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**Research Article**

# Note on Electrogravitational Stability of a Double Fluids Interface

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**Abstract:** This study deals with the axisymmetric stability of the interface between two incompressible Selfgravitating non-conducting fluids in the presence of an electric field. The pressure in the unperturbed state is not constant because the Selfgravitating force is a long-range force. The dispersion relation is derived and discussed. Some previous reported works may be obtained as limiting cases from the presented work with appropriate choices. The study under consideration has strong correlation with instabilities of sun spots and other physical phenomena.

**Keywords:** Double fluids interface, electrogravitational, incompressible, stability

## INTRODUCTION

In recent years, the selfgravitating instability of fluid models has a popular area of research. Stability of fluid layers or cylinders have gained considerable importance because of their applications in industries and biophysical laboratories, such as medical applications of electrohydrodynamic and Magnetohydrodynamic stabilities as injection of drugs inside the vessels, electric shock to treat the heart attack and effect of the magnetic resonance on blood flow. The theory of self-gravitating instability of a full fluid jet surrounded by gravitational medium of negligible inertia is due to Chandrasekhar and Fermi (1953). Chandrasekhar (1981) made a complete analysis for such a problem under the influence of the self-gravitating force and surface tension separately or with other forces by using the normal mode analysis. In the recent decades, many advanced works concerning stability of different models influenced by several external forces have been documented by many researchers. Sudo *et al*. (2010) have developed the generator of the capillary magnetic fluid jet by conducting the experiments on magnetic fluid sloshing and found that the magnetic fluid droplets and capillary jet are formed by the generator. Hasan (2011) has studied the stability of an oscillating streaming fluid cylinder subject to the combined effect of the capillary, self-gravitating and electrodynamic forces in all modes of perturbation. Hasan (2012) has studied the instability of a full fluid cylinder surrounded by self-gravitating tenuous medium pervaded by transverse varying electric field under the combined effect of the capillary, self-gravitating and electric forces for all modes of perturbation. Ellingsen and Brevik (2012) have discussed the behaviour of a dielectric fluid-fluid interface in the presence of a strong electric field from a point charge and line charge, respectively, both statically and, in the latter case, dynamically. Chand (2012a) has investigated the rotation in a magnetized ferrofluid with internal angular momentum, heated and soluted from below saturating a porous medium and subjected to a transverse uniform magnetic field. Chand (2012b) has discussed the triple-diffusive convection in a micropolar ferromagnetic fluid layer heated and soluted from below and considered in the presence of a transverse uniform magnetic field. Yin *et al*. (2013) have studied a linear stability analysis for thermal convection in a two-layer system composed of a fluid layer overlying a porous medium saturated with an oldroyd-B fluid heated from below.Ezzat *et al*. (2015) have studied some mathematical models of generalized magneto-thermo-viscoelasticity for isotropic media by using Laplace-transform technique. Balsara *et al*. (2016) have discussed the two fluids interact collectively with the full set of Maxwell's equations and obtained a solution strategy for that coupled system of equation. Hasan (2016a) has studied the linear stability of self-gravitating compound dielectric immiscible jets under the influence of an axial electric field. Hasan (2016b) has discussed the self-gravitating instability of a rotating fluid layer sandwiched between semi-infinite

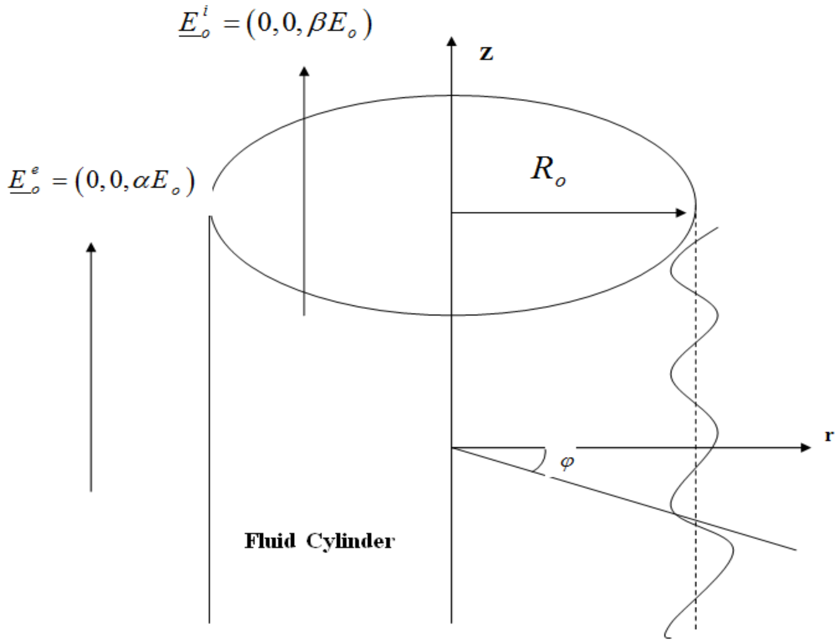


Fig. 1: Sketch for gravitational dielectric fluid cylinder

layers of a fluid with a different density. Hasan (2017) has discussed the stability of the a dielectric selfgravitating streaming fluid cylinder.

The purpose of the present work is studying the stability of the interface between two incompressible self-gravitating non-conducting fluids in the presence of an electric field. In the basic state, both fluids are at rest and the interface is the cylindrical surface. We have considered the linearized equations, derived the dispersion relation on utilizing normal modes and then analyzed it. The study under consideration has strong correlation with instabilities of sunspots and other physical phenomena.

## FORMULATION OF THE PROBLEM AND BASIC STATE

We consider a gravitational dielectric fluid cylinder of density $\rho^i$ (radius $R_o$) with dielectric constant $\varepsilon^i$ and self-gravitating potential $V^i$. The model is assumed to be streaming uniformly with velocities $\underline{u}_o$ = (0, W, U). The cylinder is embedded into a gravitational, dielectric different fluid (of density $\rho^e$) with dielectric constant $\varepsilon^e$ and self-gravitating potential $V^e$. The fluids are penetrated by the longitudinal electric fields $\underline{E_o^i}$= $(0,0,\beta E_o)$ and $\underline{E_o^e}$= $(0,0,\alpha E_o)$where $E_o$ is the electric field intensity in the fluid while$\alpha$ and $\beta$ are parameters. Both of$\underline{E_o^i}$and $\underline{E_o^e}$ are taken along the cylindrical coordinates (r, φ, z) with the z - axis coinciding with the axis of the cylinder as shown in Fig. 1. The fluids are assumed to be incompressible, inviscid and homogenous. Each of these fluids is subjected to the self-gravitating and electrodynamic forces. We assume in the basic state that there are no surface charges at the fluid-fluid interface (this is because the electric field is continuous across the interface) and consequently the surface charge density will be considered to be zero during the perturbation. No volume charges are assumed to be present in the bulk of the fluids in general.

The required equations for studying such kind of problems are the combination of ordinary fluid dynamic equations with those of Newtonian self-gravitating and electrodynamic Maxwell's equations. They are given as follows:

$$\rho^{i,e}\left(\frac{\partial}{\partial t}+\underline{u}^{i,e}\cdot\nabla\right)\underline{u}^{i,e} = -\nabla\Pi^{i,e} \qquad (1)$$

$$\nabla\cdot\underline{u}^{i,e} = 0 \qquad (2)$$

$$\nabla^2 V^{i,e} = 4\pi G\,\rho^{i,e} \qquad (3)$$

$$\nabla\cdot\left(\varepsilon^{i,e}\underline{E}^{i,e}\right) = 0 \qquad (4)$$

$$\nabla\wedge\underline{E}^{i,e} = 0 \qquad (5)$$

$$\Pi^{i,e} = P^{i,e} + \rho^{i,e}V^{i,e} + \left(\varepsilon^{i,e}/2\right)\left(\underline{E}\cdot\underline{E}\right)^{i,e} \qquad (6)$$

Here $\underline{E_o^{i,e}}$ are the intensity of the electric field, $P^{i,e}$ are the kinetic pressure of the fluid, $\underline{u^{i,e}}$ are the velocity vectors, G is the gravitational constant and $(\Pi^{i,e}-\rho^{i,e}V^{i,e})$ are the total hydro-electrodynamic pressures which are the sum of the kinetic and electrodynamic pressures.

The unperturbed state is studied, the differential equations are solved and the constants of integrations are identified by utilizing the continuity conditions and the balance of the total electrodynamic pressure across the unperturbed interface $r = R_o$. The variables in the unperturbed state are given by:

$$\underline{E}_o^i = (0, 0, \beta E_o) \ , \ \underline{E}_o^e = (0, 0, \alpha E_o) \tag{7}$$

$$V_o^e = \pi G \rho^e r^2 + 2\pi G R_o^2 \left(\rho^e - \rho^i\right)\left[\ln\left(\frac{r}{R_o}\right) - \frac{1}{2}\right] \tag{8}$$

$$V_o^i = \pi G \rho^i r^2 \tag{9}$$

$$P_o^e = -\pi G \rho^e \left[\rho^e r^2 + 2R_o^2\left(\rho^e - \rho^i\right)\left[\ln\left(\frac{r}{R_o}\right) - \frac{1}{2}\right]\right] + \frac{\alpha^2 \varepsilon^e}{2} E_o^2 \tag{10}$$

$$P_o^i = -\pi G \rho^i \left[\rho^i \left(r^2 - R_o^2\right) + \rho^e R_o^2\right] + \frac{\beta^2 \varepsilon^i}{2} E_o^2 \tag{11}$$

where, from now on, the subscript o characterizes the variables in the unperturbed state.

## LINEARIZATION

For small wave disturbances acting along the fluid-fluid interface at $r = R_o$propagating in the positive z-direction and based on the linear perturbation technique, every variable quantity Q(r, φ, z ; t)may be expressed as its unperturbed value plus a fluctuation part, viz.,:

$$Q(r, \varphi, z; t) = Q_o(r) + \delta(t) Q_1(r, \varphi, z) \tag{12}$$

where, Q(r, φ, z ; t)stands for each $\underline{E}^{i,e}$, $P^{i,e}$, $\underline{u}^{i,e}$, $V^{i,e}$ and the radial distance of the fluid cylinder with the subscript 1 indicates the perturbed quantities. The amplitude δ(t)of the perturbation at time t is described by:

$$\delta(t) = \delta_o \ e^{\sigma t} \tag{13}$$

where $\delta_o$ is the initial ($\delta = \delta_o$ at t = 0) amplitude and σ is the growth rate, asσ (=iω) is imaginary then (ω/2π)is the frequency of the oscillation. The fluid-fluid perturbed interface radial distance is assumed to be:

$$r = R_o + R_1 \tag{14}$$

With:

$$R_1 = \delta_o \ e^{[\sigma t + ikz]} \tag{15}$$

where, k (any real number) is the longitudinal wavenumber . $R_1$ is the elevation of the surface wave measured from the unperturbed position. The perturbed equations are solved and the required boundary conditions are applied, we finally obtained the dispersion relation:

$$\begin{aligned}(\sigma + imW + ikU)^2 = {} & \frac{x I_m'(x) K_m'(x) \rho^i}{\left[I_m(x) K_m'(x) - \rho I_m'(x) K_m(x)\right]} \\ & \left[4\pi G(1-\rho)\left((1-\rho) I_m(x) K_m(x) - \frac{1}{2}(2\rho+1)\right)\right. \\ & \left. - \frac{E_o^2 (\beta - \alpha\varepsilon)^2 I_m(x) K_m(x)}{\varepsilon^i \left(\rho^i\right)^2 R_o^2 \left[I_m'(x) K_m(x) - \varepsilon I_m(x) K_m'(x)\right]}\right]\end{aligned} \tag{16}$$

where, $\rho(= \rho^e/\rho^i)$ is the densities ratio of the self-gravitating dielectric fluids and $\varepsilon(= \varepsilon^e/\varepsilon^i)$ is the ratio of the dielectric constants of fluids.

## GENERAL DISCUSSIONS AND LIMITING CASES

The relation (16) is the dispersion relation of a dielectric fluid cylinder ambient with a different dielectric fluid; each is acting upon the self-gravitating, inertia and electrodynamic forces. It relates the growth rate σ, or rather the oscillation frequency $\omega$, the modified Bessel functions $I_0(x)$ and $K_0(x)$ and their derivatives, the densities ratio $\rho(= \rho^e/\rho^i)$, the dimensionless longitudinal wave number $x(=kR_o)$ and with the fundamental quantity $(4\pi\rho^i G)^{-1/2}$ as a unit of time. By means of the relation (16) the ordinary stability, marginally stability and instabilities of the present system could be identified. The marginal stability may be obtained by just setting σ = 0in equation (16) that at which transition from oscillation to instabilities occur. This happens directly e.g., as $E_o = 0$ if ρ = 1, i.e., we have an homogenous medium of uniform density $\rho^i = \rho^e$ and this is physically plausible.

The most logical and simultaneously surprising point that we have to stress on it and as a duty to find the physical interpretation for it is the following. One has a feeling that is due to the fact that we assumed there are no volume charges present in the electric field: no surface charges are present at the interfaces in both unperturbed and perturbed states.

Since the problem under consideration, for some extent, is more general, other recent reported results may be recovered as limiting cases with appropriate choices.

- Some results may be obtained as limiting case if we assume that $\rho^e = 0$, W = 0, U = 0 and m = 0 in the compatibility condition:

$$\sigma^2 = \frac{xI_0'(x)K_0'(x)}{\left[I_0(x)K_0'(x)\right]}\left[4\pi\rho^i G\left(I_0(x)K_0(x)-\frac{1}{2}\right) - \frac{E_o^2(\beta-\alpha\varepsilon)^2 I_0(x)K_0(x)}{\rho^i R_o^2\left[I_0'(x)K_0(x)-\varepsilon I_0(x)K_0'(x)\right]}\right] \tag{17}$$

- If we pose that $\rho^e = 0$ and $E_o^{i,e} = 0$, the dispersion relation (16) degenerates to:

$$\sigma^2 = 4\pi G\rho^i\left[\frac{xI_1(x)}{I_0(x)}\right]\left(I_0(x)K_0(x)-\frac{1}{2}\right) \tag{18}$$

This is exactly the same as the dispersion relation derived by Chandrasekhar and Fermi (1953), on utilizing the energy method, in examining and studying the dynamical behaviour of the spiral arm of galaxy.

## STABILITY DISCUSSION

**Selfgravitating instability:** The axisymmetric instability and oscillations of a purely self-gravitating dielectric full fluid cylinder embedded in a dielectric self-gravitating medium of negligible inertia may be determined by discussing the general relation (18) see Chandrasekhar (1981). The analytical and numerical analysis of that relation reveal to the following conclusions. The self-gravitating fluid cylinder is stable forx $\geq$ 1.0667 and unstable if x< 1.0667 where the equality is corresponding to the sausage marginal stability. For more investigations and details about the instability of such a case we may refer to the pioneering works in refs. Chandrasekhar and Fermi (1953) and Chandrasekhar (1981).

In order to investigate the present general case of a fluid cylinder embedded in a different fluid we have to study first the behaviour of the modified Bessel's functions. By an appeal to the recurrence relations:

$$I_0'(x) = I_1(x), K_0'(x) = -K_1(x) \tag{19}$$

And for $x \neq 0$ that $I_0(x)$ is monotonic increasing while $K_0(x)$ is monotonic decreasing, we may see that:

$$I_0'(x) > 0 \ , \ K_0'(x) < 0 \tag{20}$$

$$\left(xI_0'(x)/I_0(x)\right) > 0 \ , \ \left(xK_0'(x)/K_0(x)\right) < 0 \tag{21}$$

Using these inequalities we find, as $x \neq 0$, that the fraction:

$$\frac{xI_0'(x)K_0'(x)}{\left[I_0(x)K_0'(x)-\left(\rho^e/\rho^i\right)I_0'(x)K_0(x)\right]} \tag{22}$$

Is positive definite and never changing sign.

In view of the foregoing results, the determination of the sign of $\sigma^2$in the dispersion relation (16) is dependent on the sign of:

$$\left(1-\rho^e/\rho^i\right)\left[\left(1-\rho^e/\rho^i\right)I_0(x)K_0(x)-\frac{1}{2}\right] \tag{23}$$

However, it is well known, for $x \neq 0$, that:

$$\left(I_0(x)K_0(x)\right)>\frac{1}{2} \qquad \text{or} \qquad \left(I_0(x)K_0(x)\right)<\frac{1}{2} \tag{24}$$

Based on the values of x.

Therefore the identification of the sign of $(\sigma^2/4\pi\rho^i G)$ is dependent on the density ratio $\rho^e/\rho^i$ of the gravitational dielectric fluids. If we involve ourselves in investigating this general case, we have the following different cases:

i. We assume that $\rho^e = \rho^i$ i.e., we have a dielectric homogenous self-gravitating medium. In such a case, Eq. (16) gives $(\sigma^2/4\pi \rho^i G)^{1/2}$ for all $x \geq 0$. Therefore, we predict that the system is marginally stable for all (short and long) wavelengths and this is intuitively clear according to Newtonian gravitational principle.
ii. Suppose that $\rho^e<\rho^i$ i.e., the dielectric gravitational fluid cylinder is more dense than density of the surrounding dielectric fluid. In such a case the quantity $\sigma/(4\pi\rho^i G)^{1/2}$ is imaginary as the restriction:

$$\left(\left(\rho^i-\rho^e\right)I_0(x)K_0(x)\right)\geq\left(\rho^i/2\right) \tag{25}$$

Is satisfied, taking into account that $(\rho^i = \rho^e)$ is positive definite. Therefore we deduce, in such a case in which $(\rho^i>\rho^e)$, that the self-gravitating system is stable if the restriction (25) is satisfied (where the equality corresponds to the neutral stability) and vice versa.

(iii) The case as $(\rho^e = \rho^i)$ where the density of the external gravitational dielectric fluid is more dense than the dielectric fluid cylinder, is the most dangerous case because there in no any stable state in such case. Using (20) together with:

$$\left(\rho^i-\rho^e\right)<0 \tag{26}$$

We get:

$$\left[\left(\left(\rho^i-\rho^e\right)I_0(x)K_0(x)\right)-\left(\rho^i/2\right)\right]<0 \tag{27}$$

For the criterion (16) one may easily prove that $\sigma/(4\pi \rho^i G)^{1/2}$is real for each non-zero value of x. This means that the system will be unstable for all (short and long) wavelengths in axisymmetric mode of perturbation m = 0.

Indeed, the two cases (i) as $\rho^e = \rho^i$ and (ii) as $\rho^e<\rho^i$ of the discussing the relation (16) gave plausible results, but the results of the last case (iii) as $\rho^e>\rho^i$ that the system is unstable for all wavelengths are surprising and very strange. That is may be because the essential prerogative of the self-gravitating instability studies is to understand the dynamical behaviour of the spiral arm of galaxy (Chandrasekhar and Fermi, 1953). However this may be logic as we will see from the following physical interpretation. The spiral arm of the galaxy may be idealized as an infinite cylindrical column and assuming as here we consider it is not stationary in the unperturbed state.

(i) If the density of the matter of the galaxy's arm is equal to that of the surrounding matter, it is neutral stable for all wavelengths.
(ii) If the medium in which the arm exists is more dense than the matter of the galaxy's arm, then it will be unstable or stable according to the restriction:

$$\left(2\left(\rho^i-\rho^e\right)I_0(x)K_0(x)\right)<0 \tag{28}$$

And this is a logic situation.
(iii) If the matter in which the streaming galaxy's arm has been disturbed is more dense than the galaxy's arm density. This means that the galaxy is going through a matter more dense than the density of its matter.

Of course, in this case the arm will be purely gravitational unstable for all perturbed wavelengths and will broken up and destroyed. In particular the latter will be very quick and faster in the case in which the arm is streaming in the initial state than the stationary one and this is our present situation. This can be easily thought and realized in the province of astrophysics and planetary domains which is a logical and true situation, nowadays.

**Electrodynamic stability:** As we neglect streaming and the influence of self-gravitating force, the electrodynamic stability criterion is given from (16) in the form:

$$\sigma^2 = -\left(\frac{\varepsilon^i E_o^2}{\rho^i R_o^2}\right)\left[\left(\frac{x I_0'(x) K_0'(x)}{\left[I_0(x) K_0'(x) - \rho I_0'(x) K_0(x)\right]}\right)\left(\frac{(\beta - \alpha\varepsilon)^2 I_0(x) K_0(x)}{\left[I_0'(x) K_0(x) - \varepsilon I_0(x) K_0'(x)\right]}\right)\right] \tag{29}$$

The determination of the sign of $\sigma^2$ in the dispersion relation (29) is dependent on the sign of:

$$\left(\frac{(\beta - \alpha\varepsilon)^2 I_0(x) K_0(x)}{\left[I_0'(x) K_0(x) - \varepsilon I_0(x) K_0'(x)\right]}\right)\Bigg] \tag{30}$$

Therefore, the identification of the sign of $\sigma^2$ is dependent on the sign of the term ($I_0(x)$ $K_0(x)$). In view of the identities (19), (20), (22) and (24), it is found that the quantity (30) must be positive for all $x \neq 0$ values. Investigations and analysis of the relation (29) reveal that the influence of the interior and exterior electric fields have stabilizing effect for all values of $x(x \neq 0)$.

**Electrogravitational stability:** In such a case of electro gravitational stability, the system is acted by the combined effect of self-gravitating and electrodynamic forces. Its dispersion relation is given by Eq. (16). As we have seen in the foregoing sub-cases that the electric force and the self-gravitating force are stabilizing or destabilizing according to restrictions. So it is difficult to identify the unstable domains analytically. However, this can be carried out upon discussing the general relation (16) numerically.

## NUMERICAL DISCUSSION

In order to determine the combined effect of the gravitational and electrodynamical forces, the relation (16) has been formulated in dimensionless form:

$$\frac{\sigma^2}{4\pi G \rho^i} = \frac{x I_0'(x) K_0'(x)}{\left[I_0(x) K_0'(x) - \rho I_0'(x) K_0(x)\right]}\left[(1-\rho)\left((1-\rho) I_0(x) K_0(x) - \frac{1}{2}(2\rho+1)\right) - M_1 \frac{(\beta - \alpha\varepsilon)^2 I_0(x) K_0(x)}{\left[I_0'(x) K_0(x) - \varepsilon I_0(x) K_0'(x)\right]}\right] \tag{31}$$

With

$$M_1 = \left(\frac{E_o}{E_s}\right)^2 \quad , \quad E_s = 2\sqrt{\frac{\pi G}{\varepsilon^i}}\rho^i R_o \tag{32}$$

The relation (31) inserted in the computer and computed. This has been done for several values of ρ and ε. The numerical data for different values of $M_1$, the instability domains which are associated with $\sigma/(4\pi\rho^i G)^{1/2}$ and those of stability corresponding to $\omega/(4\pi \rho^i G)^{1/2}$ are collected and tabulated and presented graphically (Fig. 2 to 12). There are many features and properties of interest in this numerical analysis as we see in the following:

- For $\rho = 0.2$ and $\varepsilon = 0.2$ corresponding to $M_1$ = 0.1, 0.3, 0.5, 0.7 AND 1.0 it is found that the electrogravitational unstable domains are $0 < x < 0.568001$, $0 < x < 0.380499$. $0 < x < 0.27072$, $0 < x < 0.26335$

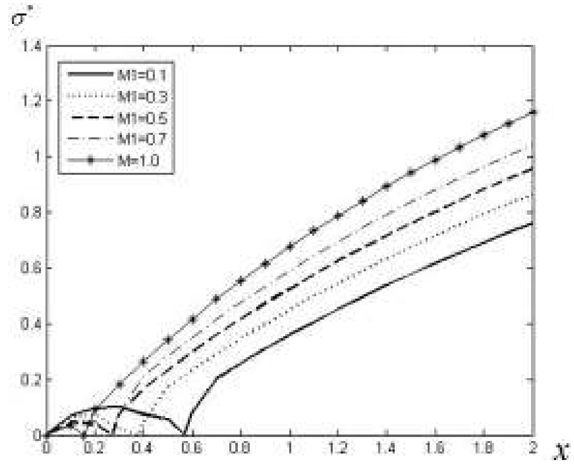


Fig. 2: Electrogravitational stable and unstable domains for $\rho = 0.2$ and $\varepsilon = 0.2$

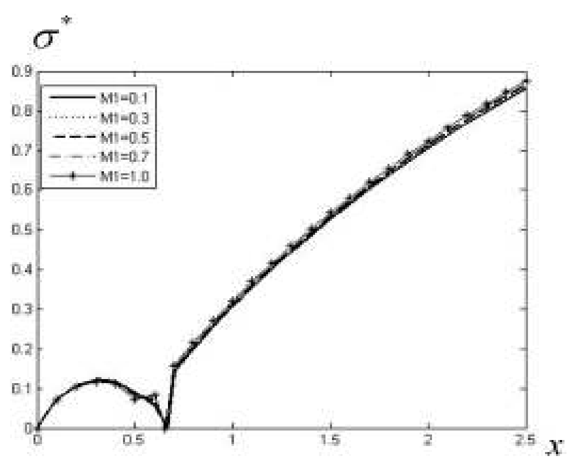


Fig. 3: Electrogravitational stable and unstable domains for $\rho = 0.2$ and $\varepsilon = 0.8$

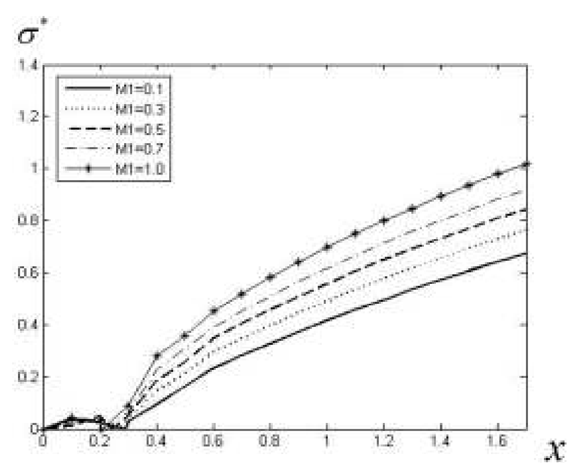


Fig. 4: Electrogravitational stable and unstable domains for $\rho = 0.4$ and $\varepsilon = 0.2$

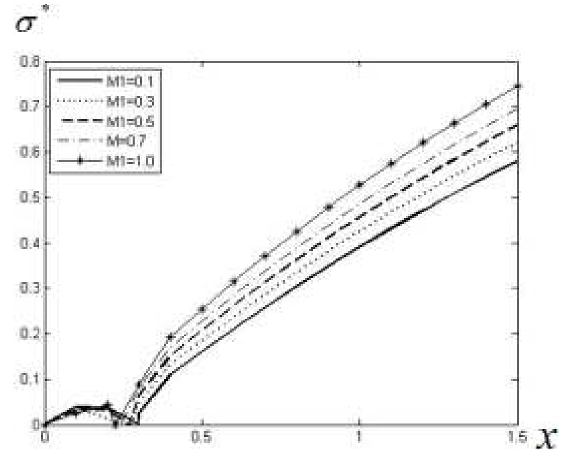


Fig. 5: Electrogravitational stable and unstable domains for $\rho = 0.4$ and $\varepsilon = 0.8$

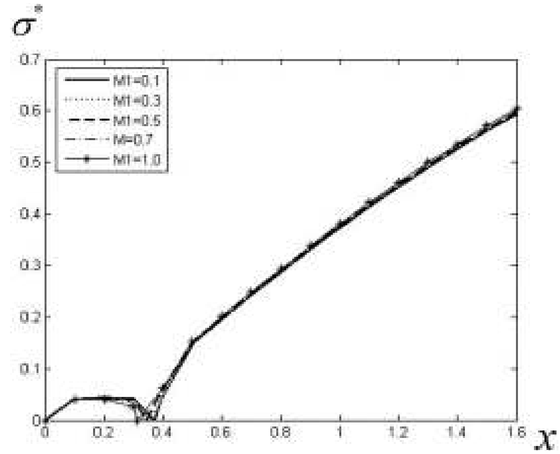


Fig. 6: Electrogravitational stable and unstable domains for $\rho = 0.4$ and $\varepsilon = 1.2$

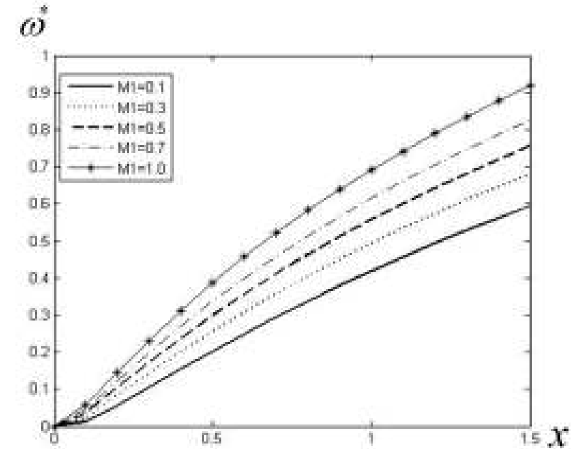


Fig. 7: Electrogravitational stable domains for $\rho = 0.5$ and $\varepsilon = 0.2$

and $0 < x < 0.1545$, the neighboring stable domains are $0.568001 \le x < \infty$, $0.380499 \le x < \infty$, $0.27072 \le x < \infty$, $0.26335 \le x < \infty$ and$0.1545 \le x < \infty$, where the equalities correspond to the marginal stability states (Fig. 2).

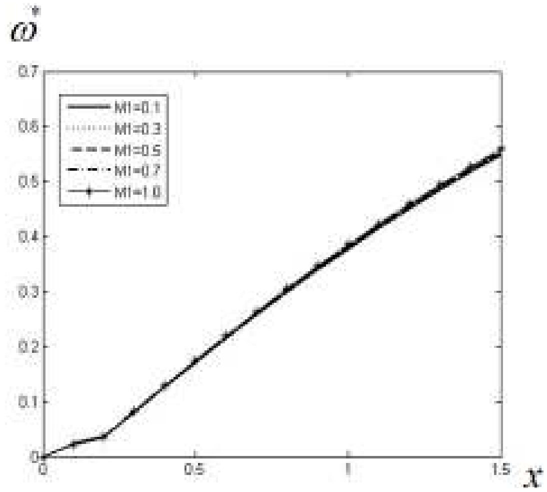


Fig. 8: Electrogravitational stable domains for ρ = 0.5 and ε = 1.2

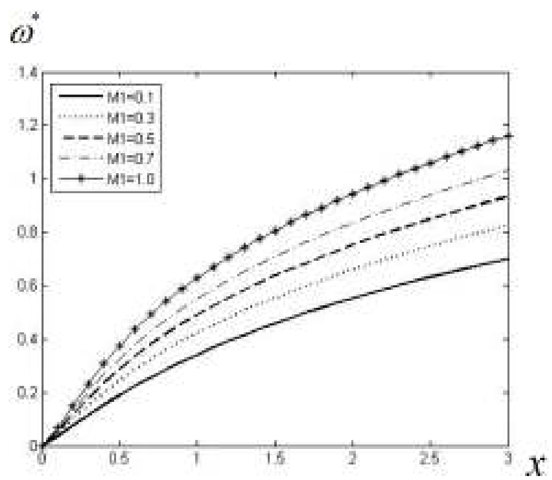


Fig. 9: Electrogravitational stable domains for ρ = 0.8 and ε = 0.2

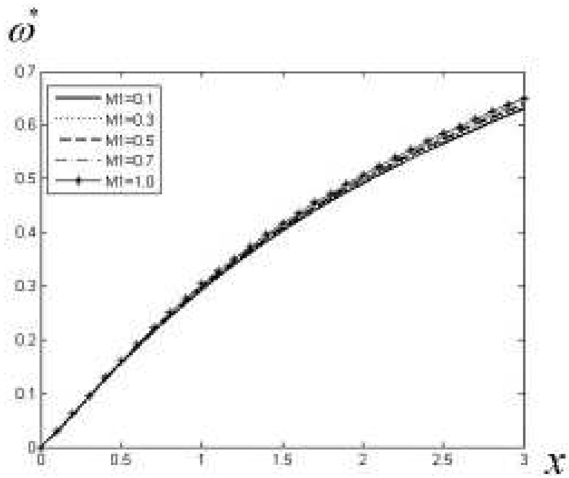


Fig. 10: Electrogravitational stable domains for ρ = 0.8 and ε = 1.2

- For ρ = 0.2and ε = 0.8 corresponding to $M_1$ = 0.1, 0.3, 0.5, 0.7 and 1.0 it is found that the electrogravitational unstable domains are 0 < x < 0.670116, 0 < x < 0.669265, 0 < x < 0.66783, 0 < x < 0.66665, and 0 < x < 0.65347, the neighboring stable domains are 0.670116 ≤ x < ∞, 0.669265 ≤ x <∞, 0.66783 ≤ x <∞, 0.66665 ≤ x < ∞ and0.65347 ≤ x < ∞, where the equalities correspond to the marginal stability states (Fig. 3).
- For ρ = 0.4and ε = 0.2 corresponding to $M_1$ = 0.1, 0.3, 0.5, 0.7 and 1.0 it is found that the electrogravitational unstable domains are 0 < x < 0.288514, 0 < x < 0.288696, 0 < x <0.266514, 0 < x< 0.24406, 0 < x < 0.22176 and 0 < x < 0.20432, the neighboring stable domains are 0.288696 ≤ x <∞, 0.266514 ≤ x < ∞, 0.24406 ≤ x<∞,0.22176 ≤ x < ∞ and0.20432 ≤ x <∞, where the equalities correspond to the marginal stability states (Fig. 4).
- Forρ = 0.4and ε = 0.8 corresponding to $M_1$ = 0.1, 0.3, 0.5, 0.7 and 1.0 it is found that the electrogravitational unstable domains are0 < x < 0.29784, 0 < x < 0.227352, 0 < x < 0.26447, 0 < x <0.24125, and 0 < x < 0.22145, the neighboring stable domains are 0.29784 ≤ x < ∞,0.27352 ≤ x <∞, 0.26447 ≤ x <∞, 0.24125 ≤ x <∞ and0.22145 ≤ x < ∞, where the equalities correspond to the marginal stability states (Fig. 5).
- For ρ = 0.4and ε = 1.2 corresponding to $M_1$ = 0.1, 0.3, 0.5, 0.7 and 1.0 it is found that the electrogravitational unstable domains are 0 <x < 0.37097, 0 < x < 0.36042, 0 <x< 0.34231, 0 < x < 0.3245and 0 <x < 0.31123, the neighboring stable domains are 0.37097≤ x<∞, 0.36042 ≤ x <∞, 0.34231 ≤ x <∞, 0.3245 ≤ x <∞ and0.31123 ≤ x < ∞, where the equalities correspond to the marginal stability states (Fig. 6).
- Forρ 0.5 andε = 0.2 corresponding to $M_1$ = 0.1, 0.3, 0.5, 0.7 and 1.0 it is found that the electrogravitational fluid cylinder is completely stable not only for short wavelengths but also for very long wavelengths (Fig. 7).
- For ρ = 0.5and ε = 1.2 corresponding to M1 = 0.1, 0.3, 0.5, 0.7 and 1.0 it is found that the electrogravitational fluid cylinder is completely stable not only for short wavelengths but also for very long wavelengths (Fig. 8).
- For ρ = 0.8andε = 0.2 corresponding to $M_1$ = 0.1, 0.3, 0.5, 0.7 and 1.0 it is found that the electrogravitational fluid cylinder is completely stable not only for short wavelengths but also for very long wavelengths (Fig. 9).
- For ρ = 0.8and ε = 1.2 corresponding to $M_1$ = 0.1, 0.3, 0.5, 0.7 and 1.0 it is found that the electrogravitational fluid cylinder is completely stable not only for short wavelengths but also for very long wavelengths (Fig. 10).

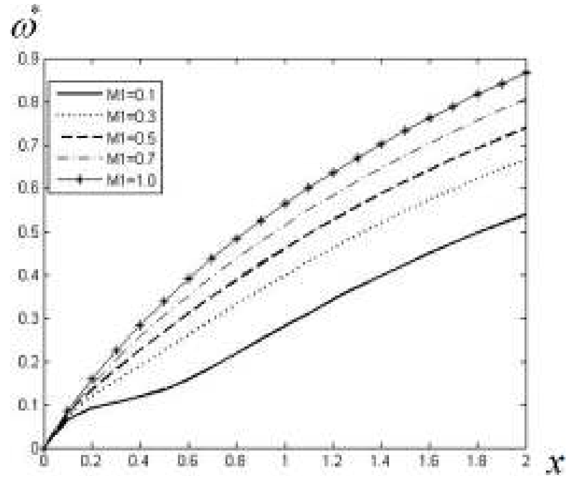


Fig. 11: Electrogravitational stable domains for $\rho = 1.2$ and $\varepsilon = 0.2$

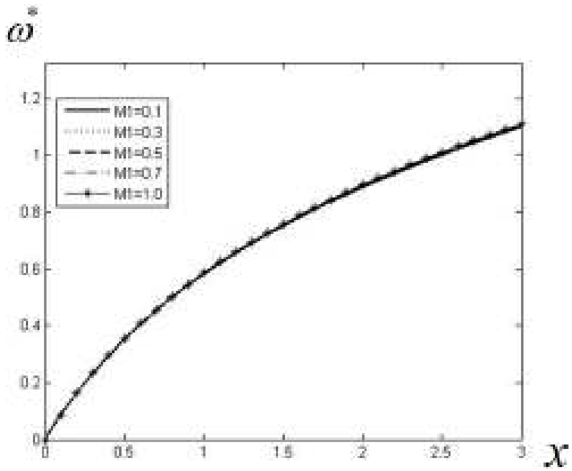


Fig. 12: Electrogravitational stable domains for $\rho = 1.2$ and $\varepsilon = 0.8$

- For $\rho = 1.2$and $\varepsilon = 0.2$ corresponding to $M_1 = 0.1$, 0.3, 0.5, 0.7 and 1.0 it is found that the electrogravitational fluid cylinder is completely stable not only for short wavelengths but also for very long wavelengths (Fig. 11).
- For $\rho = 1.2$and $\varepsilon = 0.8$ corresponding to $M_1 = 0.1$, 0.3, 0.5, 0.7 and 1.0 it is found that the electrogravitational fluid cylinder is completely stable not only for short wavelengths but also for very long wavelengths (Fig. 12).

## CONCLUSION

From the numerical discussions we may deduce the following.

For the same values of $\rho(=\rho^e/\rho^i)\rho = 0.5$say,it is found that the unstable domains are increasing with increasing $M_1$ values. This means that the influence of the electric forces have destabilizing effect for all short and long wavelengths.

However, as ($\rho > 0.5$), it is found that the model becomes completely stable not only for short wavelengths but also for very long wavelengths. This means that the density ratio ($\rho^e/\rho^i$) has a stabilizing effect. Also the $\varepsilon$ ratio $\varepsilon$ (= $\varepsilon^e / \varepsilon^i$) has a stabilizing influence.

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