribution:

SimE algorithm:

Require: A set of finite intervals \mathcal{L} and \mathcal{U}
 where $\mathcal{L} \ni \{l_1, l_2, \dots, l_n\}$, $\mathcal{U} \ni \{u_1, u_2, \dots, u_n\} | n > 0$
 $\{\mathbb{T} - \text{set of triangular MFs.}$
 $\mathbb{T} \ni \{\text{trimf}_1, \text{trimf}_2, \text{trimf}_3 \dots \text{trimf}_k\}$
 $\{\mathbb{G} - \text{set of gaussian MFs.}$
 $\mathbb{G} \ni \{\text{gaussmf}_1, \text{gaussmf}_2, \text{gaussmf}_3 \dots \text{gaussmf}_k\}$
 $\{\mathbb{T}r - \text{set of trapezoidal MFs.}$
 $\mathbb{T}r \ni \{\text{trapmf}_1, \text{trapmf}_2, \text{trapmf}_3 \dots \text{trapmf}_k\}$
 $\{\mathbb{B} - \text{set of bell MFs.}$
 $\mathbb{B} \ni \{\text{bellmf}_1, \text{bellmf}_2, \text{bellmf}_3 \dots \text{bellmf}_k\}$

1: Construct fuzzy sets:

- 1.1: $\mathbb{T} \leftarrow \text{constrimf}(\mathcal{L}, \mathcal{U})$
- 1.2: $\mathbb{G} \leftarrow \text{consgaussmf}(\mathcal{L}, \mathcal{U})$
- 1.3: $\mathbb{T}r \leftarrow \text{constrapmf}(\mathcal{L}, \mathcal{U})$
- 1.4: $\mathbb{B} \leftarrow \text{consbellmf}(\mathcal{L}, \mathcal{U})$

2: Calculate overlapping area OA:

- 2.1: $\Delta_{tri} \leftarrow OA_{\text{polygon}}(\mathbb{T})$
- 2.2: $\Delta_{gauss} \leftarrow OA_{\text{curve}}(\mathbb{G})$
- 2.3: $\Delta_{trap} \leftarrow OA_{\text{polygon}}(\mathbb{T}r)$
- 2.4: $\Delta_{bell} \leftarrow OA_{\text{curve}}(\mathbb{B})$

3: Compare Area: $(\Delta_{tri}, \Delta_{gauss}, \Delta_{trap}, \Delta_{bell})$

SimE works in 3 steps: First, fuzzy sets are created using different MFs (such as Triangular, Trapezoidal, Bell and Gaussian) for the same input data. Second, similarity between the fuzzy sets is computed based on area of overlapping. Third, overlapping area between the fuzzy sets for each membership function is analyzed.

Step 1: Parameters (\mathcal{L} and \mathcal{U}) needed for constructing fuzzy sets using triangular MF are obtained from the input data. The membership function specific parameters are extracted from \mathcal{L} and \mathcal{U} .

For triangular MF, the parameters are starting point (a), end point (b) and center (c). The mapping of parameters for a triangular MF is defined in *constrimf*(\mathcal{L}, \mathcal{U}) function and illustrated in Fig. 2.

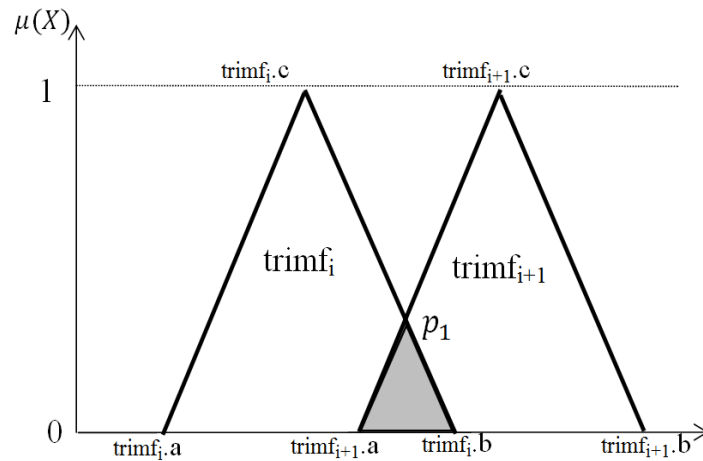


Fig. 2: Parameters of triangular MF

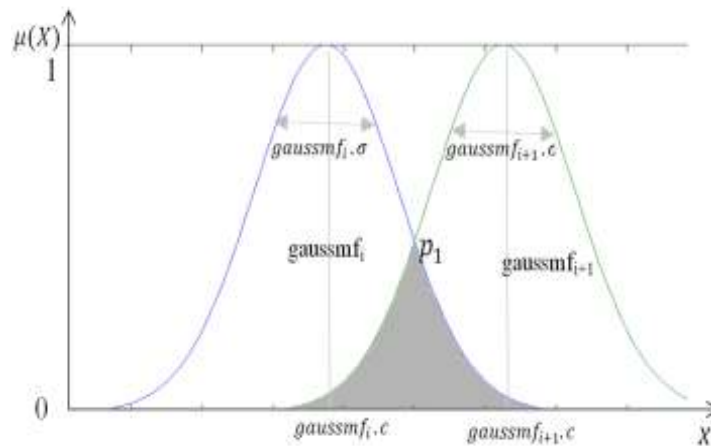


Fig. 3: Parameters of Gaussian MF

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constrimf(L,U)
/*Construction of Triangular MF*/
1: foreach l_i in L, u_i in U do
2: trimf_i.a ← l_i
3: trimf_i.b ← u_i
4: trimf_i.c ← (l_i+u_i)/2
5: T ← trimf_i
6: endfor
7: return(T)
    
```

For Gaussian MF, the parameters are variance (σ) and center (c). The parameter-mapping for Gaussian MF is presented in Fig. 3 and is defined in *consgaussmf(L,U)* function:

```

consgaussmf(L,U)
/*Construction of Gaussian MF*/
1: for each l_i in L, u_i in U do
2: gaussmf_i.sigma ← (l_i-u_i)^2/12
3: gaussmf_i.c ← (l_i+u_i)/2
    
```

```

4: G ← gaussmf_i
5: end for
6: return(G)
    
```

For trapezoidal MF, the parameters are starting point (a), end point (d) and two shoulder points (b and c). The mapping of parameters for the trapezoidal MF is defined in *constrapmf(L,U)* function. Furthermore the parameters are represented in Fig. 4.

```

Constrapmf(L,U)
/*Construction of Trapezoidal MF*/
1: for each l_i in L, u_i in U do
2: trapmf_i.a ← l_i
3: trapmf_i.b ← 1/4(l_i - u_i)
4: trapmf_i.c ← 3/4(l_i - u_i)
5: trapmf_i.d ← u_i
6: Tr ← trapmf_i
7: end for
8: return(Tr)
    
```

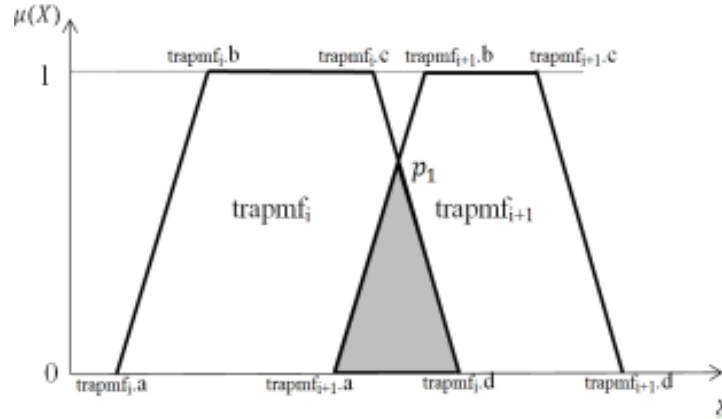


Fig. 4: Parameters of trapezoidal MF

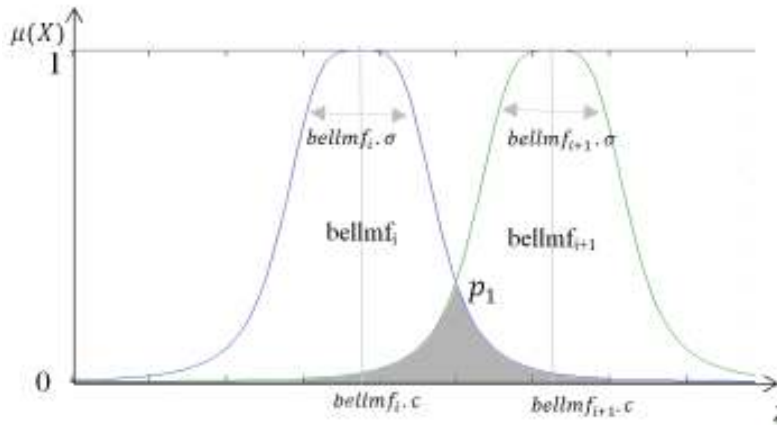


Fig. 5: Parameters of bell MF

For bell MF, the parameters are center (c), slope of the cross over point (b) and variance (σ). The parameters mapping of bell MF is defined in *consbellmf*(\mathcal{L}, \mathcal{U}) function and illustrated in Fig. 5.

consbellmf(\mathcal{L}, \mathcal{U})

/*Construction of Bell-Shaped MF*/

- 1: for each $l_i \in \mathcal{L}, u_i \in \mathcal{U}$ do
- 2: $bellmf_i.\sigma \leftarrow \frac{(l_i - u_i)^2}{12}$
- 3: $bellmf_i.b \leftarrow 4$
- 4: $bellmf_i.c \leftarrow \frac{l_i + u_i}{2}$
- 5: $\mathbb{B} \leftarrow bellmf_i$
- 6: end for
- 7: return(\mathbb{B})

Step 2: Similarity is computed for the generated fuzzy sets. Similarity is measured in terms of the area of overlapping. For computing the area of overlapping between fuzzy sets, two separate algorithms are used: The first algorithm is for computing area under curves and the second is for computing overlapping area of polygonal shapes. The former is defined by the function OA_{curve} (MIF) and the latter is defined by the

function.

$OA_{polygon}$ (MIF)

Step 3: The area of overlapping for fuzzy sets is compared for different membership functions.
 $OA_{polygon}$ (MIF)

Require: Set of membership functions

MIF: $MIF \ni \{T, Tr\}$

{P is the set of points on the perimeter of the overlapping region:

$P \ni \{p_1, p_2, p_3\}$

- 1: If MIF is T then
- 2: for each *trimf* in T do
- 3: $p_1 \leftarrow$ intersection of line joining ($trimf_i.c, trimf_i.b$) and ($trimf_{i+1}.a, trimf_{i+1}.c$)
- 4: $p_2 \leftarrow trimf_{i+1}.a$
- 5: $p_3 \leftarrow trimf_i.b$ /* $P \ni \{p_1, p_2, p_3\}$ */
- 6: $A = A + Area_T(P)$
- 7: end for
- 8: end if
- 9: If MIF is Tr then
- 10: for each *trapmf* in Tr do

```

11: if(trapmfi+1.b ≥ trapmfi.c) then
12: p1 ← intersection of
    line joining (trapmfi+1.a, trapmfi+1.b)
    and(trapmfi.c, trapmfi.d)
13: p2 ← trapmfi+1.a
14: p3 ← trapmfi.d/*P ∃ {p1, p2, p3}*/
15: A = A + AreaT(P)
16: end if
17: if(trapmfi+1.b < trapmfi.c) then
18: p1 ← trapmfi+1.a
19: p2 ← trapmfi+1.b
20: p3 ← trapmfi.c
21: p4 ← trapmfi.d/*P ∃ {p1, p2, p3, p4}*/
22: A = A + AreaT(P)
23: end if
24: end for
25: end if
26: return (A)
    
```

$OA_{curve}(\mathbb{B})$
 Require: Set of membership functions
 MIF : MIF ∃ {G, B}
 {p is the point of intersection of the curves}

```

1: If MIF is G then
2: for each gaussmf in G do
3: θ1 = ∫-∞p e(-1/2 * (x - gaussmfi+1.c) / gaussmfi+1.σ)2) dx
4: θ2 = ∫p∞ e(-1/2 * (x - gaussmfi.c) / gaussmfi.σ)2) dx
5: Area = Area + θ1 + θ2
6: end for
7: end if
8: If MIF is B then
9: for each bellmf in B do
10: θ1 = ∫t2p (1 / (1 + |(x - bellmfi+1.c) / bellmfi+1.σ|2b)) dx
11: θ2 = ∫pu1 (1 / (1 + |(x - bellmfi.c) / bellmfi.σ|2b)) dx
12: Area = Area + θ1 + θ2
13: end for
14: end if
15: return Area
    
```

Table 1: Interval limits for data distribution 1

Intervals	Lower limit (L)	Upper limit (U)
Interval 1	0.0	1.00
Interval 2	0.4	3.00
Interval 3	1.0	7.14
Interval 4	3.0	10.0

Table 2: Interval limits for data distribution 2

Intervals	Lower limit (L)	Upper limit (U)
Interval 1	0	20
Interval 2	18	32
Interval 3	28	44
Interval 4	39	63
Interval 5	60	85

RESULTS AND DISCUSSION

To analyze the performance of the proposed approach, experiments were performed over two data distribution. The inputs to construct the fuzzy sets are the lower limits and the upper limits of the intervals over a range of real numbers. Fuzzy sets are constructed by extracting the parameters for different membership functions such as Triangular, Gaussian, trapezoidal and Bell. Then overlapping areas of fuzzy sets using different MF are computed and analyzed. Fuzzy sets with minimum overlapping area are considered to be less similar. Table 1 and 2 present the information about the two data distributions (ranges and intervals) used to test the SimE.

Figure 6 to 9 present the fuzzy sets generated for data distribution 1. The fuzzy sets are characterized by triangular, Gaussian, trapezoidal and bell membership function respectively. Figure 10 to 13 present the corresponding overlapping area between the sets. The overlapping area for fuzzy sets with triangular and trapezoidal membership functions is computed by $OA_{polygon}$ function and the overlapping area for Gaussian and Bell membership functions is computed by OA_{curve} function. In the function $OA_{polygon}(MIF)$, the fourth conditional statement -if(trapmf_{i+1}.b < trapmf_i.c)- does not occur in practical situations;

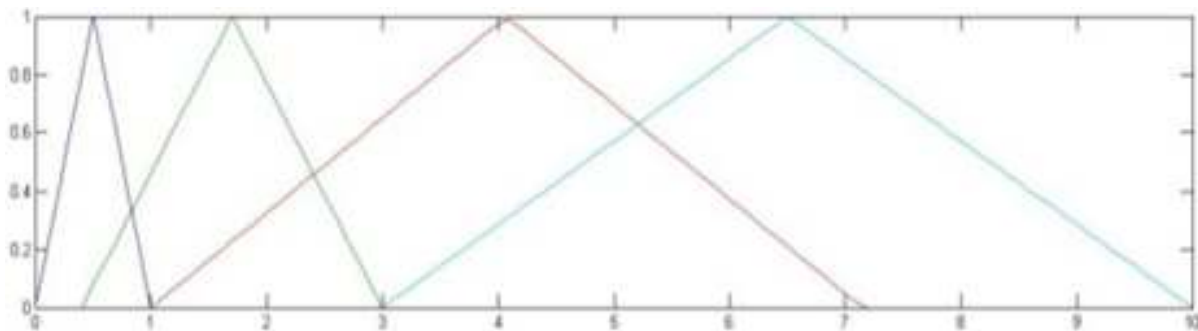


Fig. 6: Fuzzy sets using triangular membership function for data distribution 1

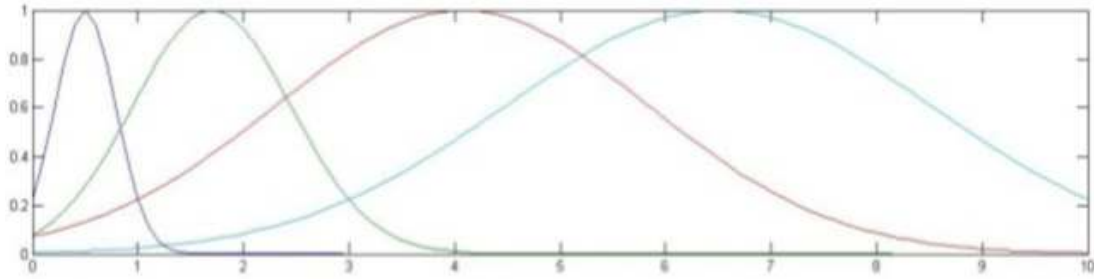


Fig. 7: Fuzzy sets using Gaussian membership function for data distribution 1

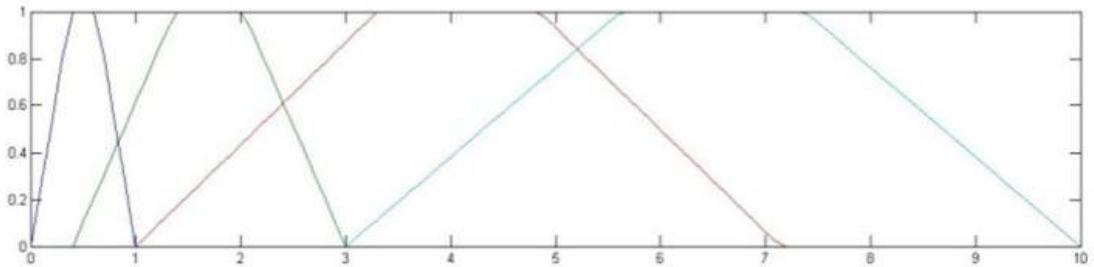


Fig. 8: Fuzzy sets using trapezoidal membership function for data distribution 1

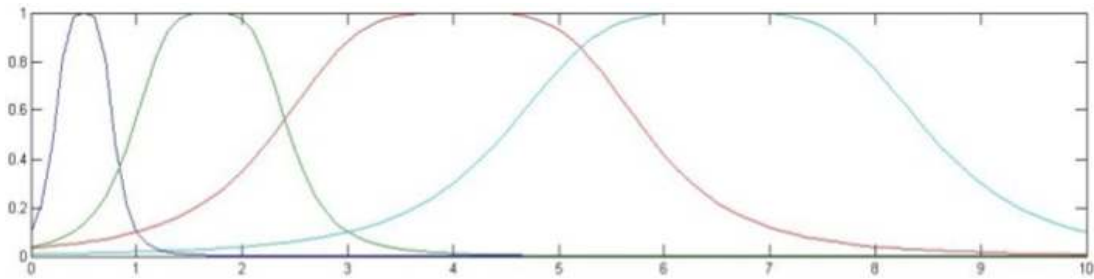


Fig. 9: Fuzzy sets using bell membership function for data distribution 1

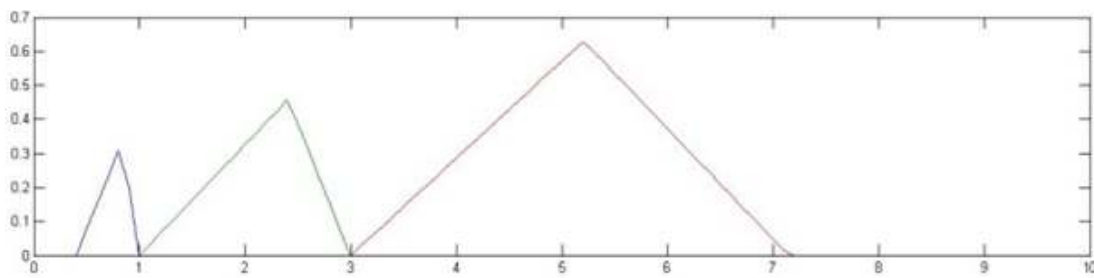


Fig. 10: Area of overlapping using triangular membership function for data distribution 1

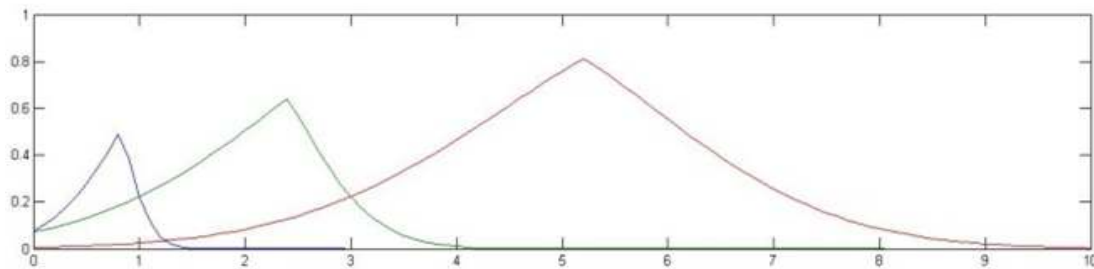


Fig. 11: Area of overlapping using Gaussian membership function for data distribution 1

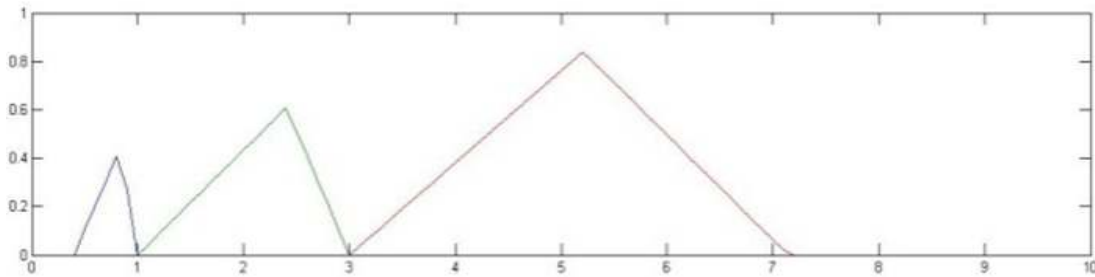


Fig. 12: Area of overlapping using trapezoidal membership function for data distribution 1

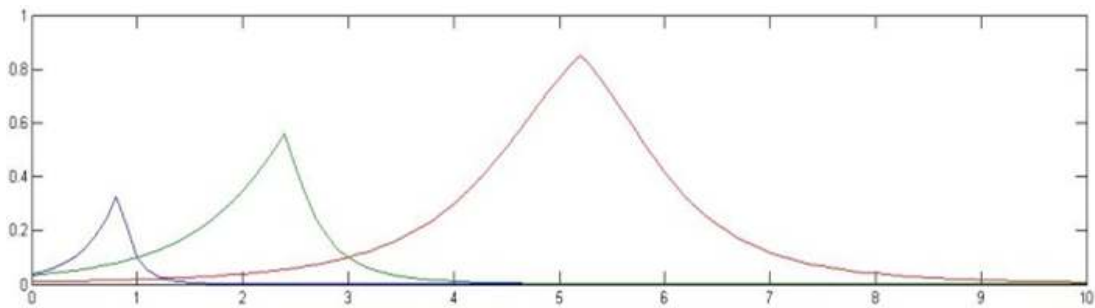


Fig. 13: Area of overlapping using bell membership function for data distribution 1

Table 3: Overlapping area for data distribution 1

Area of overlapping	Membership functions			
	TriMF	TrapMF	GaussMF	BellMF
Area 1	0.0969	0.1292	0.3028	0.1656
Area 2	0.4572	0.6097	1.0508	0.6478
Area 3	1.3061	1.7414	2.4775	1.8243
Total area	1.8602	2.4803	3.8311	2.6377

Table 4: Overlapping area for data distribution 2

Area of Overlapping	Membership functions			
	TriMF	TrapMF	GaussMF	BellMF
Area 1	0.1000	0.1333	1.5585	0.8480
Area 2	0.5179	0.6905	2.2168	1.2203
Area 3	0.6250	0.8333	2.8066	1.5323
Area 4	0.1633	0.2178	2.2933	1.2482
Total area	1.4062	1.8749	8.8752	4.8488

however, it is computed for theoretical purposes. This computation may be used for repairing fuzzy sets when they are randomly generated or evolved using an evolutionary algorithm.

Table 3 and 4 present the area of overlapping between the adjacent fuzzy sets and also the total area of that fuzzy variable. From the experimental results it can be observed that fuzzy sets with triangular MF have a minimum overlapping area compared to fuzzy sets characterized by other MFs.

CONCLUSION

Set theory based similarity estimators are well known and also frequently used in the design and development of fuzzy systems. The expressiveness of geometric models for fuzzy system design is still not

completely exploited by the fuzzy logic community. The proposed SimE is just yet another fuzzy system-design evaluator based on the similarity between fuzzy sets. The requirement of background domain knowledge for designing fuzzy systems based on geometric models, such as SimE, is less when compared to set theoretic models. SimE can be applied for designing fuzzy systems when the parameters of the fuzzy membership functions are not well-defined. Furthermore, there are several works in literature that use evolutionary approaches for generation of fuzzy sets. Evolutionary Fuzzy systems (Amaral *et al.*, 2002; Celikyilmaz and Burhan Turksen, 2007) are becoming more accepted in engineering and design. Random fuzzy sets are initially created and iteratively improved to evolve an optimal fuzzy system. In such scenarios, SimE can be used to obtain a combination of fuzzy sets with optimal similarity and overlapping area. Geometric area of similarity between fuzzy sets decides how efficiently uncertainty and vagueness is represented and handled by the fuzzy system. SimE can be used to design systems with optimal similarity for different membership functions.

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