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Research Article

A Study of Non-linear, Non-Darcy Newtonian Liquid Flow and Heat Transfer Through Vertical Channel Using Mixed Boundary Conditions on Temperature

[1, 3]Nur Asiah Mohd Makhatar, [2]P.G. Siddheshwar, [3]Habibis Saleh and [3]Ishak Hashim
[1]Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA, 40450 Shah Alam, Selangor, Malaysia
[2]Department of Mathematics, Bangalore University, Central College Campus, Bangalore 560 001, India
[3]School of Mathematical Sciences, Universiti Kebangsaan Malaysia, 43600 UKM Bangi Selangor, Malaysia

Abstract: In this study, we analyzed the flow and heat transfer within a fully-developed non-linear, non-Darcy flow through a sparsely packed chemically inert porous medium in a vertical channel by considering Dirichlet, Neumann and Robin boundary conditions. A numerical solution by using Runga-Kutta method that was obtained for the Darcy-Forchheimer-Brinkman momentum equation is used to analyze the heat transfer. The Biot number influences on velocity and temperature distributions are opposite in regions close to the left wall and the right wall. Neumann condition is seen to favor symmetry in the flow velocity whereas Robin and Dirichlet conditions skew the flow distribution and push the point of maximum velocity to the right of the channel. A reversal of role is seen between them in their influence on the flow in the left-half and the right-half of the channel. This leads to related consequences in heat transport. Viscous dissipation is shown to aid flow and heat transport. The present findings reiterate the observation on heat transfer in other configurations that no significant change was observed in Neumann condition, whereas the changes are too extreme in Dirichlet condition. It is found that Robin condition is the most stable condition.

Keywords: Dirichlet, heat transfer, neumann, porous medium, robin boundary conditions, vertical channel

INTRODUCTION

Due to the presence of porous media in diverse engineering applications including packed-bed catalytic reactor, geothermal reservoirs, drying of porous solid and regenerative heat exchangers and interest in fundamental studies of heat and mass transfer in porous media has increased significantly. Few studies in wall bounded mixed convection through vertical annuli and channels filled with porous medium have been investigated. Based on the study by Parang and Keyhani (1987) on a vertical annulus employing Darcy-Brinkman model, the Brinkman term is found to give a negligible effect to the flow when Darcy number *Da* is very small. Some important reported works on the problem include those of Tao (1960), Aung and Worku (1986), Incropera (1986), Cheng *et al*. (1990), Javeri (1976), Barletta (1998), Zanchini (1998), Barletta and Zanchini (1999), Grosan and Pop (2007), Pop *et al*. (2010) and Saleh *et al*. (2013). Except Javeri (1976) and Zanchini (1998), all others considered Dirichlet boundary condition on temperature.

A survey of the existing literature on this topic has revealed that relatively few investigations have been conducted on mixed convection in vertical porous channels. Reported studies related to localized heating at the boundaries of vertical parallel plate channels (Lai *et al*., 1988) or on vertical annuli were mostly concerned with Darcy flow regime. Only one study was reported so far on non-Darcy flow and heat transfer in a vertical channel with uniform heating at the walls (Hadim, 1994). Vafai and Kim (1995) have presented a numerical study based on the Darcy-Brinkman-Forchheimer model for the forced convection in a composite system containing fluid and porous regions. Knupp and Lage (1995) have studied generalized Forchheimer-extended Darcy flow model to the permeability case via a variation principle. Marpu (1995) have studied the Forchheimer and Brinkman extended Darcy flow model on natural convection in a vertical cylindrical porous medium. Singh and Thorpe (1995) presented a comparative analysis of flow models on natural convection in a fluid flow over a porous layer. Nield (1996) obtained a closed-form solution of

Corresponding Author: Nur Asiah Mohd Makhatar, Faculty of Computer and Mathematical Sciences, University Teknologi MARA, Shah Alam, Selangor, Malaysia

the Brinkman-Forchheimer equation for different values of Darcy number using the Brinkman-Forchheimer model and stress jump boundary condition at the porous interface. Kuznetsov (1997) obtained an analytical solution for the steady fully developed flow in a composite region. Nakayama (1998) has presented a unified treatment of Darcy-Forchheimer boundary layer flows. Singh *et al*. (2011) studied this problem for natural convection flow in a vertical channel partially filled with a porous medium using Brinkman-Forchheimer extended Darcy model.

Boundary conditions are essential along the entire boundary or part of the boundary in order to solve differential equations. Three types of boundary conditions are compared in this study, comprising the Dirichlet (first kind), Neumann (second kind) and Robin (third kind). In Dirichlet condition, the value is set of the unknown function itself, whereas in Neumann condition the gradient of the function is set in a direction normal to the boundary. The Robin condition sets the value of a combination of the unknown function and its normal gradient that is linear in the unknown function. Among the earlier systematic comparisons of the effect of these boundary conditions were by Novy *et al*. (1991), where the Robin-type condition proves the best as it requires the least work to achieve a given accuracy. Hence, the Robin condition is applied for the default case in this study. Papanastasiou *et al*. (1992) introduced and tested a new outflow boundary condition, called free boundary condition. Other related study was done by Ryan *et al*. (2010), where Neumann and Robin boundary conditions with a novel method in two dimensional cases were modeled. Later, Sikarudi and Nikseresht (2015) proposed a new approach in aiding the implementation of Robin and Neumann boundary conditions and the method is proven to be less sensitive to particle disorder. Meanwhile, Helgadóttir *et al*. (2015) did a study involving a straightforward approach in imposing the mixed Dirichlet-Neumann-Robin boundary conditions, producing a symmetric positive definite linear system and second-order accurate solutions and a robust method in challenging configurations.

As there is no experimental investigation on the subject dealt with in the present work, no experimental verification of the theoretical results could be done. The goal of this present work is the following:

- To consider the effect of Robin temperature boundary condition on the flow and heat transfer in the non-linear, non-Darcy flow.
- To ascertain whether viscous dissipation plays its classical role on flow and heat transfer in the flow.
- To make a comparison between the extent of heat transfers facilitated by the Dirichlet, Neumann and Robin boundary conditions.

Though the governing equations seem simple, clearly they are not analytically tractable due to the use of a more realistic Robin boundary condition on temperature. Hence, shooting method is used to solve the flow and temperature distributions.

MATERIALS AND METHODS

Physical configuration: Mixed convection non-linear, non-Darcy flow is investigated in this study. We consider under the Boussinesq-Oberbeck approximation, the steady flow of a Newtonian fluid in a parallel plate vertical channel of width L. The x-axis lies on the axial plane of the channel and its direction is opposite to the gravitational field, g. The y-axis is perpendicular to the channel walls and the channel is assumed to occupy the region of space $-\frac{L}{2} \leq Y \leq \frac{L}{2}$. The flow has a uniform upward vertical velocityU_0, at the channel entrance. The physical configuration is as shown in Fig. 1. As customary, the Boussinesq-Oberbeck approximation and the equation of state:

$$\rho = \rho_0[1 - \beta(T - T_0)] \quad (1)$$

Are adopted, in which T_0 is the reference temperature.

It is assumed the X-component of $U(Y)$ is the only non-zero component of the velocity field U. Thus, the continuity equation is

$$\frac{\partial U}{\partial X} = 0 \quad (2)$$

The momentum balance equations along X and Y directions are modified for porous liquid using Zanchini (1998) model to obtain:

$$\beta g(T - T_0) = \frac{1}{\rho_0}\frac{\partial P}{\partial X} - \nu\frac{d^2U}{dY^2} + \frac{\nu}{K}U + \frac{C_F}{\sqrt{K}}U^2 \quad (3)$$

$$\frac{\partial P}{\partial Y} = 0 \quad (4)$$

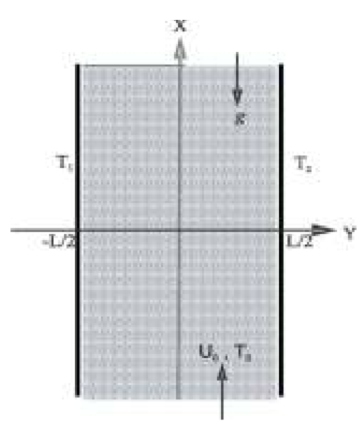


Fig. 1: Physical configuration of vertical channel containing fully-developed mixed convection non-linear, non-Darcy flow

where,

ν : The kinematic viscosity

K : The permeability of porous medium,

C_F : The inertial coefficient and $P = p + \rho_{0gX}$. In view of Eq. (4), (3) can be rewritten as:

$$T - T_0 = \frac{1}{\beta g \rho_0}\frac{dP}{dX} - \frac{\nu}{\beta g}\frac{d^2U}{dY^2} + \frac{\nu}{\beta g K}U + \frac{C_F}{\beta g\sqrt{K}}U^2 \quad (5)$$

Differentiating Eq. (5) with respect to X and then separately with respect to Y, we get:

$$\frac{\partial T}{\partial X} = \frac{1}{\beta g \rho_0}\frac{d^2P}{dX^2} \quad (6)$$

$$\frac{\partial T}{\partial Y} = -\frac{\nu}{\beta g}\frac{d^3U}{dY^3} + \frac{\nu}{\beta g K}\frac{dU}{dY} + \frac{2C_F}{\beta g\sqrt{K}}U\frac{dU}{dY} \quad (7)$$

$$\frac{\partial^2 T}{\partial Y^2} = -\frac{\nu}{\beta g}\frac{d^4U}{dY^4} + \frac{\nu}{\beta g K}\frac{d^2U}{dY^2} + \frac{2C_F}{\beta g\sqrt{K}}\left[U\frac{d^2U}{dY^2} + \left(\frac{dU}{dY}\right)^2\right] \quad (8)$$

The boundary conditions on U are taken as follows:

$$U\left(-\frac{L}{2}\right) = U\left(\frac{L}{2}\right) = 0 \quad (9)$$

The boundary conditions on the temperature field are assumed to be the following:

$$-k_{nl}\left.\frac{dT}{dX}\right|_{Y=-\frac{L}{2}} = h_1\left[T_1 - T\left(X, -\frac{L}{2}\right)\right] \quad (10)$$

$$-k_{nl}\left.\frac{dT}{dX}\right|_{Y=\frac{L}{2}} = h_2\left[T\left(X, \frac{L}{2}\right) - T_2\right] \quad (11)$$

where, h_1 and h_2 are constants. Using Eq. (7), (10) and (11) are rewritten as:

$$\left.\frac{d^3U}{dY^3}\right|_{Y=-\frac{L}{2}} = \frac{\beta g h_1}{k\nu}\left[T_1 - T\left(X, -\frac{L}{2}\right)\right] \quad (12)$$

$$\left.\frac{d^3U}{dY^3}\right|_{Y=\frac{L}{2}} = \frac{\beta g h_2}{k\nu}\left[T\left(X, -\frac{L}{2}\right) - T_2\right] \quad (13)$$

It is easily verified that Eq. (12) and (13) imply that $\partial T/\partial X$ is zero both at $Y = -L/2$ and at $Y = L/2$. Since Eq. (6) ensures that $\partial T/\partial X$ does not depend on Y, it is concluded that $\partial T/\partial X$ is zero everywhere. Therefore, the temperature T depends only on Y, i.e., $T=T(Y)$. Thus, on account of Eq. (6), we may write:

$$\frac{\partial P}{\partial X} = A \quad (14)$$

The conservation of energy equation in the presence of viscous dissipation is taken to be:

$$\frac{d^2T}{dY^2} = -\frac{\mu}{k}\left(\frac{dU}{dY}\right)^2 \quad (15)$$

We now define $D=2L$ which is the hydraulic diameter and the reference velocity U_0 and the reference temperature T_0, given by

$$U_0 = -\frac{AD^2}{48\mu}, T_0 = \frac{T_1+T_2}{2} + S\left(\frac{1}{Bi_1} - \frac{1}{Bi_2}\right)(T_2 - T_1)$$

where,

$$Bi_1 = \frac{h_1 D}{k} \text{ and } Bi_2 = \frac{h_2 D}{k} \quad (16)$$

Equations (8), (9), (10), (11) and (15) can be written in a dimensionless form by employing the following dimensionless parameters:

$$u = \frac{U}{U_0}, \theta = \frac{T - T_0}{T_2 - T_1}, y = \frac{Y}{D}, Gr = \frac{g\beta\Delta T D^3}{\nu^2},$$
$$Br = \frac{\mu(U_0)^2}{k\Delta T}, Pr = \frac{\nu}{\alpha}, GR = \frac{Gr}{Re}, R_T = \frac{T_2 - T_1}{\Delta T},$$
$$F = ReC_F Da, \alpha = \frac{k}{\rho_0 C_p}, Re = \frac{U_0 D}{\nu}, Da = \frac{D}{\sqrt{K}},$$
$$S = \frac{Bi_1 Bi_2}{Bi_1 Bi_2 + 2Bi_1 + 2Bi_2} \quad (17)$$

The non-dimensional governing equations and boundary conditions are:

$$GR\frac{d^2\theta}{dy^2} = -\frac{d^4u}{dy^4} + Da^2\left(\frac{d^2u}{dy^2}\right) + 2F\left[u\frac{d^2u}{dy^2} + \left(\frac{du}{dy}\right)^2\right] \quad (18)$$

$$\frac{d^2\theta}{dy^2} = -Br\left(\frac{du}{dy}\right)^2 \quad (19)$$

$$u(-1/4) = u(1/4) = 0 \quad (20)$$

$$\left.\frac{d^2u}{dy^2}\right|_{y=-\frac{1}{4}} - \frac{1}{Bi_1}\left.\frac{d^3u}{dy^3}\right|_{y=-\frac{1}{4}} = -48 + \frac{R_T GR}{2}S\left(4 + \frac{1}{Bi_1}\right) \quad (21)$$

$$\left.\frac{d^2u}{dy^2}\right|_{y=\frac{1}{4}} + \frac{1}{Bi_2}\left.\frac{d^3u}{dy^3}\right|_{y=\frac{1}{4}} = -48 - \frac{R_T GR}{2}S\left(4 + \frac{1}{Bi_2}\right) \quad (22)$$

$$\left.\frac{d\theta}{dy}\right|_{y=-\frac{1}{4}} = Bi_1\left[\theta\left(-\frac{1}{4}\right) + \frac{R_T S}{2}\left(1 + \frac{4}{Bi_1}\right)\right] \quad (23)$$

$$\left.\frac{d\theta}{dy}\right|_{y=\frac{1}{4}} = Bi_2\left[-\theta\left(\frac{1}{4}\right) + \frac{R_T S}{2}\left(1 + \frac{4}{Bi_2}\right)\right] \quad (24)$$

The dimensionless form of velocity and temperature profiles thus depend on five parameters:

the mixed convection parameter $GR = Gr/Re$, the Brinkman number Br, the temperature difference ratio R_T, the Darcy number Da, the Forchheimer number F and the Biot numbers Bi_1 and Bi_2. Following the work of Zanchini (1998), the Nusselt numbers calculated at the left and right vertical channels are given by:

$$Nu_1 = \frac{1}{R_T[\theta(1/4) - \theta(-1/4)] + (1 - R_T)} \frac{d\theta}{dy}\bigg|_{y=-1/4}$$

$$Nu_2 = \frac{1}{R_T[\theta(1/4) - \theta(-1/4)] + (1 - R_T)} \frac{d\theta}{dy}\bigg|_{y=1/4} \qquad (25)$$

Equations (18) and (19) yield a 4^{th} order non-linear ordinary differential equation in U but needs two additional boundary conditions on U to be generated. This was impossible to do in view of third type boundary condition on θ. Hence, the coupled system of Eq. (18) and (19) subject to conditions (20)-(24) is solved by Runge-Kutta method with shooting method.

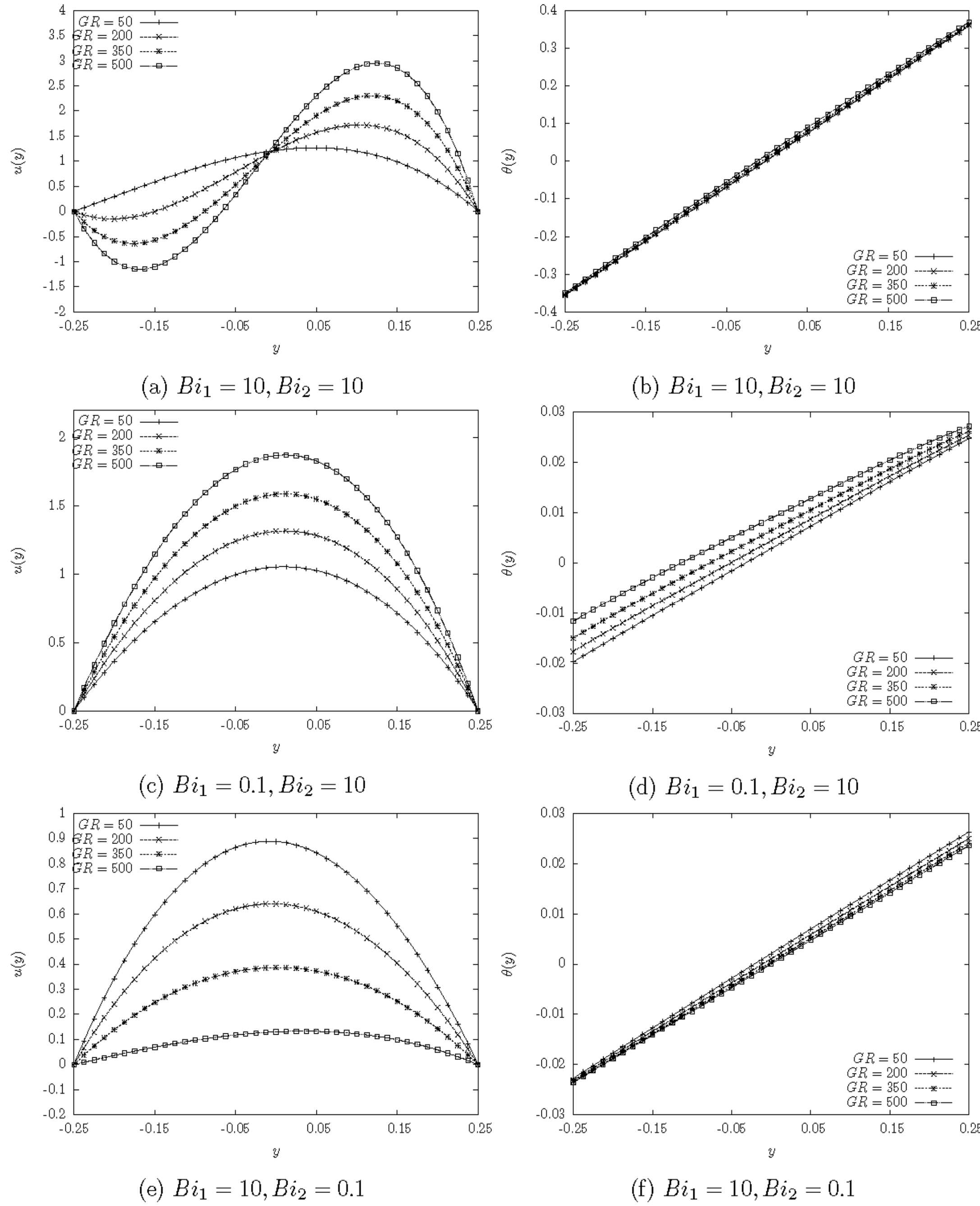


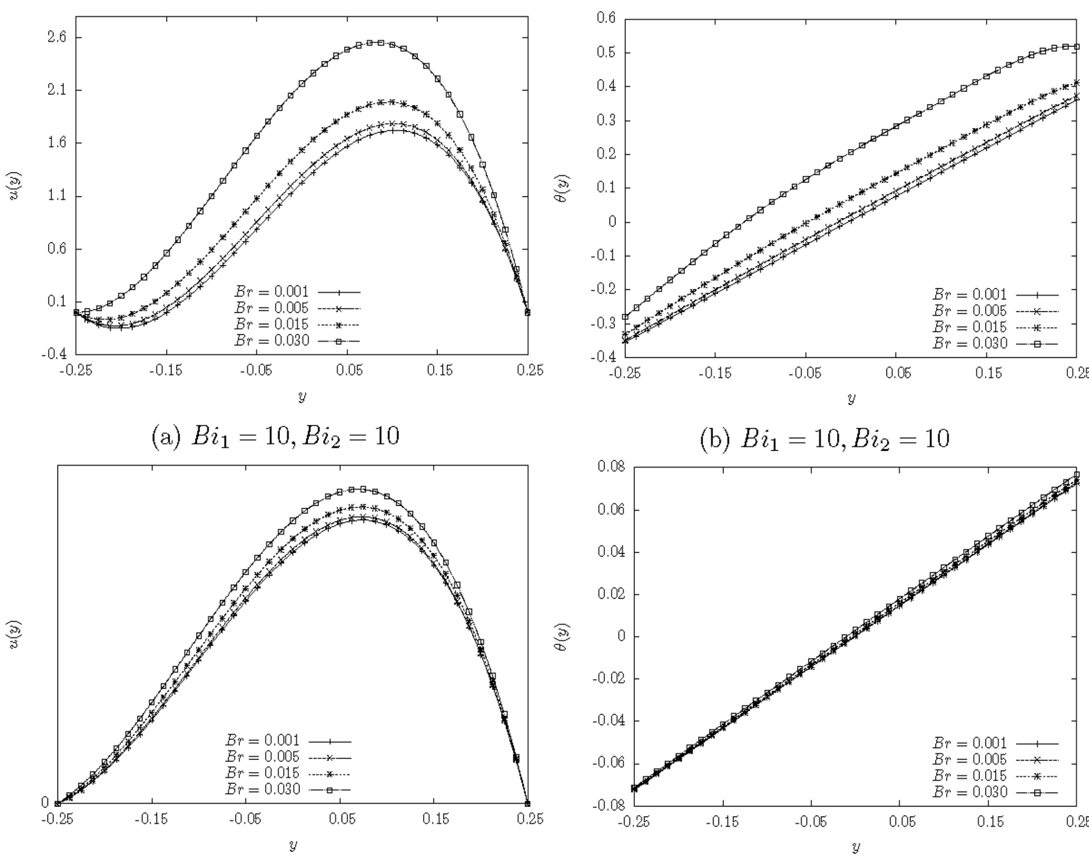
Fig. 2: Velocity profiles (left) and temperature profiles (right) of non-linear, non-Darcy flow for various values of GR and different combination of Biot numbers, Bi_1 and Bi_2 with $Br = 0.001, F = 1$ and $Da = 2$

RESULTS AND DISCUSSION

The studies involves non-linear, non-Darcy flow through sparsely packed chemically inert porous medium, focusing on investigating the effects of the mixed convection parameter GR = *Gr/Re*, the Brinkman number *Br*, the Darcy number *Da*, the Forchheimer number F, Biot numbers Bi_1 and Bi_2 and the Nusselt numbers *Nu*. The fixed value selected for all cases is $R_T = 1$.

VELOCITY AND TEMPERATURE PROFILES

Various values of *GR* with different combination of Biot numbers: Fig. 2 represents the velocity and the temperature profiles respectively for non-linear, non-Darcy flow of various values of mixed convection parameter GR and different combination of Biot numbers Bi_1 and Bi_2 with $Br = 0.001, F = 1$ and $Da = 2$. Looking into Fig. 2a, where the Biot numbers are equal, it can be observed that as the intensity of the mixed convection increases, the velocity distribution within the channel becomes less uniform. As the liquid made contact with the colder wall, it shrinks accordingly. With this reverse flow occurs due to the increase of density and the reduce of buoyancy force in the upward direction. With high value of both Biot numbers and when $GR \geq 200$ (high value of mixed convection parameter), reflow phenomenon occurs. However, having different combination of Biot numbers as in Fig. 2c and e shows that the velocity profile is symmetrical throughout the channel and there is no existence of reversal flow. When $Bi_1 < Bi_2$, as in Fig. 2c, the velocity profile increases with the increase of *GR*. When $Bi_1 > Bi_2$, as in Fig. 2e, the velocity profile decreases as the value of GR increases. Meanwhile, looking at the temperature profile in Fig. 2b, we can observe almost no change with high values of Bi_1 and Bi_2. However, looking at the temperature profiles with unequal Biot numbers in Fig. 2d and f, more obvious change can be seen in the temperature profiles. The effect of *GR* on the temperature profile is most operative when the value of Bi_1 is smaller and Bi_2 is larger and is more apparent at the colder wall. Meanwhile, when $Bi_1 > Bi_2$, the value of temperature decreases with the increase of the value of *GR* and the changes is slightly more prominent at the warmer wall.



(a) $Bi_1 = 10, Bi_2 = 10$

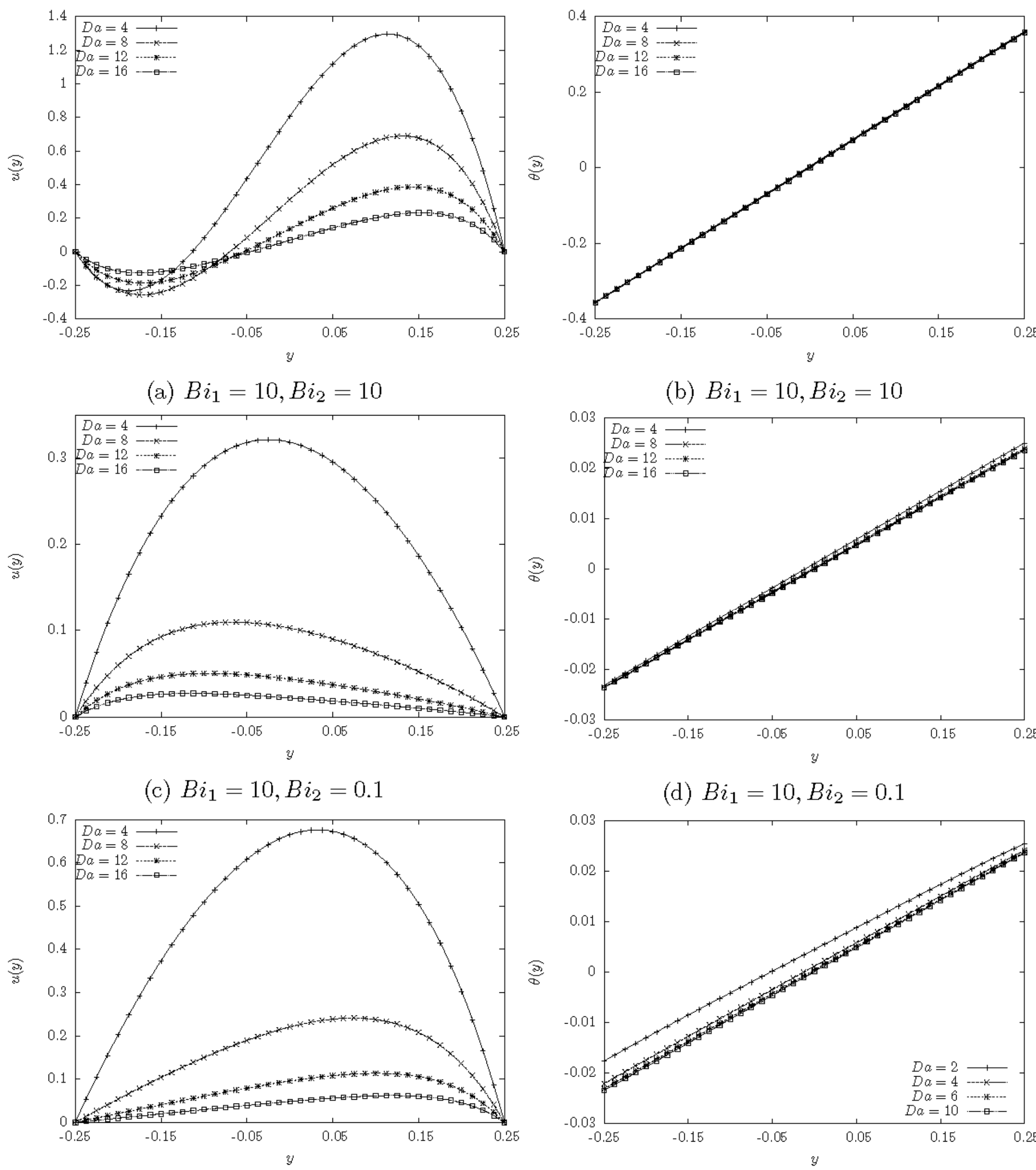
(b) $Bi_1 = 10, Bi_2 = 10$

(c) $Bi_1 = 0.1, Bi_2 = 10$

(d) $Bi_1 = 0.1, Bi_2 = 10$

Fig. 3: Velocity profiles and temperature profiles of non-linear, non-Darcy flow for various values of Br and different combination of Biot numbers, Bi_1 and Bi_2 with $Br = 0.001, GR = 200$ and $Da = 2$

Effects of *Br* on velocity and temperature profiles with different combination of Biot numbers: Fig. 3 presents effects of various levels of Brinkman number on dimensionless velocity and temperature profiles. Obviously in Fig. 3a, increasing viscous dissipation increases the velocity and decreases the flow reversal. The intensity of flow reversal enhanced with smaller value of *Br*. This is a natural consequence because with the increase of *Br*, there is an enlarging in buoyancy effect due to dissipation. The stronger viscous dissipation causes higher fluid temperature, which resulted in the increase of *GR* and therefore yields an increase of the fluid velocity. This result is similar with what was obtained by Umavathi and Sheremet (2016). However, Fig. 3c shows that unequal Biot numbers resulted to no occurrence of flow reversal. This shows that having high value on both Biot numbers enhances the occurrence of flow reversal. Meanwhile, in Fig. 3d, reflecting to the heating effect due to viscous dissipation on the thermal field, increasing the value of Br enhances the temperature profile. There is almost no obvious change in the temperature profile when $Bi_1 < Bi_2$. However, having high value of both Biot numbers give significant change in the temperature profile and it is more prominent towards the hotter wall.



(a) $Bi_1 = 10, Bi_2 = 10$ (b) $Bi_1 = 10, Bi_2 = 10$

(c) $Bi_1 = 10, Bi_2 = 0.1$ (d) $Bi_1 = 10, Bi_2 = 0.1$

(e) $Bi_1 = 0.1, Bi_2 = 10$ (f) $Bi_1 = 0.1, Bi_2 = 10$

Fig. 4: Velocity profiles and temperature profiles of non-linear, non-Darcy flow for various values of Da and different combination of Biot numbers, Bi_1 and Bi_2 with $Br = 0.001, GR = 200$ and $F = 1$

Effects of Da on velocity and temperature profiles with different combination of Biot numbers: The dimensionless velocity and dimensionless temperature distributions, with various levels of Darcy numbers and a constant mixed convection parameter $GR = 200$, the Brinkman number $Br = 0.001$ and the Forcheimer number $F = 1$ are shown in Fig. 4. Looking at the velocity profiles, reversal flow only occurs when both the Biot numbers are high, as in Fig. 4a. Increasing the Darcy number enhances the flow reversal and suppresses the maximum velocity towards the hotter wall. However, for the large Da, flow reversal decreases. This means that a sufficiently low Darcy number weakens the velocity at each position including flow reversal near the colder wall. When Bi_1 is low and Bi_2 is high, the maximum velocity skew towards the colder wall as Da increases and when $Bi_1 > Bi_2$, the maximum velocity moves towards the hotter wall as Da increases. For all the cases, velocity profile decreases with the increase of the value of Da. The reason for such behavior is that higher Darcy number Da has lower resistance to the fluid flow. Observing the corresponding temperature profiles, most significant changes can be noticed when $Bi_1 > Bi_2$ as in Fig. 4f and it is more prominent at the colder wall. Lower Darcy number shows a higher temperature profile and it

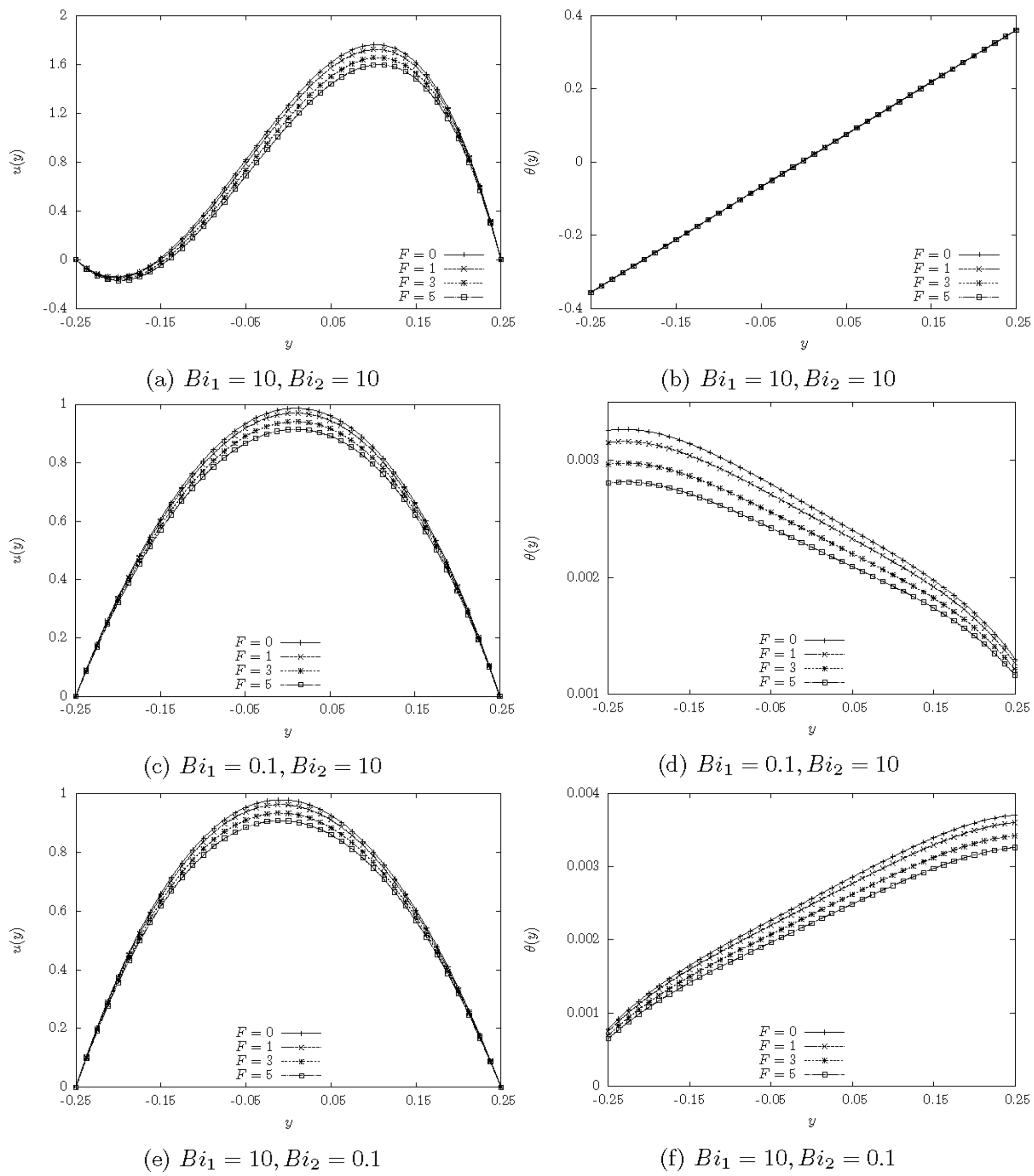
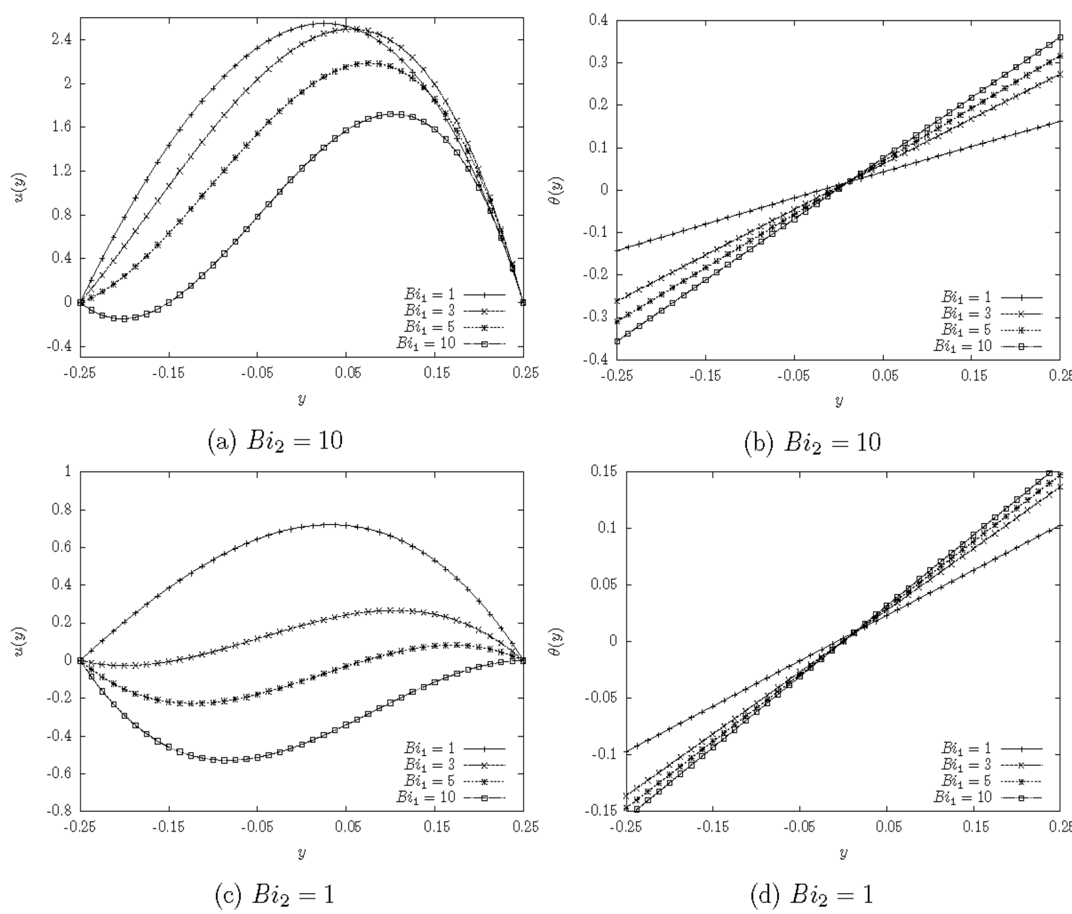


Fig. 5: Velocity profiles and temperature profiles of non-linear, non-Darcy flow for various values of F and different combination of Biot numbers, Bi_1 and Bi_2 with $GR = 200, Br = 0.001$ and $Da = 2$

became more consistent as Darcy number gets higher. However though, almost no changes on the temperature profile when Biot numbers are equally high as in Fig. 4b and when $Bi_1 < Bi_2$, only slight changes is noticeable which is more obvious at the warmer wall.

Effects of F on velocity and temperature profiles with different combination of Biot numbers: In order to understand the effect of inertia (Forcheimer number F) on the flow, both the velocity and temperature profiles are plotted in Fig. 5 with different combination of Biot numbers, Bi_1 and Bi_2 with $Br = 0.001, GR = 200$ and $Da = 2$. Observing Fig. 5a, since the drag in the medium reduces the flow, as a result, the maximum magnitude of the velocity is expected to decrease with the increase of F. Similar relation is also reported by Kumar *et al*. (2011). Asymmetric velocity profiles occur when the flow tends to force convection, resulting the buoyancy effect to decrease. The velocity near the central region slows down due to the existence of inertia effect, leading to a more uniform velocity distribution. The asymmetric velocity profiles occurs with unequal Biot numbers, as in Fig. 5c and e and it illustrate the effect of nonuniform buoyancy force caused by the asymmetric thermal boundary conditions. Meanwhile, looking into the temperature profiles of Fig. 5b, d and f, when Biot numbers are equally high, it does not give any significant effect on the temperature profiles relative to its effects on the maximum magnitude at center of the channel. However, when $Bi_1 < Bi_2$, the maximum magnitude of the temperature decrease with the increase of F and the parabolic profile becomes flat towards the warmer wall. From the Forcheimer number F, the inertia forces add on resistance mechanism which results in reduction of flow field. Mean while, when $Bi_1 > Bi_2$, the temperature profile increase with the increase of F and more operative at the warmer wall.

Effects of Biot numbers on velocity and temperature profiles: The effects of Biot number on colder wall $Bi_1$and hotter wall Bi_2 are illustrated graphically in Fig. 6 and 7 respectively. Both the velocity profiles in Fig. 6 decrease with the increase of Bi_1 and become symmetrical with low value of Bi_1. For higher value of Bi_2as in Fig. 6a, backflow only starts to occur with large valueofBi$_1$. However, with lower value of Bi_2 as



(a) $Bi_2 = 10$ (b) $Bi_2 = 10$

(c) $Bi_2 = 1$ (d) $Bi_2 = 1$

Fig. 6: Velocity profiles and temperature profiles of non-linear, non-Darcy flow for various values of Bi_1 and different variation of Bi_2 with $GR = 200, Br = 200, Br = 0.001, F = 1$ and$Da = 2$

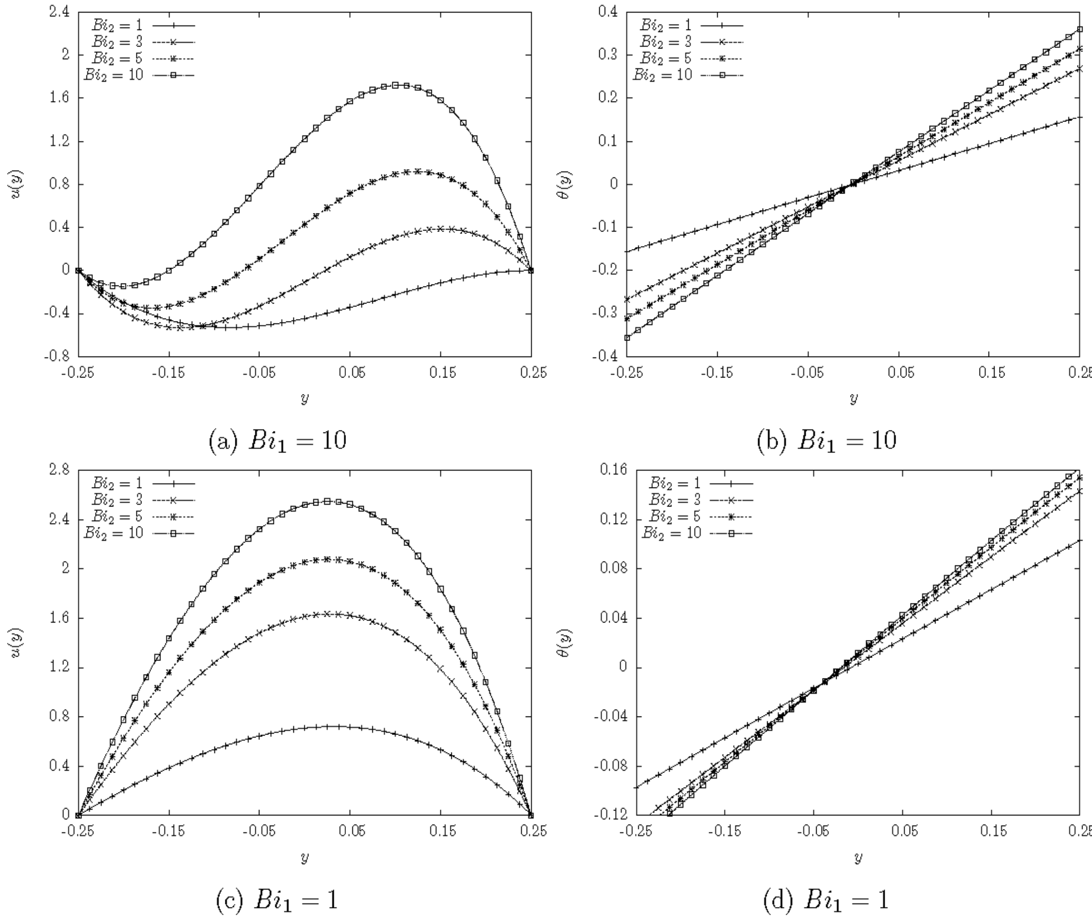
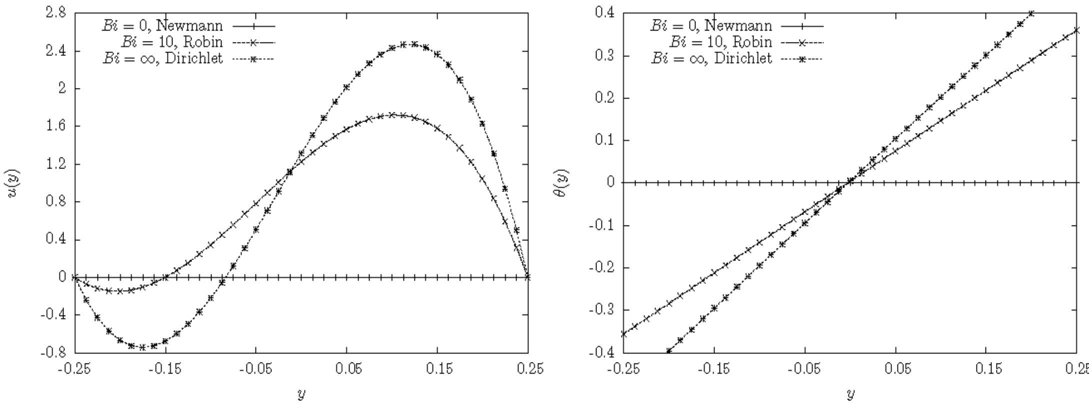


Fig. 7: Velocity profiles and temperature profiles of non-linear, non-Darcy flow for various values of Bi_2 and different variation of Bi_1 with $GR = 200, Br = 0.001, F = 1$ and$Da = 2$

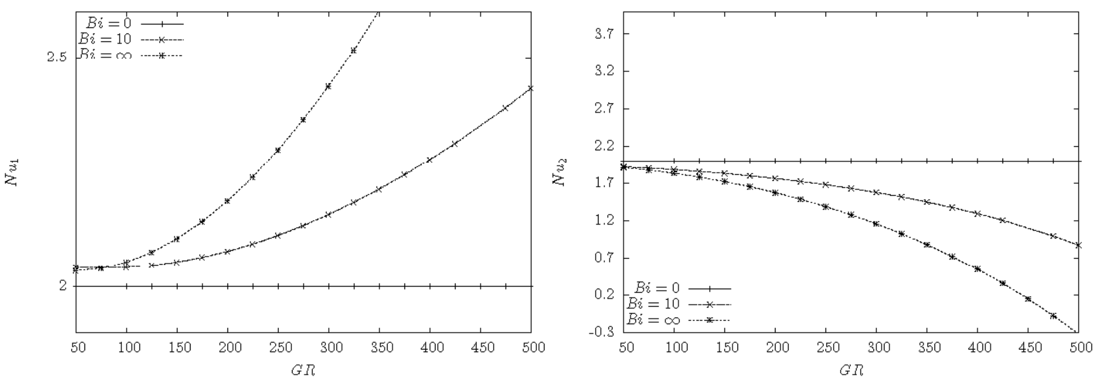
in Fig. 6c, the concavity of the curve in the velocity profile changes even for a small value of Bi_1, i.e., $Br = 0.001$. This change occurs along with point of inflection at the center zone of the channel, which indicates the occurrence of back flow near to the colder wall. Meanwhile, both the velocity profiles in Fig. 7 increase with the increase of Bi_2. With large value of Bi_1 in Fig. 7a, back flow occurs for all cases of Bi_2 and the intensity increases with the increase of Bi_2 However, with small value of Bi_1 in Fig. 7c, there is no occurrence of back flow and the graph is symmetrical for all cases of Bi_2. The corresponding temperature profiles are in Fig. 6b and 6d for the effects of Biot numbers on colder wall Bi_1. With high value of Bi_2as in Fig. 6b, a more rapid change occurs for higher values of Bi_1 Meanwhile, with low value of Bi_2 in Fig. 6d, there is no significant change for higher values of Bi_1. For both cases, the the temperature decrease with the increase of Biot numbers from the left wall towards mid-channel and increase with the increase of Biot numbers from mid-channel towards the right wall. It is interesting to note that the thermal resistance of the channel decreases and convective heat transfer to the fluid on the right wall increases as Biot number increases. Similar result is obtained for the case of the effects of Biot numbers on hotter wall Bi_2 in Fig. 7b and d.

Velocity and temperature profiles for mixed boundary conditions: Fig. 8 shows the influence of three kinds of boundary conditions on temperature, $Bi = 0$ (Neumann), $Bi = 10$ (Robin) and $Bi \to \infty$ (Dirichlet) on the velocity and temperature profiles. The Biot number Bi is the ratio of the thermal resistance of the channel to the fluid thermal resistance. Hence, for Neumann case with $Bi = 0$ (without Biot number), the hot side of the channel is totally insulated and no convective heat transfer to the cold wall, hence both the velocity and temperature profiles are constant. Meanwhile, the channel thermal resistance reduces as the convection Biot number increases. This resulting the significant increase of the peak velocity and the velocities in the neighbourhood of the peak towards the hotter wall. The stronger buoyancy forces induced as a result of the increase in the strength of the convective process on the channel triggered this effect. Hence, the higher value of Biot number (Dirichlet condition) indicates higher magnitude of the velocity as well as temperature profile at the vicinity of the right wall of the channel.



(a) (b)

Fig. 8: (a) Velocity profile and (b) temperature profile of non-linear, non-Darcy flow for three kinds of boundary conditions on temperature ($Bi = 0$-Neumann, $Bi = 10$-Robin and $Bi \to \infty$-Dirichlet) where $Bi_1 = Bi_2 = Bi, GR = 200, Br = 0.001$, $F = 1$ and $Da = 2$



(a) (b)

Fig. 9: Effect of mixed convection parameter (*GR*) on Nusselt number values, (a) $Nu_1$and (b) Nu_2 for three kinds of boundary conditions on temperature ($Bi = 0$-Neumann, $Bi = 10$-Robin and $Bi \to \infty$-Dirichlet) where $Bi_1 = Bi_2 = Bi$, $Br = 0.001$, $F = 1$ and$Da = 2$

HEAT TRANSFER EVALUATION

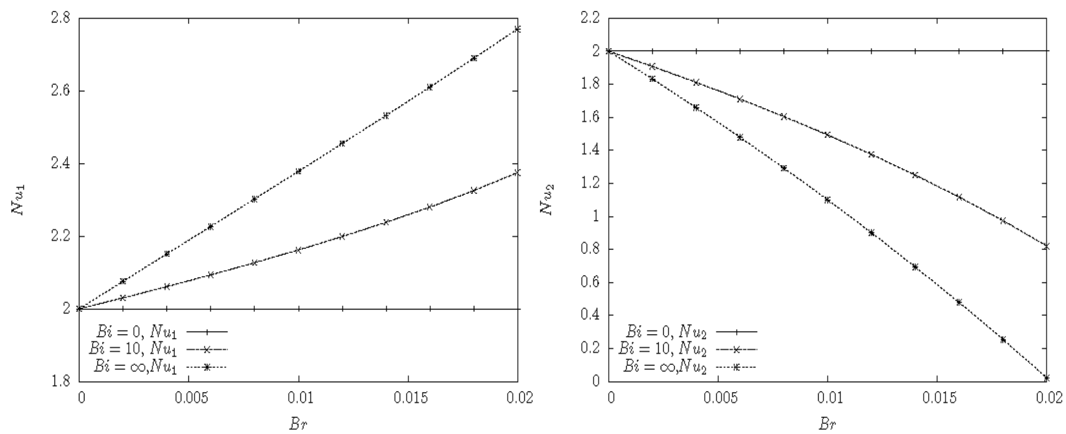
To maximize the heat transfer efficiency with a smooth flow, the viscosity as well as the mixed convection parameter should be considered as important physical properties of a porous medium. The effects of *GR*, *Br*, *F* and *Da* on the rate of heat transfer at both walls for three kinds of boundary conditions on temperature are displayed at Fig. 9 to 15. Nusselt number, *Nu* represent the heat transfer rate with Nu_1 referring to the colder wall and Nu_2referring to the warmer wall. When $Nu_2 > 0$, heat-transfer direction at the hot wall is from the channel wall to the liquid and when $Nu_2 < 0$, heat-transfer direction at the hot wall is from the liquid to the channel wall. Similarly, When $Nu_1 > 0$, heat-transfer direction at the cold wall is from the liquid to the wall and when $Nu_1 < 0$, heat-transfer direction at the cold wall is from the wall to the liquid. For all the cases, Nusselt numbers both on the cold wall $Nu_1$and hotter wall Nu_2 do not show any significant change for the case of Neumann boundary condition ($Bi = 0$). This proves the importance of the existence of Biot numbers on heat transfer performance. Besides, the Robin-type condition proves best as it requires the least work to achieve a given accuracy and it gives the most accurate solution at fixed cost. The Robin condition appears to be superior to the Neumann condition and the Dirichlet condition seems unrealistic on sufficiently large mixed convection parameter *GR*. The findings are supported by what was discovered by Novy *et al*. (1991).

Effects of *GR* on Nusselt numbers: The variation of Nu_1 and Nu_2 as a function of the Mixed Convection parameter *GR* for three different kinds of boundary conditions on temperature is plotted in Fig. 9. As shown in Fig. 9a, the buoyancy force acts to increase the fluid velocity when *GR* increases, thus will enhance the convection heat transfer, resulting the increase of Nusselt number for the case of Robin and Dirichlet boundary conditions. Meanwhile, Nusselt number at the warmer wall Bu_2 is a decreasing function of *GR*, with the most extreme changes occurs in Dirichlet condition (large value of Biot number). However, *GR* gives no significant change in terms of the Nusselt number values for the case of Neumann boundary condition.

Effects of *Br* on Nusselt numbers: Figure 10a plots the variation of the heat transfer rate on the colder wall Nu_1 with the Brinkman number *Br* for three different kinds of boundary conditions on temperature. It is shown in the Fig. that increasing *Br* tends to accelerate the fluid flow, thus raising the heat transfer rate of the fluid flowing through the vertical channel. An opposite condition is noticeable for the hotter wall Nu_2, in Fig. 10b.

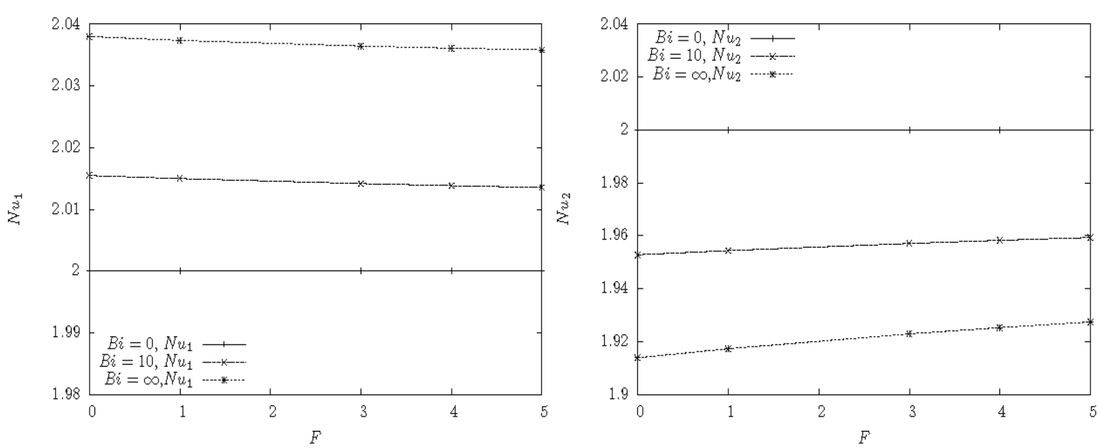
Effects of *F* on Nusselt numbers: Forcheimer number *F* has a very minor effect on the Nusselt numbers both on the cold wall Nu_1 in Fig. 11a and on the hotter wall Nu_2 in Fig. 11b. With the increase of *F*, there is a slow increment of Nu_2 and a slow decrement of Nu_1 for Robin and Dirichlet boundary conditions on temperature. It can be concluded that *F* effect on Nusselt numbers is negligible. The result is similar as obtained by Chen *et al*. (2000).

Effects of *Da* on Nusselt numbers: Fig. 12 represent the heat transfer variation on the cold wall Nu_1 and hot wall Nu_2 as a function of *Da*. The value of Nu_2 increase with the increase of *Da* for the Robin and



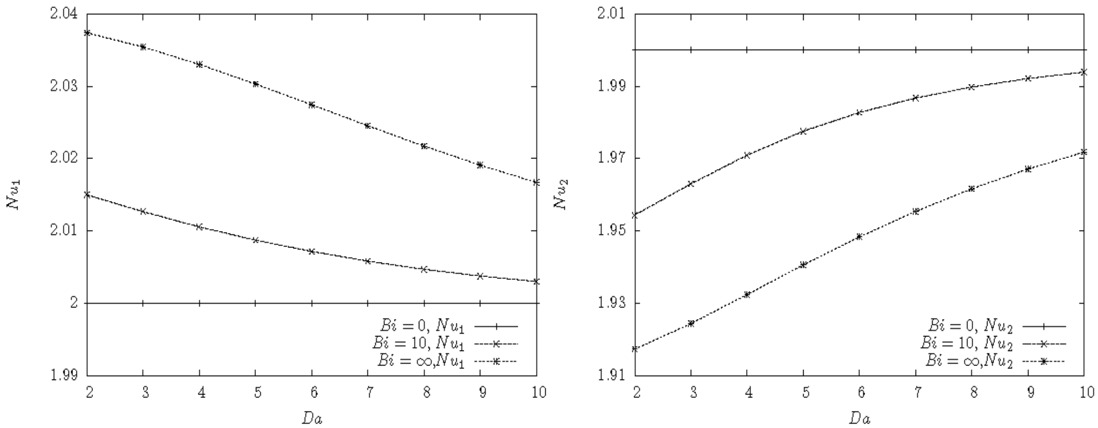
(a) (b)

Fig. 10: Effect of viscous dissipation (*Br*) on Nusselt number values, (a): $Nu_1$and (b): Nu_2 for three kinds of boundary conditions on temperature ($Bi = 0$-Neumann, $Bi = 10$-Robin and $Bi \to \infty$-Dirichlet) where $Bi_1 = Bi_2 = Bi$, $GR = 200$, $F = 1$ and$Da = 2$



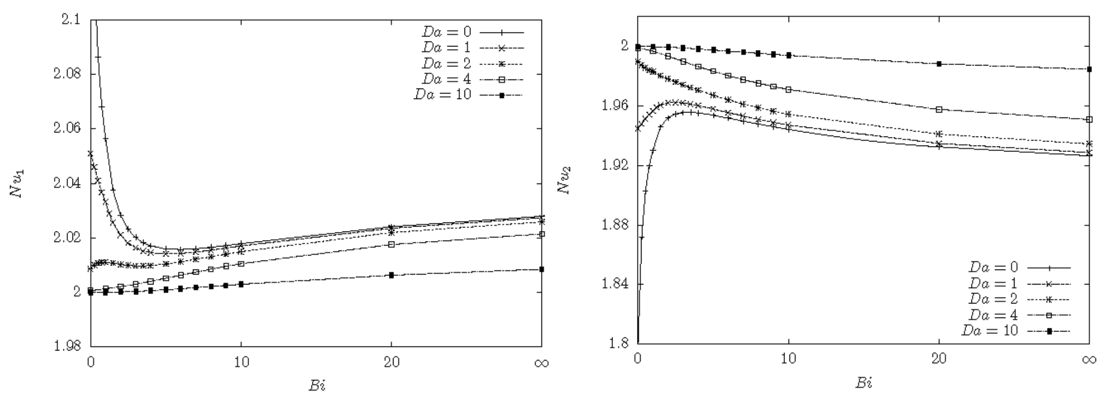
(a) (b)

Fig. 11: Effect of Forchheimer drag term (*F*) on Nusselt number values, (a): $Nu_1$and (b): Nu_2 for three kinds of boundary conditions on temperature ($Bi = 0$-Neumann, $Bi = 10$-Robin and $Bi \to \infty$-Dirichlet) where $Bi_1 = Bi_2 = Bi$, $GR = 200$, $Br = 0.001$ and $Da = 2$



(a) (b)

Fig. 12: Effect of Darcy number (*Da*) on Nusselt number values, (a): Nu_1 and (b): Nu_2 for three kinds of boundary conditions on temperature ($Bi = 0$-Neumann, $Bi = 10$-Robin and $Bi \to \infty$-Dirichlet) where $Bi_1 = Bi_2 = Bi,\ GR = 200,\ Br = 0.001$ and $F = 1$



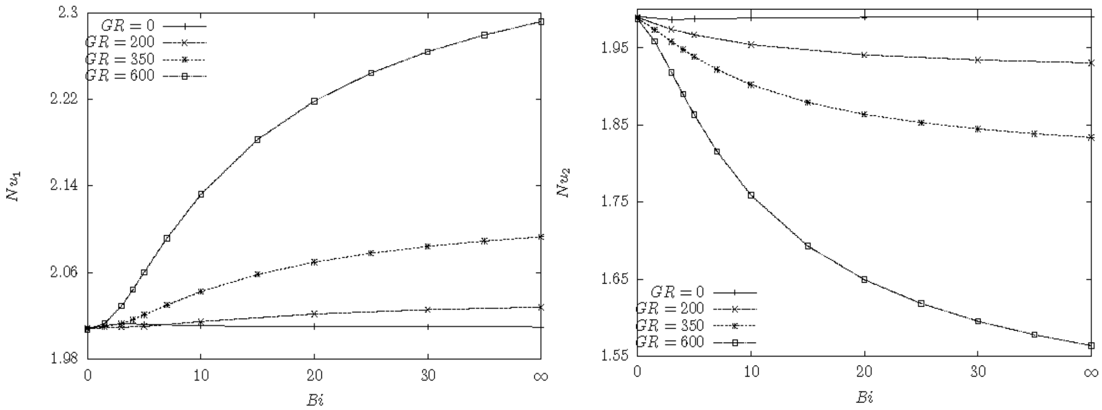
(a) (b)

Fig. 13: Effect of Biot numbers *Bi* (where $Bi_1 = Bi_2 = Bi$), on Nusselt number values, (a): $Nu_1$and (b): $Nu_2$for various Darcy numbers *Da*, with $GR = 200,\ Br = 0.001$ and$F = 1$

Dirichlet conditions. In these cases, *GR* and hence the buoyancy forces are fixed. A high *Da* leads to a high permeability. Hence, as the fluid flows through the porous medium, the fluid experiences a relatively smaller resistance which leads to higher speed and, end up an increase in the heat transfer rates. An opposite effect occurs on the colder wall Nu_1. In both cases, lower value of *Da* resulting the penetrating capability of a porous medium diminishes and induces more drag force on fluid. Hence, lower value of *Da* give a less significant change in the Nusselt numbers.

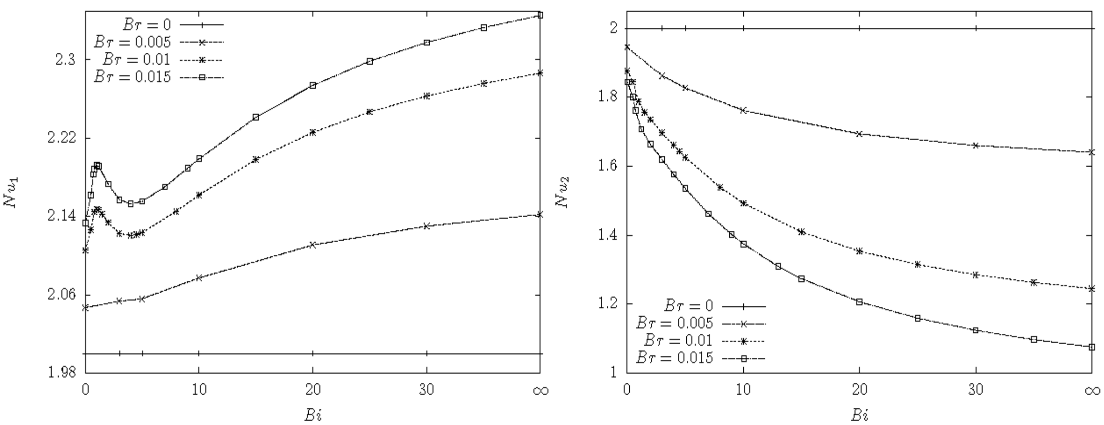
Effects of Biot number *Bi* on Nusselt numbers *Nu* for various Darcy number *Da*: The effect of the Biot number *Bi* on the heat transfer rate on the left wall Nu_1 and right wallNu_2 with various value of Darcy numbers *Da* is illustrated in Fig. 13. Since *Da* by defination is inversely proportional to the square of the permeability *K*, heat transfer rate at the colder wall decrease abruptlyin Neumann condition *Bi* = 0), before reaching a constant rate when *Da* is higher. Meanwhile, the peak Nusselt number Nu_2 value is seen to increase significantly on the left wall before reaching a constant value when *Da* is higher. For both cases, with the increase of *Da*, Nu_1 and Nu_2 values remain consistent in Robin condition, but for Dirichlet condition ($Bi \to \infty$), it shows a rather significant decrease inNu_2as in Fig. 13a and increase in Nu_2 as in Fig. 13b.

Effects of Biot number *Bi* on Nusselt numbers *Nu* for various Mixed Convection parameter *GR*: The rate of heat transfer at both walls on Biot number *Bi* with the variation of mixed convection parameter *GR* is displayed in Fig. 14. With the increase of *GR*, the Nusselt number at cold wall Nu_1 is an increasing function of *Bi* due to the increase in the buoyancy ratio



(a) (b)

Fig. 14: Effect of Biot numbers *Bi* (where $Bi_1 = Bi_2 = Bi$), on Nusselt number values, (a): Nu_1 and (b): Nu_2 for various mixed convection parameter *GR*, with $Da = 2,\ Br = 0.001$ and $F = 1$



(a) (b)

Fig. 15: Effect of Biot numbers *Bi* (where $Bi_1 = Bi_2 = Bi$), on Nusselt number values, (a): $Nu_1$and (b): $Nu_2$for various values of Brinkman number *Br*, with $Da = 2,\ GR = 200$ and $F = 1$

that leads to acceleration of the fluid flow, hence raising the heat transfer rate. This is illustrated in Fig. 14a. However, in Fig. 14b, the maximum Nusselt number at the hot wall Nu_2 occurs on the left channel and drop faster with the increase of *GR*. The decrease of Nu_2 is becoming more prominent when the boundary condition moves towards the Dirichlet condition ($Bi \to \infty$).

Effects of Biot number *Bi* on Nusselt numbers *Nu* for various Brinkman number *Br*: Effect of Biot number on Nusselt numbers for various values of Brinkman number *Br* are plotted in Fig. 15. Figure 15a, for the case of Neumann condition (low *Bi*), high *Br* leads to a high viscosity rate, resulting a vast increase, then a vast decrease on the Nusselt number at the colder wall Nu_1. It can also be observed that the maximum value of Nu_1 move away from the colder wall as *Br* increases. However, Nu_1 shows a consistent increase with the increase of *Br* for Robin condition ($Bi = 10$) and a rather extreme increase with the increase of *Br* for Dirichlet condition ($Bi \to \infty$).Meanwhile, for the case of Nusselt number at the hotter wall Nu_2, with the increase of *Br* and *Bi*, Nu_2 shows a consistent decreases throughout the channel.

CONCLUSION

In this study, the flow and heat transfer within a fully-developed non-linear, non-Darcy flow through asparsely packed chemically inert porous medium in a vertical channel by considering Dirichlet, Neumann and Robin boundary conditions has been studied. The dimensionless forms of the governing equations are solved using Runge-Kutta method with shooting technique. The major findings are as follows:

- The velocity distribution become less uniform by increasing mixed convection parameter.
- The temperature is constant by increasing mixed convection parameter for all parameters except Brinkman number for Robin and Dirichlet condition.
- At Robin and Dirichlet condition, the intensity of flow reversal is enhanced by increasing Darcy number and Forcheimer number. However, for large Darcy number, flow reversal decreases.
- Both Nusselt numbers are constant by changing mixed convection parameter, Brinkman number, Forcheimer number and Darcy number for the case of Neumann boundary condition. The Robin boundary condition appears to be superior to Neumann boundary condition and Dirichlet boundary condition seems unrealistic for sufficiently large mixed convection parameter and Brinkman number.
- It is believed that the Robin boundary condition deserve a more widespread use as it gives a more satisfactory and realistic result, in comparison with the Dirichlet condition and Neumann condition.

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Conflict of interest: The authors certify that there are no financial/relevant interest that may have influenced the development of the manuscript.

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