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Research Article

On the Simplification of Flexural Failure Prediction of Glass Reinforced Concrete: An Eco-friendly Beam Structure-under 4 Point Bending Loads

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Abstract: This study is aimed to explore how to simply predict the failure of glass reinforced concrete. A lack of design by analysis using FEM has a plenty extensive technique, particularly for reinforced concrete structure cases. Their ingredient are complicated hence induce time costly analysis caused by DOF booming due to an severe computation concerned size of grains, reinforcement, pull out bonds, meso-scale properties and initial cracks, cover spalling and delaminations. Some real concrete based softwares were costly and hard to be run because their complexity input needed. An eco-friendly beam structure using glass strips waste as reinforcement called GLARC (Glass Reinforced Concrete) has been proposed and need to be simply analysis, something convenient but quiet proper for design aims, hence the failure behavior should be easy to predict and its criterion should be known well. Contrast to the deformations and strains that have a strong affiliation with material model, the stresses are fairly free from material model because actually they are loads. Hence standard linear FEM software adequate enough to use as initial analysis GLARC system or its design aims. In this study, a GLARC beam was used as typical example and 4 points bending loads was applied. Initially, the failure will be occurs first at the tensioned part of the concrete and then the next failure sequences were concrete cracks, then glass strip breaks or slip to concrete, before its failure. The maximum deflection around loading zone was 0.815 mm, this show that another nonlinear responses have not been taken into account. This simplified analysis will be a proper-useful way to design a GLARC structural system.

Keywords: Eco-friendly beam structure, flexural failure prediction, GLARC, glass-concrete hybrid, glass strip pieces, glass waste, simplified analysis

INTRODUCTION

Nowadays, due to the development of computational analysis sciences growing rapidly, an approach standard such as DBA (Design by Analysis) build upon finite element procedures that come in entire engineering aspects unite both of theoretic and design field implementation utilize various programming code.

There are many models for analyzing reinforced concrete structures. From flat shell element and idealized layered model shell elements and also discrete techniques combination of solid and beam elements. In the discrete modeling by Irawan and Maekawa (1997), Paccola and Coda (2008) and Danesh *et al*. (2008), concrete is modeled using 3D solid elements while its reinforcement bars are modeled using truss or beam elements. The last kind of modeling needs more degree of freedom, than others. In the layered shell model, concrete is divided into one set consist of 3 layers (top-middle-bottom), while its rebars were combined into one set together with filled concrete, consist of 2 layers (compression-tension). Indeed, layered model is simpler but it represents unrealistic configuration of steel rebars. Phuvoravan and Sotelino (2005) develop a new nonlinear shell element by combining 4 nodal Kirchhoff shell element for concrete and 2 nodal Euler beam for steel rebars, using rigid link between them. Bongochgetsakul and Tanabe (2002) also develop shell element truss-like continuum equivalent for cyclic loading. This study does not describe and simulate each of sophisticated nonlinear shell elements, but rather than implement researcher's idealization by using simple software to investigate discrete model accuracy under static loads linearly both material and geometry. This is adequate to design reinforced concrete shell based on analysis.

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LITERATURE REVIEW

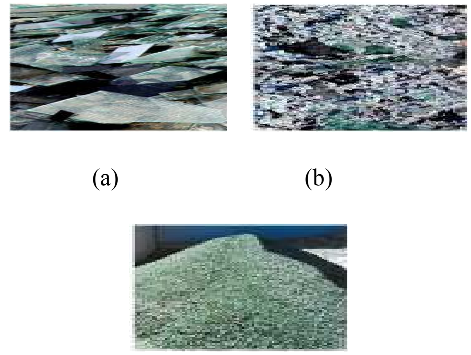
Glass material and glass waste: Glass is formed by super cooling a molten mixture of sand (silicon dioxide), soda ash (sodium carbonate) and/or limestone to form a rigid physical state. Glass aggregate is a product of recycled mixed glass from manufacturing and post consumer waste (Illinois Department of Transportation, 2002). Usually, when a molten material is cooled, a point is reached (the freezing point) where spontaneous nucleation and growth of crystals occurs and the material rapidly solidifies in a crystalline state (Donald, 2010).

The earliest development in glass making of which we have a reasonably documented description seems to be the invention of glass of lead by Ravenscroft around 1673-1676 (Moody, 1988, 1989). From very distant times it had been common to make much European glass using sand and an alkali flux obtained from plant ash. It is important to recognize that glass makers do not use the word melting in its strict scientific sense; most glasses are melted at temperatures below the melting points of their major refractory constituent, most often silica. Their thermal expansion coefficients are similar to those of commercial soda-lime-silica glasses. If a glass can be produced by normal melting that will nearly always be the preferred method on grounds of cost and scale of production: when it will produce a satisfactory product there is little reason to choose a more restricted and expensive method. However, alternatives to normal melting are very active research topics at present (Zarzycki, 1991).

Nearly all of these glasses have little commercial use, primarily because of poor chemical durability. More than 90% of the commercial tonnage is oxide silicate glasses, which are generally multi-component. These systems are high-silica glasses (including vitreous silica, which is the only single-component system), soda lime silicates, sodium borosilicate, aluminosilicates and lead silicate/borates. The balance tonnage comprises borate and phosphate glasses and several nonoxide glasses such as heavy metal fluorides, glassy metals, chalcogenides and amorphous semiconductors (Harper, 2001).

Glass waste and its values in applications: The U.S. produces about 10 million tonnes (11 million tons) of post-consumer glass by-product each year, with about 3.4 million tonnes (3.7 million tons) used primarily as cullet (Miller and Collins, 1976). Figure 1 shows the recycled chain of glass waste from post-consumer glass forms into glass cullets and then formed into sand grain.

Post-consumer glass can be used as a partial replacement of fine aggregate in regular and flowable



(a) (b)

(c)

Fig. 1: Recycling process from post-consumer glass waste into glass cullets and into glass sand grain; Bangil (3a and 3b) and Mid America (3c) recycling

concrete, as well as in CLSM, with Class F coal flyash in amounts needed to control ASR expansion (Naik, 2002). Concrete Materials Research at Columbia University summarized that an economic analysis has been performed to show that a 10% glass substitution for sand and cement can be commercially viable in the New York Metropolitan area. In particular, it was found that the block machine throughput was improved by about 6%, presumably because the glass particles improved the flow properties of the concrete mix (Concrete Materials Research, 2000).

Glass aggregate has been investigated by many state DOTs including New York, Washington and Pennsylvania. Since the 1960s, Washington DOT has used a portion of glass aggregate in bituminous concrete pavements. This aggregate material is also used in backfill for foundations, pipe bedding and other applications not subject to heavy repeated loading. When glass is properly crushed, this material exhibits coefficient of permeability similar to coarse sand. Also, the high angularity of this material, compared to rounded sand, may enhance the stability of asphalt mixes. In general, glass is known for its heat retention properties, which can help decrease the depth of frost penetration (Illinois Department of Transportation, 2002).

Glass mechanical properties: The mechanical properties of concrete and glass among the other materials depicted in Fig. 2.

Figure 3 shows that concrete material have fracture strength around 1-10 MPa and its embodied energy was 2.10^3 MJ/m^3. It is clear that concrete is a brittle material because it has a relatively low stress crack, while glass fracture strength relatively high as 100 MPa and its embodied energy larger than that in concrete around 7.10^4 MJ/m^3 (Ashby, 2005).

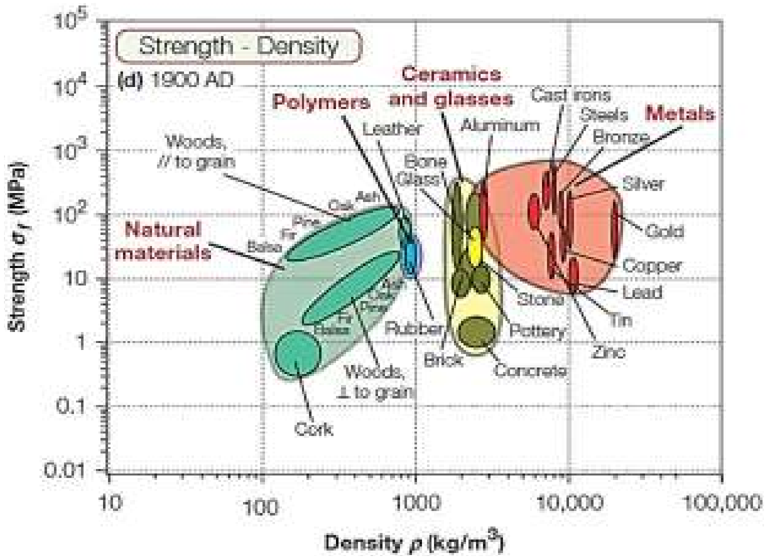


Fig. 2: Strength and density of concrete and glass among the other materials (Ashby, 2005)

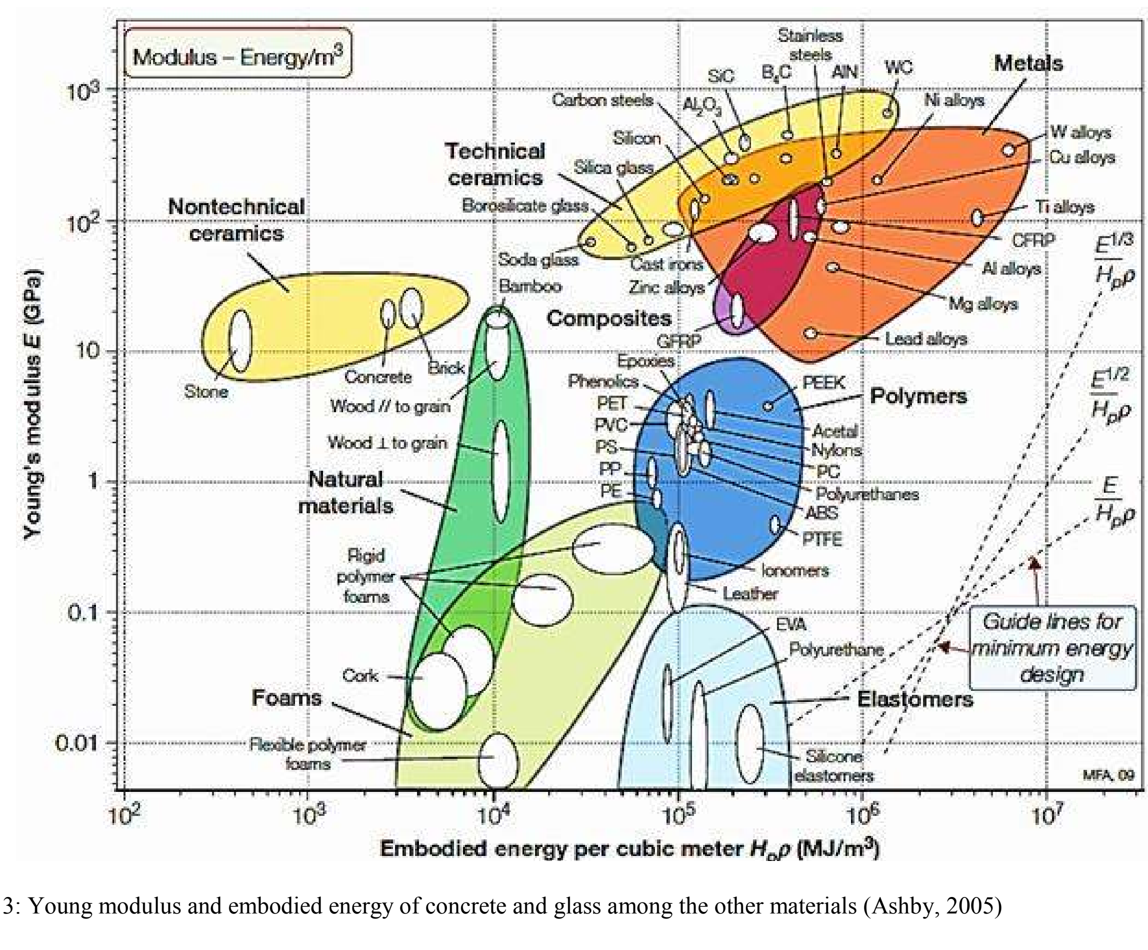


Fig. 3: Young modulus and embodied energy of concrete and glass among the other materials (Ashby, 2005)

In Fig. 4, the relationship of elastic modulus versus cost per unit volume of concrete among the other materials. Concrete Young modulus seems good enough in a relatively \$100/m^3 lower than the other costs above \$100/m^3.

The theoretical strength of amorphous materials is in excess of 7000 MPa (~1,000,000 psi), due to strong covalent bonds. For example, silica glass (SiO_2) has strong tetrahedral bonds between the silicon and oxygen atoms. Indeed, strengths of 3500 to 14,000 MPa

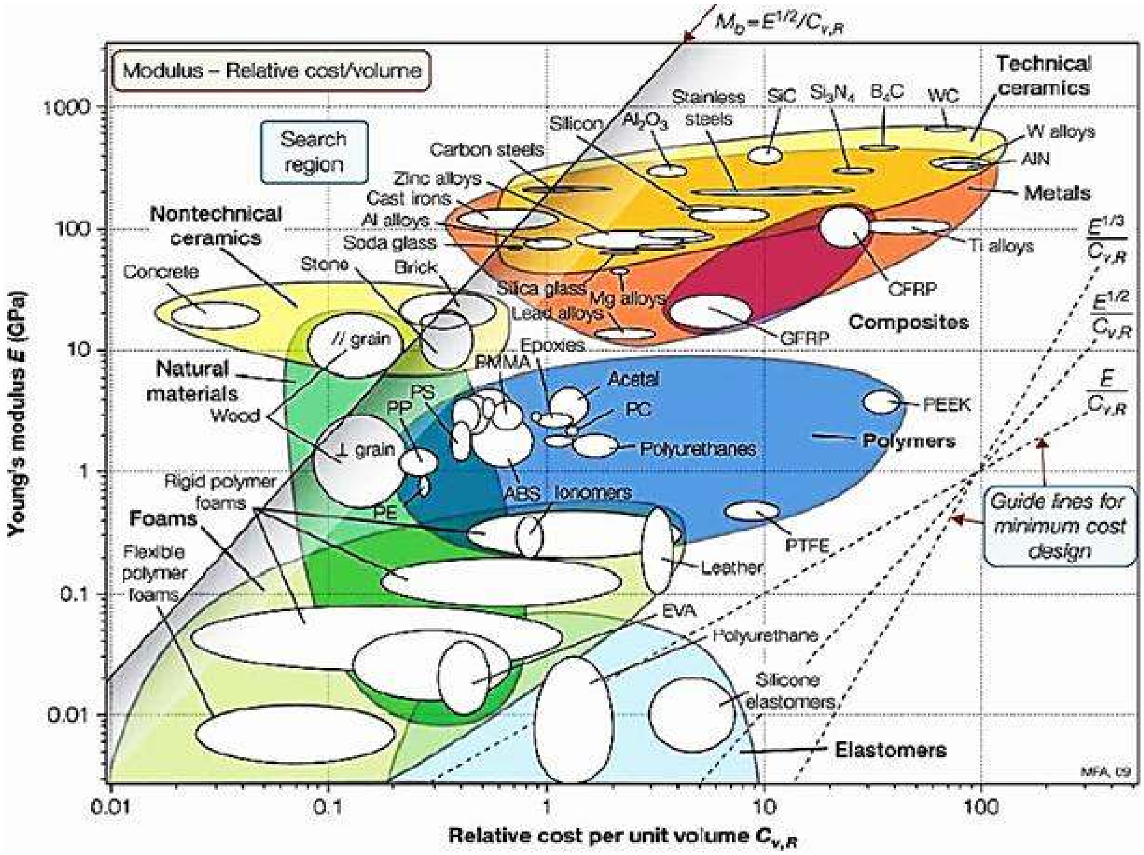


Fig. 4: The relationship of Young modulus versus cost per unit volume (Ashby, 2005)

(500,000 to 2,000,000 psi) have been attained1 by chemical polishing with hydrofluoric acid solutions and by flame-polishing small silica rods. One might consider the only "defect" in such materials to be the interstitial molecular spacing, which is on the sub-nanometer level. Recent advances in nanotechnology further bear this out. However, manufacturing processes, such as those involving generating, grinding, or lapping, are such that defects of much larger proportion-on the micron level-are introduced, which greatly reduces strength. Griffith noting that failure of glass occurred orders of magnitude below its theoretical atomic strength, was the first to postulate that there were microscopic cracks in every material and that these cracks were larger than the interatomic distance. Griffith further hypothesized that these cracks lowered the overall strength of the material. He presented experimental results on glass to prove this by introducing defects of various sizes and showing that it was these effects that determined the strength of the glass. Griffith's work is the basis of modern fracture mechanics, which describes the failure of glass, ceramics and other materials (Pepi, 2014).

Crack in concrete section (Broms method, Broms (1965c)): An approach from Broms (1965a) to take into account the tensile stresses in a concrete section due to pullout force. The specimen that Brom considered is a concrete beam with 2 initial adjacent cracks where the investigation was focused between adjacent cracks in the constant moment zone and pullout bonds between rebar ends (Piyasena, 2002).

External force that applied in rebar position is uniform line load through the width of beam. The amount of these pulling bonds are assumed as 1.5 total tensile force that are transferred by the rebar in a fracture zone (Broms, 1965b). The elastic analytical results show that longitudinal tensile force is described in the concrete as an ellipse between two adjacent flexural cracks with the centre in rebar location such as described in Fig. 5.

Although Raab (1966) has questioned the validity of this assumption in above analysis, Broms (1965b) uses this results to state that the new cracks will be occurs in the mid of two adjacent cracks only if the crack width much larger than twice of concrete cover measured from the mid of rebars (Piyasena, 2002).

Finite element: All real life objects are continuous. Means there is no physical gap between any two consecutive particles. As per material science, any object is made up of small particles, particles of molecules, molecules of atoms and so on and they are bonded together by force of attraction. Solving a real

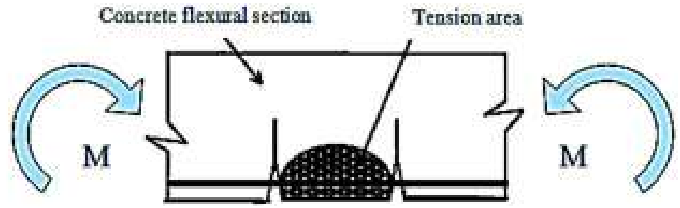


Fig. 5: Beam with 2 initial adjacent cracks predicted by Broms (1965a)

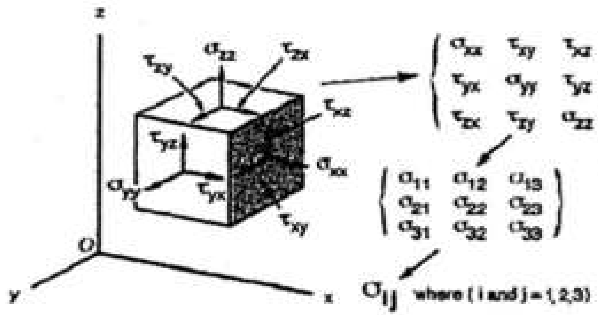


Fig. 6: Stress tensor of the 3D continuum global element (Kriz, 2000)

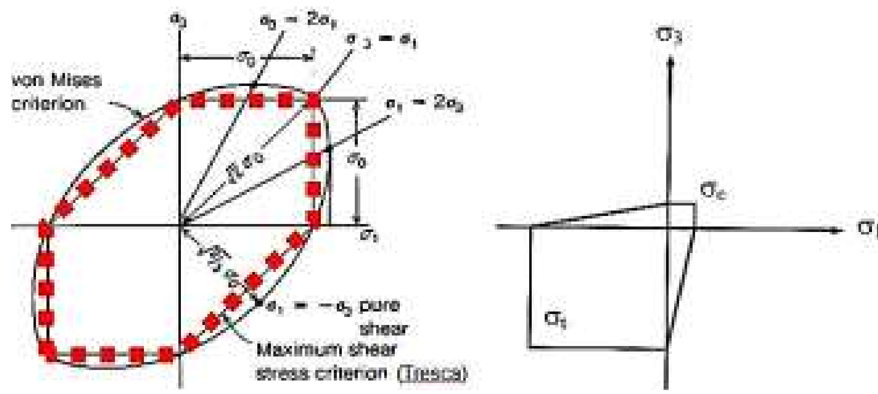


Fig. 7: (a): Comparison von Mises and Tresca failure criterion theories of isotropic materials; (b): Schematic two-dimensional representation of Mohr-Coulomb failure criterion (Chen and Han, 2007)

life problem with continuous material approach is difficult and basic of al1 numerical methods is to simplify the problem by discretizing (discontinuation) it. In simple words nodes work like atoms and with gap in between filled by an entity called as element. Calculations are made at nodes and results are interpolated for elements (Gokhale *et al*., 2008).

Figure 6 shows global structural stress tensor in a massive 3D cubic element. The stresses in cube sides can be described using 6 stress components, where 3 normal stresses σ_x, σ_y, σ_z and 3 couples of shear stresses as $\tau_{xy}= \tau_{yx}$, $\tau_{xz}= \tau_{zx}$, $\tau_{yz}= \tau_{zy}$.

Brittle failure criterion: Before explained the brittle failure such as Mohr-Coulomb criterion, it need to explore ductile Tresca (maximum shear stress) criterion as the closest criterion to Mohr-Coulomb. Plastic flow starts when the maximum shear stress in a complex state of deformation reaches a value equal to the maximum shear stress at the onset of flow in uniaxial tension or compression (Meyers and Chawla, 2009). The maximum shear stress is given by following expression and figured out in Fig. 7a):

$$\tau_{\max} = \frac{\sigma_{11} - \sigma_{33}}{2} = \frac{\sigma_{\max} - \sigma_{\min}}{2} \tag{1}$$

This criterion corresponds to taking the differences between σ_1 and σ_3 and making it equal to the flow stress in uniaxial tension (or compression).

The tensile strength of concrete is approximately one-tenth of its compressive strength. Such is also the case for many brittle materials. Therefore, the Tresca criteria have to be modified to incorporate this behavior.

Table 1: Glass waste mechanical properties

Stiffness	Symbols	Value	Strength	Symbols	Value
Elastic modulus	E_{11}	70 GPa	Tensile strength	X_T	70 MPa
Shear (plane) modulus	G_{12}	27 GPa	Compression strength	X_C	800 MPa
Poisson ratio	ν_{12}	0.22	Flexural strength	X_F	45 MPa

From many literatures and researches

This Mohr-Coulomb criterion is simply the equivalent of the Tresca criterion with different tensile and compressive strengths. Figure 7b shows the Mohr-Coulomb criterion in a schematic fashion. The criterion for failure is a maximum shear stress; the compressive strength σ_c is much higher than the tensile strength σ_t (Meyers and Chawla, 2009).

Classical shell theory: This model, CST, is the well-known basic theory in plate/shell analysis, that have been discussed in the plate and shell and early finite element analysis literatures (Timoshenko and Woinowsky-Krieger, 1959; Szilard, 1974; Cook, 1981; Weaver and Johnson, 1984; Reddy, 1984). This theory based on the assumption that the normal plane is remain straight and perpendicular before and after deformation. Thus, this model ignores transverse shear deformation in *xz* and *yz* plane that occurs between lamina and this is a CST's lack of accuracy. But this model has enough accuracy for thin shell design (Rochman *et al*., 2014). The CST model rotation can be formulated as:

$$\frac{dw}{dx} = -\psi_x ; \frac{dw}{dy} = \psi_y \tag{2}$$

And the displacement field of CST model in the *x*, *y* and *z* direction is:

$$\begin{aligned} u(x,y,z) &= u_o(x,y,z) + z.\frac{\partial w}{\partial x}(x,y,z) \\ v(x,y,z) &= v_o(x,y,z) + z.\frac{\partial w}{\partial x}(x,y,z) \\ w(x,y,z) &= w_o(x,y,z) \end{aligned} \tag{3}$$

The strain of CST model is described as Eq. (4) where $\kappa_I = \partial^2 w / \partial i^2$:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^{\ 0} \\ \varepsilon_y^{\ 0} \\ \gamma_{xy}^{\ 0} \end{Bmatrix} + z.\begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{4}$$

The stress {σ} of this model found from strain {ε} become:

$$\begin{aligned} \sigma_x &= \frac{E}{1-\nu^2}(\varepsilon_x + \nu.\varepsilon_y) \qquad \sigma_y = \frac{E}{1-\nu^2}(\nu.\varepsilon_x + \varepsilon_y) \\ \tau_{xy} &= 2.G.\varepsilon_{xy} = G.\gamma_{xy} \qquad G = \frac{E}{2.(1+\nu)} \end{aligned} \tag{5}$$

Further detail can be found in Rochman *et al*. (2014).

MATERIALS AND METHODS

The study was conducted in 4th April 2016 at Structural Laboratory of State Polytechnics of Malang.

Mechanical properties of glass waste: Glass waste density was around 2600 kg/m^3 and tensile, shear and Poisson ratio respectively for both stiffnesses and strengths are showed in Table 1.

Structural properties of GLARC girder: A glass hybrid structural girder with rectangular shape is investigate through a non-destructively testing with a 4 points concentrated load (with 2 point load centralized distance 0.15 L = of 30 cm to keep pure bending response without any shear stress in the middle) with a maximum value of 200 kN. The girder length was 1.2 m (clear support distance is 1 m), the beam depth was 200 mm and 100 mm width. The geometric of structural GLARC beam is described in Fig. 8.

This glass-concrete hybrid girder is made of concrete with crushed recycled aggregate and refined crushed recycled as sand using tipical steel reinforcement and tension reinforcement from longitudinal glass strip pieces. This system called GLARC (glass reinforced concrete) and hence the structural properties of GLARC as depict in Table 2.

In order to get accurate and representative results, a stress analysis needs to be globally performed in the real application of GLARC structural system. This analysis is required to predict the range of service loads, the stress concentration within a local corresponding cross section and its failure behavior in order to predict the specimen dimensions and loading capacity of the equipment. This is related to the range of monotonic load given hence remain responsive to loadings but still gain data completion, before its collapse. Comparison done with GLARC girder FEM model with concrete as solid elements and glass strips as plate/shell elements such as a Fig. 9.

In this discussion a discrete system is used to model GLARC beam, 3D solid elements is used to model concrete parts, 2D beam elements to model steel reinforcement and 2D shell elements to model glass strip pieces. This model was analysed using 6666 nodal joints and total 6028 elements consist of 628 beam elements, 400 shell elements and 5000 solid elements and initial load for analysis given from 1 to 60 kN. In

Table 2: GLARC geometric and properties data

	Data of GLARC structural system and materials				
	Data geometris		Data propertis		
b	100	mm	E_c	21718.46	MPa
h	200	mm	□	2402.62	kg/m^3
d’	30	mm	□	10^{-5}	1/K
L	1000	mm	□	0.17	-

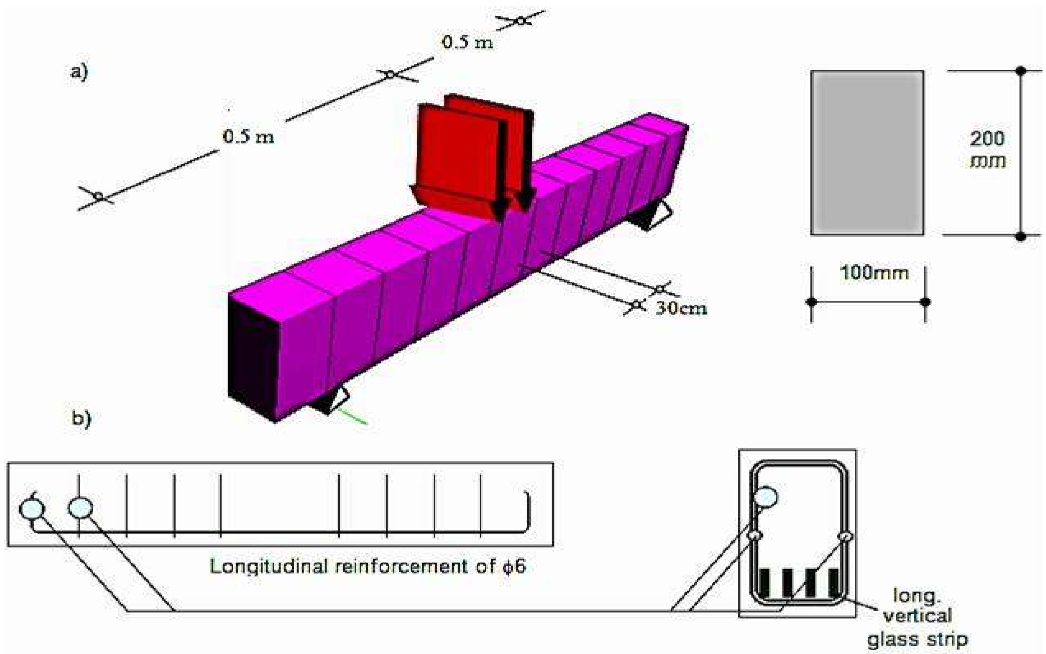


Fig. 8: (a): 3D geometric cross section (top); (b): Typical steel and glass reinforcement (bottom)

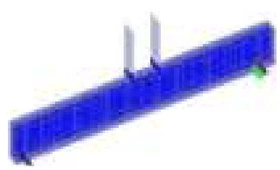
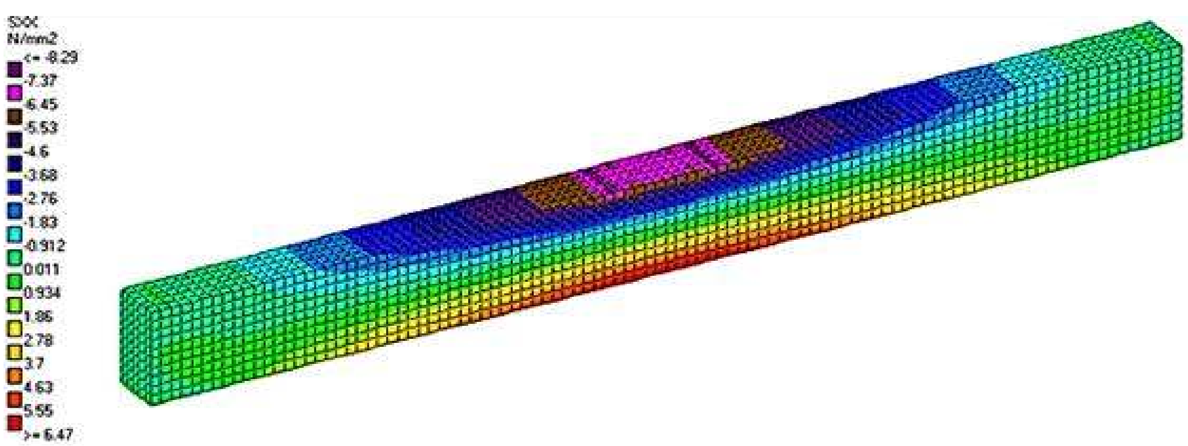


Fig. 9: Geometry of 3D FEM model of GLARC girder

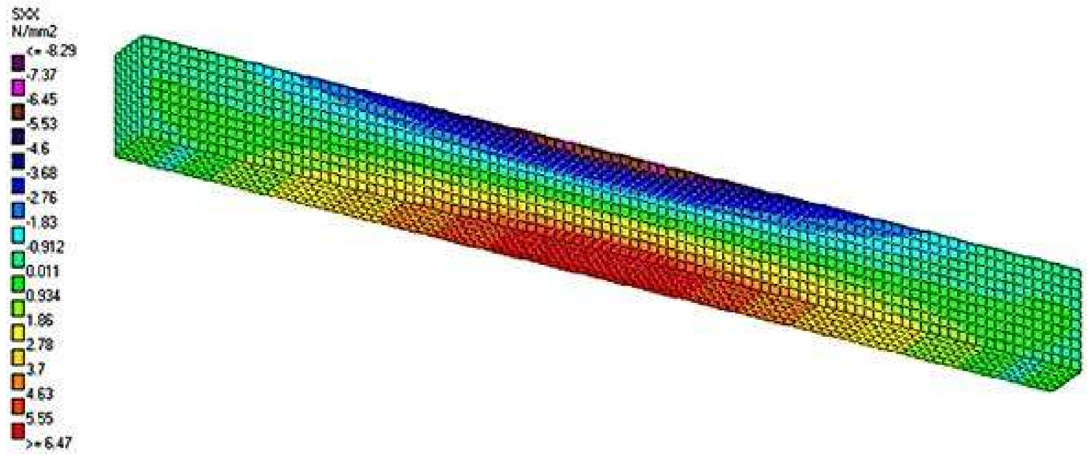
the screenshot, the results were conducted at the total loads of 12 kN.

RESULTS AND DISCUSSION

Global stress result: Forces and stresses are kind of structural responses from external loads that will be



(a)



(b)

Fig. 10: The maximum longitudinal normal stress S_{xx} is predicted occur at mid section of GLARC Girder: a) of 6.47 MPa for tension and; b) of 8.29 MPa for compression

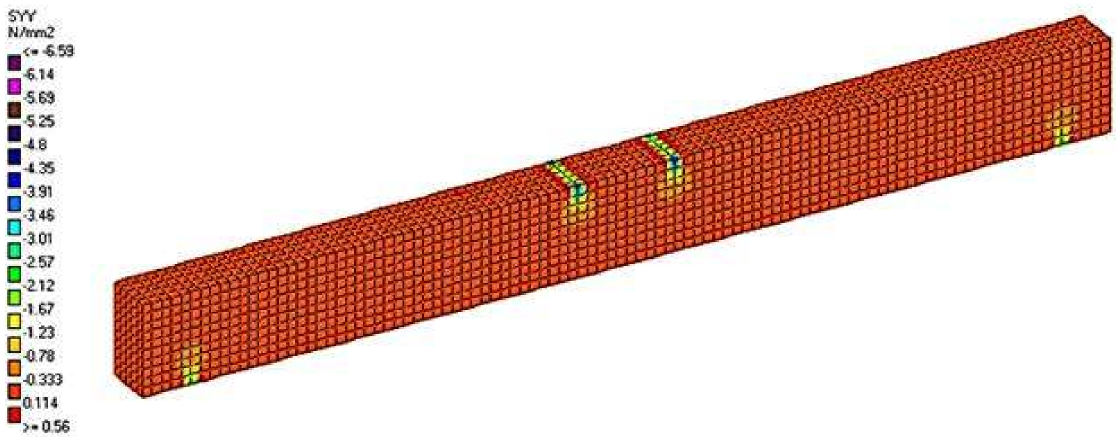


Fig. 11: The maximum normal stress at *y*-direction S_{yy} is predicted occur as 6.59 MPa at loading zone of GLARC beam

influenced by its dimensions and not depend on material specifications. Contrary with strain and deformation are kind of internal responses, hence are closely related to the material parameters that will affect the flexural capacity. For that reason, a simple 3D elastic linear isotropic analysis is enough for pre-analysis stage in order to determine and to check dimensions of a quasi-static response.

Figure 10 shows S_x stress analysis on the concrete and GLARC girder.

The compression normal stress was predicted as 8.29 MPa as presented in Fig. 11, the compressive strength of the concrete that is 25 MPa. But initial failure was predicted 6.47 MPa occurred at the mid tension zone because above the concrete tension strength that is around 2.5 MPa.

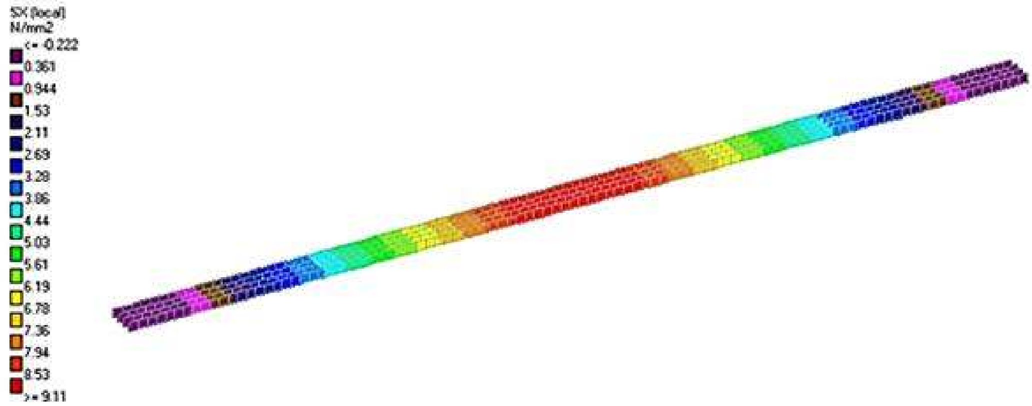
The compression normal stress was predicted 6.59 Mpa in Fig. 11 and occurs at *y*-direction loading zone as, below the concrete compressive strength that around 25 MPa.

According to Fig. 12, the maximum longitudinal normal stress S_{xx} was predicted, for glass strip piece as 9.11 MPa is below the glass tensile strength that was around 70 MPa. Initially, the failure will be occurs first at the tensioned part of the concrete and then the next failure sequences were concrete cracks, then glass strip breaks or slip to concrete, before its failure.

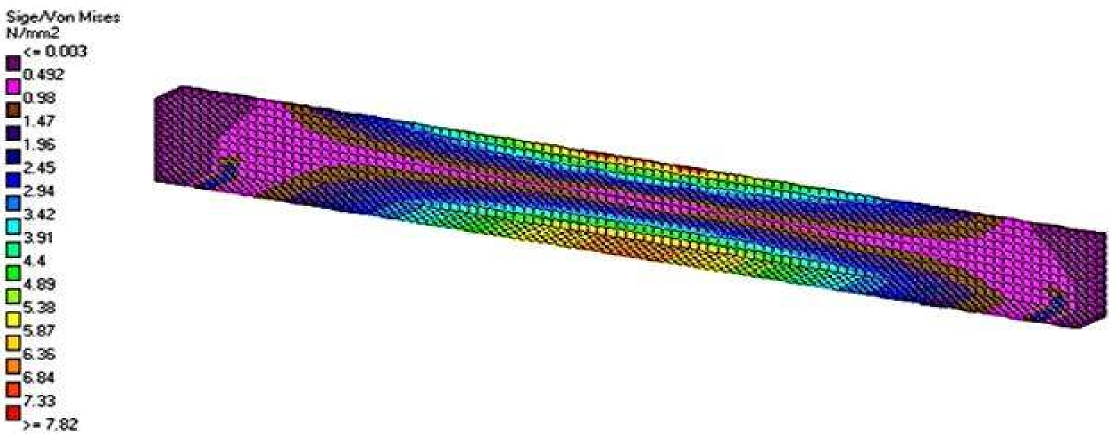
In the analysis results, the maximum deflection around loading zone was 0.815 mm, this show that simple analysis performed is linear.

This simplification was relatively poor than previously good results (1.88 mm) from ANSYS model attained by Barbosa and Ribeiro (1998) suggest that, in spite of the relative simplicity of the analyzed structure and of the employed models, satisfactory prediction of the response of reinforced concrete structures may be obtained.

These results were differ than Barbosa and Ribeiro (1998) results, because they involve Drucker-Prager discrete model and done without any glass strips



(a)



(b)

Fig. 12: (a): The maximum longitudinal normal stress Sxx at the glass strip piece was predicted as 9.11 MPa at the support area; (b): Sige/Von Mises stress for concrete solid elements

reinforcement. They also differ in length, dimension and strength. Hence, in this study, shear and large deformation effect, also crack propagation, slip between steel-concrete and glass-concrete and other nonlinear responses has not been taken into account.

CONCLUSION

The results of discrete model were good enough, accurate and meet the experimental for linear elastic results, especially for design by analysis application. For better results, some of advanced software like ANSYS and ABAQUS can be used to take into account some advanced analysis such as local buckling, large deformation, bond slip among steel, concrete and glass strip pieces, elastoplastic with strain hardening, crack propagation and other nonlinear behaviors.

The GLARC structural girder of glass waste materials that have been installed in vertical position in horizontal arrangement can be used as good flexural structures, safe, cheap and relatively easy to be implemented. A national engineer should have a comprehensive knowledge to design GLARC girder well, with special attention regard to recycling treatment especially glass type, dimension of piece strips and its volume of glass reinforcement.

CONFLICT OF INTEREST

Herewith I certify and declare that there is no conflict of interest at all with the authors in relation to the above-mentioned article and also I have no conflict of financial or relevant interest with any manufacturer of a product discussed in this study.

And I have disclosed all conflicts of interest to the Editors.

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