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Research Article

Magnetogravitodynamic Stability of Three Dimensional Streaming Velocities of Fluid Cylinder under the Effect of Capillary Force

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Abstract: The magnetohydrodynamic stability in a uniform cylinder of an incompressible inviscid fluid under the effect of self-gravitating, magnetic field and capillary forces is studied. The obtanined results were studied theoretically and numerically. The dispersion relationship was obtained and the effect of the different parameters had been discussed. The behavior of the system in terms of whether stable or unstable had been studied. The uniform streaming has a destabiliizing influence. We observe that the system gives an unstable situation where the streaming fluid under the effect of the capillary force and in the absence of the magnetic field. Also, the system gives an unstable situation where the streaming fluid under the effect of the self-gravitating and magnetic forces. But the system be more stable and the stability zone increase if the streaming fluid under the effect of the self-gravitating, magnetic and capillary forces. The curves are drawn to illustrate the areas of stability and instability.

Keywords: Capillary force, magnetohydrodynamic, self-gravitating,stability, streaming

INTRODUCTION

The electrodynamic stability of a dielectric self-gravitating streaming fluid cylinder with a different dielectric self-gravitating streaming fluid has been studied by Hasan (2017a). The magnetohydrodynamic stability of a gravitational medium with streams of variable velocity distribution for a general wave propagation in the present of the rotation forces was presented by Hasan (2017b). The stability of a fluid cylinder under the influence of the capillary force was studied in many researches (Yuen, 1968; Nayfeh and Hassan, 1971; Rayleigh, 1892). The magnetohydrodynamic (MHD) stability of an oscillating fluid with longitudinal magnetic field has been studied by Barakat (2015). Barakat (2016) studied self-gravitating stability of a fluid cylinder embedded in a bounded liquid, pervaded by magnetic field, for all symmetric and axisymmetric perturbation modes. The electrogravitational instability of an oscillating streaming fluid cylinder under the action of the self-gravitating, capillary and electrodynamic forces was presented by Hasan (2011), he used the Mathieu second order integro-differential equation in this model. The self-gravitating instability of a fluid cylinder pervaded by magnetic field and endowed with surface tension was investigated by Radwan and Hasan (2009). The instability of a self-gravitating fluid cylinder surrounded by a self-gravitating tenuous medium pervaded by transverse varying electric field is discussed under the combined effect of the capillary, self-gravitating and electric forces for all axisymmetric and nonaxisymmetric modes of perturbation was presented by Hasan (2012). Hasan and Abdelkhalek (2013) studied the Magneto-hydrodynamic stability criterion of self-gravitating streaming fluid cylinder under the combined effect of self-gravitating, magnetic and capillary forces.

DEFINITION OF THE PROBLEM

Research results were concluded in 2017. The study was conducted at Al-AzharUniversity.

Basic equations: We consider a fluid cylinder of radius R_0,with negligible motion with streaming velocity:

$$\underline{u}_0 = (V, W, U). \tag{1}$$

The internal and external magnetic fields:

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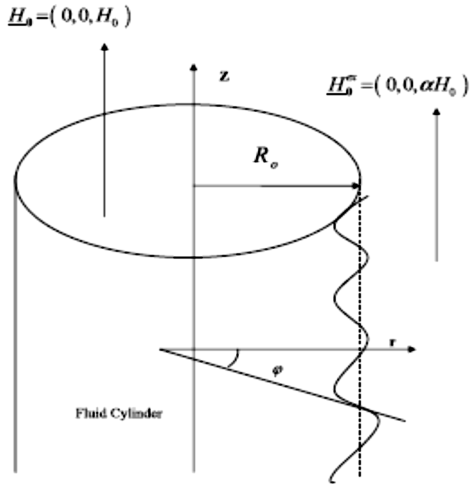

Fig. 1: Physical model and coordinates

$$\underline{H}_0=(0,0,\underline{H}_0),\quad \underline{H}_0^{ex}=(0,0,\alpha\underline{H}_0^{ex}). \qquad (2)$$

where,V, W, U are the streaming velocites. H_0 represents the intensity of the magnetic field, α is an arbitrary parameter. We shall use the cylindrical polar coordinates (r,ϕ,z) system with the axis of the cylinder coinciding with the z-axis (Fig. 1). The fluid of the cylinder under the effects of the self-gravitating, magnetic forces and capillary forces. The surrounding tenuous medium of the fluid cylinder under the effects of the magnetic forces and self-gravitating only.
The basic equations of this model are given as follows:

$$\rho\frac{\partial u}{\partial t}+\left(\underline{u}\cdot\nabla\underline{u}\right)=-\nabla P+\rho\nabla\tilde{V}+\frac{\mu}{4\pi}\left(\nabla\wedge\underline{H}\right)\wedge\underline{H}, \quad (3)$$

$$\nabla\cdot\underline{u}=0, \quad (4)$$

$$\nabla\cdot\underline{H}=0, \qquad (5)$$

$$\frac{\partial H}{\partial t}=\nabla\wedge\left(\underline{u}\wedge\underline{H}\right), \qquad (6)$$

$$\nabla^2\tilde{V}=-4\pi\rho G, \qquad (7)$$

where,$\rho$is the density, $\underline{u}$ is the velocity vector, P is the kinetic pressure, $\mu$is the magnetic field permeability coefficient and G is the gravitational constant.

The curvature pressure due to the existence of the capillary force is:

$$P_s=T(\nabla\cdot\underline{N}_s)\quad where\quad \underline{N}_s=\frac{\nabla F}{|\nabla F|}. \qquad (8)$$

Such that:

$$F(r,\phi,z)=0, \quad (9)$$

whereas T is surface tension, $\underline{N}_s$ is the unit normal vector. Basic equations of the surrounding tenuous medium:

$$\nabla\cdot\underline{H}^{ex}=0 \qquad (10)$$

$$\nabla\wedge\underline{H}^{ex}=0 \qquad (11)$$

$$\nabla^2\tilde{V}^{ex}=0 \qquad (12)$$

Here $H_0, {H_0}^{ex}$are represents the intensity of the magnetic field and$\tilde{V},\tilde{V}^{ex}$are represents self-gravitating potentials, inside and outside the fluid cylinder.

UNPERTURBED STATE

In the initial state, we get:

$$P_{0s}=\frac{T}{R_0} \qquad (13)$$

is the surface pressure due to the capillary force(Chandrasekhar, 1981).In the inital state, the self-gravitating potentials$\widetilde{V_0}$, $\widetilde{V_0}^{ex}$satisfy:

$$\nabla^2\tilde{V}_0=-4\pi\rho G \qquad (14)$$

$$\nabla^2{\tilde{V}_0}^{ex}=0 \qquad (15)$$

Where $\nabla^2=\left(\frac{\partial^2}{\partial r^2},\frac{1}{r^2}\frac{\partial^2}{\partial\phi^2},\frac{\partial^2}{\partial z^2}\right)$.

If we put $\left(\frac{\partial}{\partial\phi}=\frac{\partial}{\partial z}=0\right)$ cylindrical symmetries, we get solutions for the Eq.(14), (15) in the form:

$$\tilde{V}_0=-\pi G\rho r^2+C_1 \qquad (16)$$

$${\tilde{V}_0}^{ex}=C_2\ln r+C_3 \qquad (17)$$

Where C_1, C_2 and $C_3$are constants of integration.

In order to determine the values of this constants C_1, C_2 and C_3, we will use the conditions of the self-gravitational potential andits derivative, i.e.,($\tilde{V}_0={\tilde{V}_0}^{ex}$ and $\frac{\partial\tilde{V}_0}{\partial r}=\frac{\partial{\tilde{V}_0}^{ex}}{\partial r}$ at $r=R_0$). Let$C_1=0$, since the potential inside the cylinder is zero, therefore:

$$C_2=-2\pi G\rho {R_0}^2 \qquad (18)$$

$$C_3=-\pi G\rho {R_0}^2\left(1+2\ln\left(\frac{1}{R_0}\right)\right) \qquad (19)$$

Then, we get the final solution for the Eq. (14), (15) in the form:

$$\tilde{V}_0 = -\pi G\rho r^2 \quad (20)$$

$$\tilde{V}_0{}^{ex} = -\pi G\rho R_0{}^2(1 + 2\ln(\tfrac{r}{R_0}))(21)$$

The distribution of the fluid pressure cross the boundary surface at $r = R_0$ in the unperturbed state is given by:

$$P_0 = \left(\frac{T}{R_0}\right) + \pi G\rho^2(R_0{}^2 - r^2) + \left(\frac{\mu}{4\pi}\right) H_0{}^2(\alpha^2 - 1)(22)$$

PERTURBATION ANALYSIS

Perturbation theory leads to an expression for the desired solution in terms of a formal power series in some "small" parameter known as a perturbation series that quantifies the deviation from the exactly solvable problem, considers the effect of small disturbances for a small departure from the unperturbed state, then that the perturbed interface is described by equation:

$$r = R_0 + \varepsilon(t)R_1, \quad (23)$$

where,

$$R_1 = e^{\sigma t + i(kz + m\phi)} \quad (24)$$

$$\mathcal{E}(t) = \mathcal{E}_0 e^{\sigma t} \quad (25)$$

Here R_1, k, m represents the elevation of the surface wave, the wave number, the transverse wave number, respectively. $\varepsilon(t)$ is the amplitude of the perturbation where ε_0 is the initial amplitude and σ is the temporal amplification.

Now, we will put a mathematical expression Y in the initial state for each of the following variables$u, P, \tilde{V}, \tilde{V}^{ex}, H,\ H^{ex}$ and N_s in the form:

$$Y(r,\phi,z) = Y_0(r) + Y_1(r,\phi,z) \quad (26)$$

where Y_0 represent unperturbed quantity and Y_1 is a small increment of Y due to disturbances.

From Eq. (26), we can put the basic equations of motion (3)-(12) in the initial state as following:

$$\rho\frac{\partial \underline{u}}{\partial t} + (\underline{u}_0 \cdot \nabla \underline{u}_1) = -\nabla P_1 + \rho\boldsymbol{\nabla}\tilde{V}_1 + \frac{\mu}{4\pi}(\underline{H}_0 \cdot \nabla)\underline{H}_0 - \frac{\mu}{4\pi}\nabla(\underline{H}_0 \cdot \underline{H}_1) \quad (27)$$

$$\nabla \cdot \underline{u}_1 = 0 \quad (28)$$

$$\nabla \cdot \underline{H}_1 = 0 \quad (29)$$

$$\frac{\partial H_1}{\partial t} = (\underline{H}_1 \cdot \nabla)\underline{u}_1 - (\underline{u}_0 \cdot \nabla)\underline{H}_1 \quad (30)$$

$$\nabla^2\tilde{V}_1 = 0 \quad (31)$$

$$P_{1s} = \left(\frac{T}{R_0{}^2}\right)\left[R_1 + \left(\frac{\partial^2 R_1}{\partial\phi^2}\right) + R_0{}^2\left(\frac{\partial^2 R_1}{\partial z^2}\right)\right] \quad (32)$$

$$\nabla \cdot \underline{H}_1{}^{ex} = 0 \quad (33)$$

$$\nabla\wedge \underline{H}_1{}^{ex} = 0 \quad (34)$$

$$\nabla^2\tilde{V}_1{}^{ex} = 0 \quad (35)$$

According to the theory of wave function solutions, wecan put $Y_1(r,\phi,z,t)$in the following form:

$$Y_1(r,\phi,z,t) = y_1(r)e^{(\sigma t + i(kz+m\phi))}. \quad (36)$$

Substituting from (36) into (31) and (35), we obtain the second-order ordinary differential equations:

$$\frac{d^2}{dr^2}\tilde{V}_1(r) + \frac{1}{r}\frac{d}{dr}\tilde{V}_1(r) - \left(k^2 + \frac{m^2}{r^2}\right)\tilde{V}_1(r) = 0. \quad (37)$$

$$\frac{d^2}{dr^2}\tilde{V}_1{}^{ex}(r) + \frac{1}{r}\frac{d}{dr}\tilde{V}_1{}^{ex}(r) - \left(k^2 + \frac{m^2}{r^2}\right)\tilde{V}_1{}^{ex}(r) = 0. \quad (38)$$

Then, the solution of Eq. (31) and (35) can be written in the forms:

$$\tilde{V}_1 = C_4 I_m(kr)\, e^{(\sigma t + i(kz+m\phi))}. \quad (39)$$

$$\tilde{V}_1{}^{ex} = C_5 K_m(kr)\, e^{(\sigma t + i(kz+m\phi))}. \quad (40)$$

where,C_4 and C_5 are constants of integration$I_m(kr)$ and $K_m(kr)$are the modified Bessel functions of the first and second kind of order m. By usingEq. (36) in (27), we get:

$$\left(\sigma + V\frac{\partial}{\partial r} + \frac{imW}{r} + ikU\right)\underline{u}_1 - \frac{i\mu k H_0}{4\pi\rho}\underline{H}_1 = -\nabla\Lambda \quad (41)$$

where,

$$\Lambda = \frac{P_1}{\rho} - \tilde{V}_1 + \frac{\mu}{4\pi\rho}(\underline{H}_0 \cdot \underline{H}_1) \quad (42)$$

From Eq. (30), we get:

$$\left(\sigma + V\frac{\partial}{\partial r} + \frac{imW}{r} + ikU\right)\underline{H}_1 = ikH_0\underline{u}_1(43)$$

By using Eq. (43) in (41), we obtain:

$$[(\sigma + V\frac{\partial}{\partial r} + \frac{imW}{r} + ikU)^2 + \Omega_A{}^2]\underline{u}_1 = -\left(\sigma + V\frac{\partial}{\partial r} + \frac{imW}{r} + ikU\right)\nabla\Lambda, \quad (44)$$

where,

$$\Omega_A{}^2 = \frac{\mu k^2 H_0{}^2}{4\pi\rho} \quad (45)$$

Ω_A represent the Alfven wave frequency. Take the divergence of the two sides of the Eq. (44) and by using ($\nabla \cdot \underline{u}_1 = 0$), we obtain:

$$\nabla^2 \Lambda = 0 \quad (46)$$

Also, we can obtain the solution of Eq. (46) as follows:

$$\Lambda = C_6 I_m(kr) e^{(\sigma t + i(kz+m\phi))}. \quad (47)$$

From Eq. (32) by using Eq. (24), we get the pressure surface P_{1s} in the initial state due to the capillary in the form:

$$P_{1s} = \frac{-T}{R_0{}^2}(1 - m^2 - (kR_0)^2) e^{\sigma t + i(kz+m\phi)} \quad (48)$$

The scalar magnetic field intensity $H_1{}^{ex}$ are defined as:

$$H_1{}^{ex} = \nabla\, \Theta_1{}^{ex}. \quad (49)$$

It follows that the scalar magnetic field intensity H_1^{ex} satisfy the $Laplace's$ equations:

$$\nabla^2 \Theta_1{}^{ex} = 0. \quad (50)$$

where $\Theta_1{}^{ex}$ is a scalar function, the solution of this Eq. (50) can be written as follows:

$$\Theta_1{}^{ex} = C_7 K_m(kr)\, e^{(\sigma t + i(kz+m\phi))}. \quad (51)$$

where C_7 is a constant of integration.

BOUNDARY CONDITIONS

- The gravitational potential and its derivative must be continuous across the fluid interface:

$$\tilde{V}_1 + R_1\frac{\partial \tilde{V}_0}{\partial r} = \tilde{V}_1{}^{ex} + R_1\frac{\partial \tilde{V}_0{}^{ex}}{\partial r} \text{at} r = R_0 \quad (52)$$

$$\frac{\partial \tilde{V}_1}{\partial r} + R_1\frac{\partial^2 \tilde{V}_0}{\partial r^2} = \frac{\partial \tilde{V}_1{}^{ex}}{\partial r} + R_1\frac{\partial^2 \tilde{V}_0{}^{ex}}{\partial r^2} \text{at } r = R_0 \quad (53)$$

Substituting (20), (21), (24), (39) and (40) into (52), (53), then we will get the following two equations:
$C_4 I_m(x) = C_5 K_m(x)$, (54)

$$C_4 I'_m(x) = C_5 K'_m(x) + 4\pi\rho G. \quad (55)$$

We will solve the Eq. (54), (55) together to obtain the values of the constants C_4 and C_5, then:

$$C_4 = 4\pi\rho G R_0 K_m(x), \quad (56)$$

$$C_5 = 4\pi\rho G R_0 I_m(x), \quad (57)$$

where x(is the dimensionless longitudinal wave number)$= kR_0$ .Also the dimensionless longitudinal wave number gives as the following equation:

$$x = \frac{1}{I'_m(x)K_m(x) - K'_m(x)I_m(x)}. \quad (58)$$

- At the unperturbed surface $r = R_0$, the normal component of the velocity vector u must be suitable with the velocity of the particles:

$$\underline{u}_{1r} = \frac{\partial R_1}{\partial t} + (\underline{u}_0 \cdot \nabla)R_1. \quad (59)$$

From Eq. (24), (44) and (47) into (59), then we get:

$$C_6 = \frac{-R_0[\left(\sigma + V\frac{\partial}{\partial r} + \frac{imW}{r} + ikU\right)^2 + \Omega_A{}^2]}{xI'_m(x)}. \quad (60)$$

- At the perturbed interface $r = R_0$, the jump of the normal component of the magneticfield iszero:

$$\underline{N}_s \cdot \underline{H} - \underline{N}_s \cdot \underline{H}^{ex} = 0. (61)$$

Eq. (61) leads to,

$$\underline{H}_{1r} - \underline{H}_{1r}{}^{ex} = ikR_1H_0(1-\alpha). \quad (62)$$

Substituting from Eq. (43), (44), (47), (49) and (51) into (62), then we obtain:

$$C_7 = \frac{i\alpha H_0}{K'_m} \quad (63)$$

DISPERSION RELATION

The dispersion relation can be written as follows:

$$P_{1S} = P_1 + R_1\frac{\partial P_0}{\partial r} + \frac{\mu}{4\pi}(\underline{H}_0 \cdot \underline{H}_1) - \frac{\mu}{4\pi}(\underline{H}_0 \cdot \underline{H}_1)^{ex}. \quad (64)$$

By using Eq. (42), we can rewrite the dispersion relation (64) as following:

$$P_{1S} + \frac{\mu}{4\pi\rho}(\underline{H}_0 \cdot \underline{H}_1)^{ex} = \rho(\Lambda + \tilde{V}_1) + R_1 \frac{\partial P_0}{\partial r} \quad (65)$$

From Eq. (2), (22), (24), (39), (47), (48), (49), (50) (51), (60) and (61) into the Eq.(65), we get the dispersion relation in the form:

$$\frac{V^2x^2}{{R_0}^2}I''_m(x) + \frac{2Vx}{R_0}\left(\sigma + \frac{imW}{R_0} + ikU\right)I'_m(x) - \frac{imWV}{{R_0}^2}I_m(x) + (\sigma + \frac{imW}{R_0} + ikU)^2 I_m(x) = \frac{T}{\rho {R_0}^3}(1 - m^2 - x^2)xI'_m(x) + 4\pi\rho x G I'_m(x)\left(I_m(x)K_m(x) - \frac{1}{2}\right) + \frac{\mu {H_0}^2}{4\pi\rho {R_0}^2}(-x^2 I_m(x) + \frac{\alpha^2 x^2 I'_m(x)K_m(x)}{K'_m(x)}) \quad (66)$$

The dimensionless dispersion relation is:

$$\hat{\sigma} = \widehat{W} + \widehat{U} - \widehat{V}x\frac{I'_m(x)}{I_m(x)} + \frac{1}{(I_m(x))^{\frac{1}{2}}}[\widehat{V}^2x^2\frac{(I'_m(x))^2}{I_m(x)} - I''_m(x) + M(1 - m^2 - x^2)xI'_m(x) + x\,I'_m(x)\left(I_m(x)K_m(x) - \frac{1}{2}\right) - \widehat{W}\widehat{V}I_m(x) + \gamma(-x^2I_m(x) + \frac{\alpha^2x^2I'_m(x)K_m(x)}{K'_m(x)})]^{1/2}. \quad (67)$$

where,

$$\hat{\sigma} = \frac{\sigma}{(4\pi\rho G)^{1/2}},\ \widehat{U} = \frac{-ikU}{(4\pi\rho G)^{1/2}},\ \widehat{W} = \frac{-imW}{(4\pi\rho G)^{1/2}},$$

$$\widehat{V} = \frac{V}{(4\pi\rho G)^{1/2}},\ M = \frac{T}{4\pi\rho^2 G{R_0}^3},\ \gamma = \frac{\mu {H_0}^2}{16\pi\rho^2 G {R_0}^2}.$$

RESULTS AND DISCUSSION
CAPILLARY INSTABILITY

Suppose the magnetic field vanishes and the streaming fluid under the effect of the capillary force only. In this case, we can write the dispersion relationship as follows:

$$\hat{\sigma} = \widehat{W} + \widehat{U} - \widehat{V}x\frac{I'_m(x)}{I_m(x)} + \frac{1}{(I_m(x))^{\frac{1}{2}}}[\widehat{V}^2x^2(\frac{(I'_m(x))^2}{I_m(x)} - I''_m(x)) + M(1 - m^2 - x^2)xI'_m(x) + x\,I'_m(x)\left(I_m(x)K_m(x) - \frac{1}{2}\right) - \widehat{W}\widehat{V}I_m(x)]^{1/2}. \quad (68)$$

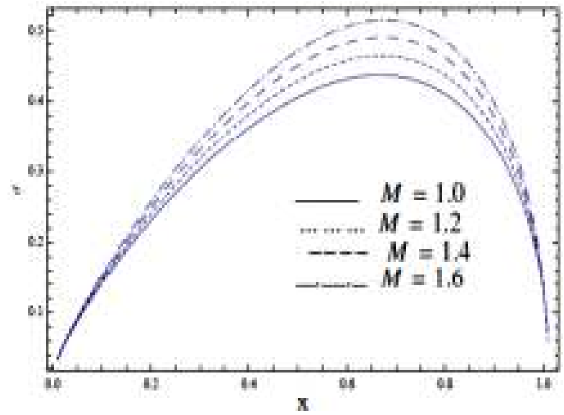

Fig. 2: Stability diagram for a system having the particulars: $\widehat{V} = \widehat{W} = \widetilde{U} = 0.01, m = 0, \gamma = 0$

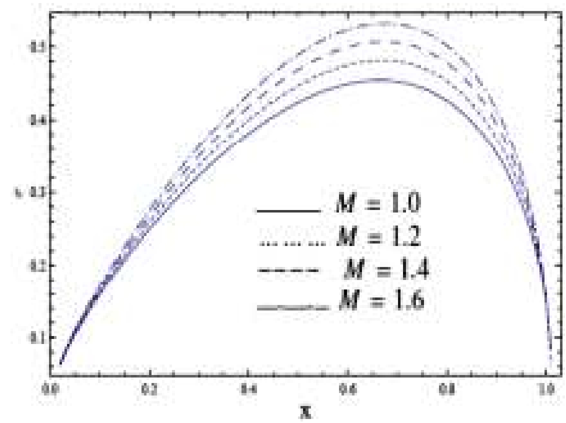

Fig. 3: Stability diagram for a system having the particulars: $\widehat{V} = \widehat{W} = \widetilde{U} = 0.02, m = 0, \gamma = 0$

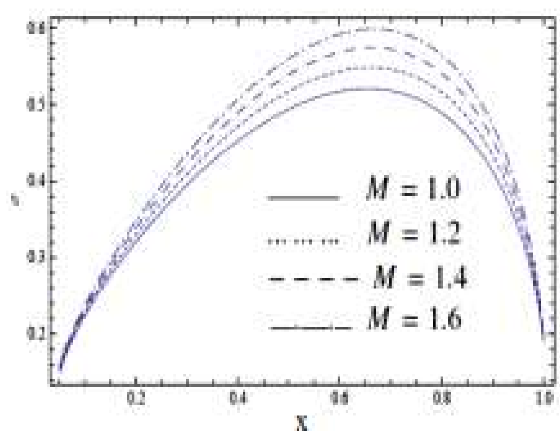

Fig. 4: Stability diagram for a system having the particulars: $\widehat{V} = \widehat{W} = \widetilde{U} = 0.06, m = 0, \gamma = 0$

Now, we begin drawing some neutral curves stability problem:

From Fig. 2, in this domain $0.0101 \le x \le 1.0101$, we get stability states. While otherwise domin, we get unstable state.

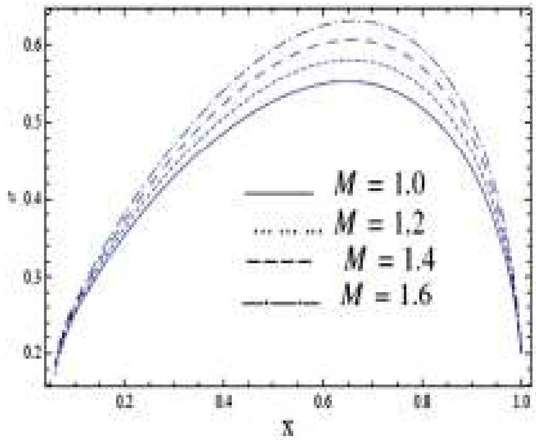

Fig. 5: Stability diagram for a system having the particulars: $\hat{V} = \widetilde{W} = \widetilde{U} = 0.08, m = 0, \gamma = 0$

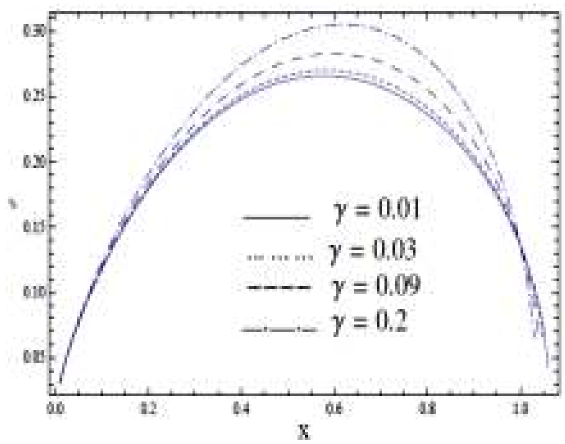

Fig. 6: Stability diagram for a system having the particulars: $\hat{V} = \widetilde{W} = \widetilde{U} = 0.01, m = 0, M = 0$

From Fig. 3, in this domain $0.0201 \leq x \leq 1.0101$, we get stability states. While otherwise domin, we get unstable state.

From Fig. 4, in this domain $0.0501 \leq x \leq 1.0101$, we get stability states. While otherwise domin, we get unstable state.

From Fig. 5, in this domain $0.0701 \leq x \leq 1.0101$, we get stability states. While otherwise domin, we get unstable state.

From Fig. 6, in this domain $(0.0101 \leq x \leq 1.0601), (0.0101 \leq x \leq 1.0501), (0.0101 \leq x \leq 1.0401), (0.0101 \leq x \leq 1.0301)$, we get stability states. While otherwise domin, we get unstable state.

MAGNETOGRAVITODYNAMIC STABILITY

Suppose that the streaming fluid under the effect of the self-gravitating and magnetic forces. In this case, we can write the dimensionless dispersion relationship as follows:

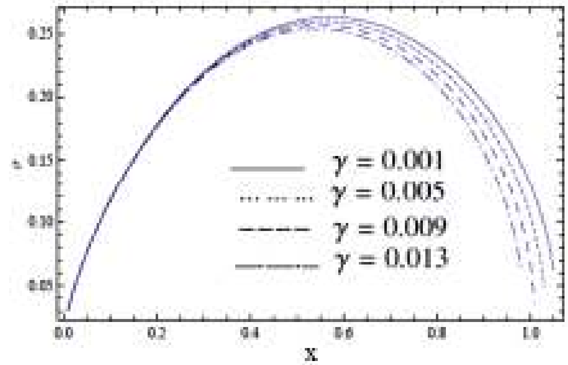

Fig. 7: Stability diagram for a system having the particulars: $\hat{V} = \widetilde{W} = \widetilde{U} = 0.01, m = 0, M = 0$

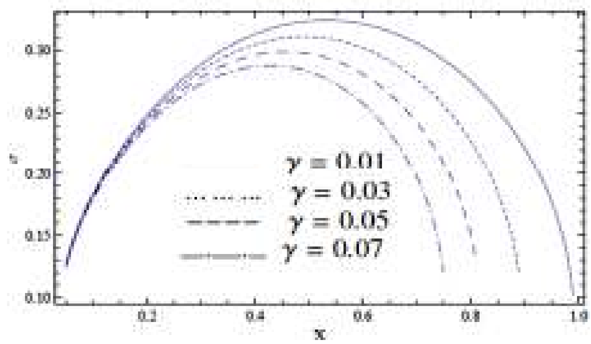

Fig. 8: Stability diagram for a system having the particulars: $\hat{V} = \widetilde{W} = \widetilde{U} = 0.05, m = 0, M = 0$

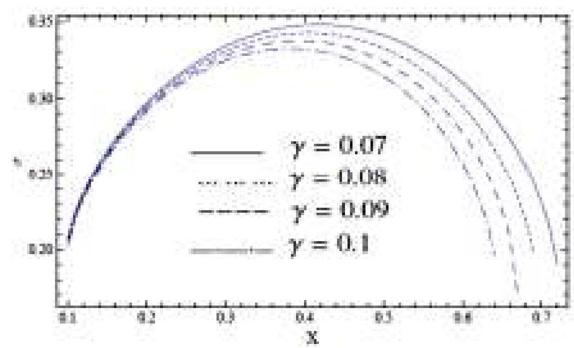

Fig. 9: Stability diagram for a system having the particulars: $\hat{V} = \widetilde{W} = \widetilde{U} = 0.09, m = 0, M = 0$

$$\hat{\sigma} = \widehat{W} + \widehat{U} - \hat{V} x \frac{I'_m(x)}{I_m(x)} + \frac{1}{(I_m(x))^{\frac{1}{2}}} [\hat{V}^2 x^2 \frac{(I'_m(x))^2}{I_m(x)} - I''_m(x)) + x\, I'_m(x)\left(I_m(x)K_m(x) - \frac{1}{2}\right) - \widehat{W}\hat{V} I_m(x) + \gamma(-x^2 I_m(x) + \frac{\alpha^2 x^2 I'_m(x) K_m(x)}{K'_m(x)})]^{1/2} \quad (69)$$

From Fig. 7, in this domain $(0.0101 \leq x \leq 1.0501), (0.0101 \leq x \leq 1.0301), (0.0101 \leq x \leq$

$1.0101), (0.0101 \leq x \leq 0.9801)$, we get stability states. While otherwise domin, we get unstable state.

From Fig. 8, in this domain $(0.0501 \leq x \leq 0.9901), (0.0501 \leq x \leq 0.8901), (0.0501 \leq x \leq 0.8101), (0.0501 \leq x \leq 0.7501)$, we get stability states. While otherwise domin, we get unstable state.

From Fig. 9, in this domain $(0.1 \leq x \leq 0.7201), (0.1 \leq x \leq 0.6901), (0.1 \leq x \leq 0.6701), (0.1 \leq x \leq 0.6401)$, we get stability states. While otherwise domin, we get unstable state.

MAGNETOGRAVITODYNAMIC CAPILLARY STABILITY

If the streaming fluid under the effect of the self-gravitating, magnetic forces and the capillary force. In this case, dispersion relationship identical with the general desperion relation (67):

$$\hat{\sigma} = \widehat{W} + \widehat{U} - \widehat{V}x\frac{I'_m(x)}{I_m(x)} + \frac{1}{\left(I_m(x)\right)^{\frac{1}{2}}}[\widehat{V}^2x^2\frac{\left(I'_m(x)\right)^2}{I_m(x)} - I''_m(x)) + M(1-m^2-x^2)xI'_m(x) + x\,I'_m(x)\left(I_m(x)K_m(x)-\frac{1}{2}\right) - \widehat{W}\widehat{V}I_m(x) + \gamma(-x^2I_m(x) + \frac{\alpha^2x^2I'_m(x)K_m(x)}{K'_m(x)})]^{1/2}$$

From Fig. 10, in this domain $(0.0101 \leq x \leq 0.7301), (0.0101 \leq x \leq 0.7601), (0.0101 \leq x \leq 0.7901), (0.0101 \leq x \leq 0.8201)$, we get stability states. While otherwise domin, we get unstable state.

From Fig. 11, in this domain $(0.0901 \leq x \leq 0.8201), (0.0801 \leq x \leq 0.8501), (0.0801 \leq x \leq 0.8701), (0.0801 \leq x \leq 0.8901)$, we get stability states. While otherwise domin, we get unstable state.

From Fig. 12, in this domain $(0.0901 \leq x \leq 0.8801), (0.0801 \leq x \leq 0.9101), (0.0801 \leq x \leq 0.9301), (0.0701 \leq x \leq 0.9401)$, we get stability states. While otherwise domin, we get unstable state.

From what we had before, we observe that the streaming velocity has a destabilizing influence, which is consistentwith all previous studies. We observethat with the increase of the M values, the system gives an unstable situation and with the continuous increase of the M values and $\gamma = 0$, the system be more unstable and the stability zone decreases (Fig. 1 to 6). Also, we observe that under the effect of the self-gravitating and magnetic forces with the increase of the γ values, the system gives an unstable situation and with the continuous increase of the γ values and $M = 0$, the system be more unstable and the stability zone decreases (Fig. 7 to 9). Finally, we conclude that in the increase of M values with the existance of the γ variable, the system be more stable and with the continuous increase of the M values with theexistence of theγ variable, the system

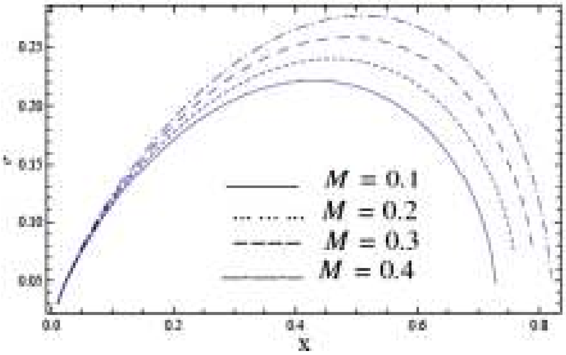

Fig. 10: Stability diagram for a system having the particulars: $\widehat{V} = \widetilde{W} = \widetilde{U} = 0.01, m = 0, \gamma = 0.1$

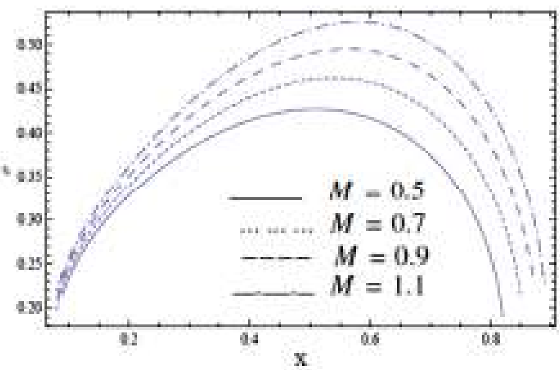

Fig. 11: Stability diagram for a system having the particulars: $\widehat{V} = \widetilde{W} = \widetilde{U} = 0.09, m = 0, \gamma = 0.1$

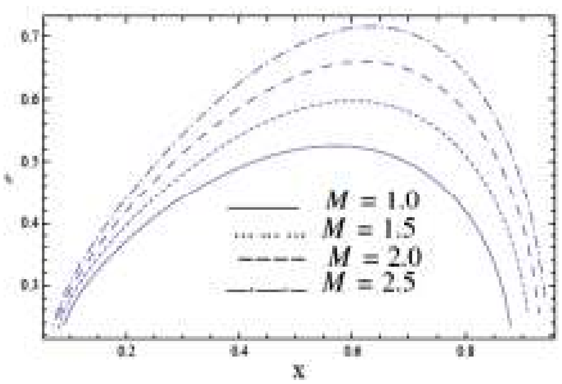

Fig.12: Stability diagram for a system having the particulars: $\widehat{V} = \widetilde{W} = \widetilde{U} = 0.1, m = 0, \gamma = 0.1$

be more unstable and the stability zone increase (Fig. 10 to 12). In addition, we found that the best value for the streaming velocities is less than one, this leads the system to be more stable, this is compatible with the results of Hasan and Abdelkhalek (2013).

CONCLUSION

In this study, we have examined the influence of the existence of self-gravitating, magnetic field and capillary forces in the stability magnetohydrodynamic in a uniform cylinder of an incompressible inviscid fluid. After we obtained the despersion relation, we plotted $(x-\sigma)$ plane and studying the effect of different variables on the process stability.

The following is a general summary of the study in this paper:

- The streaming velocity has a destabilizing influence.
- In the absence of the magnetic field and the streaming fluid under the effect of the capillary force only, we observe that the increase of the M values, the system gives an unstable situation and with the continuous increase of the M values, the system be more unstable and the stability zone decreases.
- If the streaming fluid under the effect of the self-gravitating and magnetic forces, we observe that the increase of the γ values, the system gives an unstable situation and with the continuous increase of the γ values, the system be more unstable and the stability zone decreases.
- If the streaming fluid under the effect of the self-gravitating, magnetic and capillary forces, We conclude that in the increase of M values with the existance of the γ variable,the system be more stable and with the continuous increase of the M values with the existance of the γ variable, the system be more stable and the stability zone increase.

CONFLICT OF INTEREST

The authors declare that they have no competing interest.

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